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$$y^{(4)}(t) + a_3(t)\, y^{(3)}(t) + a_2(t)\, y^{(2)}(t) + a_1(t)\, y^{(1)}(t) + a_0(t)\, y(t) = h(t).$$

Assume $a_i(t) = a_i$, which is constant

and $h(t) = 0$.

then $y_1 = y$,

$y_2 = y' = y_1'$

$y_3 = y'' = y_2'$

$y_4 = y^{(3)} = y_3'$

$$y^{(4)} = y_4' = -a_3 y_4 - a_2 y_3 - a_1 y_2 - a_0 y_1$$

$$\left(= -a_3 y^{(3)} - \cdots - a_0 y \right)$$

Construct the matrix.

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \qquad y_1' = y_2$$

$$\boxed{Y' = AY.}$$

수학적 귀납법으로 n차에 대해 증명가능.

$$Ch_A(\lambda) = \det(A - \lambda I) = \cancel{(-1)^4 (y_4' + a_3 y_4 + \cdots + a_0 y_1})$$

$$\underbrace{= \cancel{y^{(4)} + a_3 y^{(3)} + \cdots + a_0 y}}_{}\ //$$

$$(-1)^4 \left(\lambda^4 + a_3 \lambda^3 + \cdots + a_0 \right)$$

You can get eigen values, and eigen vectors.

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Then, how to solve $Y' = AY$?

(Recall : $y' = ay \Rightarrow y = e^{at} \cdot y(0)$)

$$\underbrace{Y = e^{tA} \cdot Y_0}_{\text{"}} \quad (Y_0 = Y(0))$$

What is e^{tA} ?

→ define matrix exponential as

$$e^A = \exp(A) = I + A + \frac{1}{2!}A^2 + \cdots = \sum_{k=0}^{\infty} \frac{A^k}{k!}$$

If A is diagonalizable, using λ and x you obtained from $ch_A(\lambda)$, $Q^{-1}AQ = D$ (diagonal matrix).

$$e^{tA} = e^{t(QDQ^{-1})} = Q e^{tD} Q^{-1} \quad \left(\because e^{BAB^{-1}} = Be^A B^{-1}\right)$$

taylor expansion 해서 보면 trivial.

$$= Q \begin{bmatrix} e^{t\lambda_1} & & & \\ & e^{t\lambda_2} & & \\ & & \ddots & \\ & & & e^{t\lambda_n} \end{bmatrix} Q^{-1}, \quad Q = \begin{bmatrix} \psi_1 & \cdots & \psi_n \end{bmatrix}$$

where ψ_i is an eigen vector corresponding to λ_i.

∴ You can calculate

$$Y = e^{tA} Y_0 = Q \begin{bmatrix} e^{t\lambda_1} & \\ & \ddots \end{bmatrix} Q^{-1} \begin{bmatrix} y_1(0) \\ y_2(0) \\ \vdots \end{bmatrix}$$