CHBE320 LECTURE XI
CONTROLLER DESIGN AND PID CONTOLLER TUNING

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Road Map of the Lecture XI

- **Controller Design and PID Tuning**
  - Performance criteria
  - Trial and error method
  - Continuous cycling method
  - Relay feedback method
  - Tuning relationships
  - Direct Synthesis
  - Internal Model Control (IMC)
  - Effects of modeling error

CONTROLLER DESIGN

- **Performance criteria for closed-loop systems**
  - Stable
  - Minimal effect of disturbance
  - Rapid, smooth response to set point change
  - No offset
  - No excessive control action
  - Robust to plant-model mismatch

$$\min_{K_c,\tau_I,\tau_D} \int_0^\infty (w_1 e^2(\tau) + w_2 \Delta u^2(\tau))d\tau$$

- **Trade-offs in control problems**
  - Set point tracking vs. disturbance rejection
  - Robustness vs. performance

GUIDELINES FOR COMMON CONTROL LOOPS

- **Flow and liquid pressure control**
  - Fast response with no time delay
  - Usually with small high-frequency noise
  - PI controller with intermediate controller gain
    - 0.5< K_c <0.7 and 0.2< τ_I <0.3min (Fruehauf et al. (1994))
- **Liquid level control**
  - Noisy due to splashing and turbulence
  - High gain PI controller for integrating process
    - Increase in K_c may decrease oscillation (special behavior)
  - Conservative setting for averaging control when it is used for damping the fluctuation of the inlet stream (usually P-control)
    - PI control:

      $K_c = 100\%/\Delta h,\quad \tau_I = 4V/(K_c Q_{max})\quad (\Delta h \equiv \min(h_{max} - h_{sp}, h_{sp} - h_{min}))$
    - Error-squared controller with careful tuning
  - If heat transfer is involved, it becomes much more complicated.

- **Gas pressure control**
  - Usually fast and self regulating
  - PI controller with small integral action (large reset time)
  - D mode is not usually needed.
- **Temperature control**
  - Wide variety of the process nature
  - Usually slow response with time delay
  - Use PID controller to speed up the response
- **Composition control**
  - Similar to temperature control usually with larger noise and more time delay
  - Effectiveness of derivative action is limited
  - Temperature and composition controls are the prime candidates for advance control strategies due to its importance and difficulty of control

TRIAL AND ERROR TUNING

- **Step1: With P-only controller**
  - Start with low K_c value and increase it until the response has a sustained oscillation (continuous cycling) for a small set point or load change. (K_{cu})
  - Set $K_c = 0.5K_{cu}$.
- **Step2: Add I mode**
  - Decrease the reset time until sustained oscillation occurs. (τ_{Iu})
  - Set $\tau_I = 3\tau_{Iu}$.
  - If a further improvement is required, proceed to Step 3.
- **Step3: Add D mode**
  - Increase the preact time until sustained oscillation occurs. (τ_{Du})
  - Set $\tau_D = \tau_{Du}/3$.

(The sustained oscillation should not be cause by the controller saturation)

CONTINUOUS CYCLING METHOD

- **Also called as loop tuning or ultimate gain method**
  - Increase controller gain until sustained oscillation
  - Find ultimate gain (K_{CU}) and ultimate period (P_{CU})
- **Ziegler-Nichols controller setting**
  - ¼ decay ratio (too much oscillatory)

Controller	K_C	τ_I	τ_D
P	$0.5K_{CU}$	-	-
PI	$0.45K_{CU}$	$P_{CU}/1.2$	-
PID	$0.6K_{CU}$	$P_{CU}/2$	$P_{CU}/8$

  - Modified Ziegler-Nichols setting

Controller	K_C	τ_I	τ_D
Original	$0.6K_{CU}$	$P_{CU}/2$	$P_{CU}/8$
Some overshoot	$0.33K_{CU}$	$P_{CU}/2$	$P_{CU}/3$
No overshoot	$0.2K_{CU}$	$P_{CU}/2$	$P_{CU}/3$

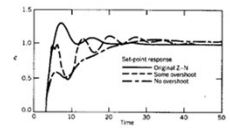
- **Examples**

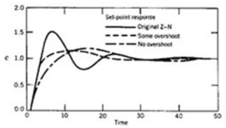
$G_p(s) = \frac{4e^{-3.5s}}{7s+1}$ $K_{CU} = 0.95$, $P_{CU} = 12$

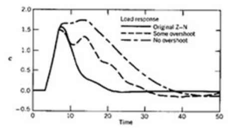
$G_p(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}$ $K_{CU} = 7.88$, $P_{CU} = 11.6$

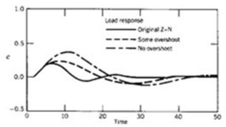
Controller	K_C	τ_I	τ_D
Original	0.57	6.0	*1.5*
Some overshoot	0.31	6.0	*4.0*
No overshoot	0.19	6.0	*4.0*

Controller	K_C	τ_I	τ_D
Original	4.73	5.8	*1.45*
Some overshoot	2.60	5.8	*3.87*
No overshoot	1.58	5.8	*3.87*









- **Advantages of continuous cycling method**
  - No a priori information on process required
  - Applicable to all stable processes

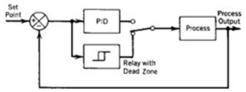
- **Disadvantages of continuous cycling method**
  - Time consuming
  - Loss of product quality and productivity during the tests
  - Continuous cycling may cause the violation of process limitation and safety hazards
  - Not applicable to open-loop unstable process
  - First-order and second-order process without time delay will not oscillate even with very large controller gain

=> Motivates Relay feedback method. (Astrom and Wittenmark)

RELAY FEEDBACK METHOD

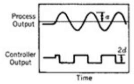
- **Relay feedback controller**
  - Forces the system to oscillate by a relay controller
  - Require a single closed-loop experiment to find the ultimate frequency information
  - No *a priori* information on process is required
  - Switch relay feedback controller for tuning
  - Find P_{CU} and calculate K_{CU}

$$K_{CU} = \frac{4d}{\pi a}$$



  - User specified parameter: d

    Decide d in order not to perturb the system too much.



  - Use Ziegler-Nichols Tuning rules for PID tuning parameters

- **Calculation of model parameters from K_{CU} and P_U**
  - Integrator-plus-time-delay model: $G(s) = \frac{Ke^{-\theta}}{s}$

$$K = \frac{2\pi}{K_{CU} P_U} \quad \theta = P_U/4$$

  - First-order-plus-time-delay model: $G(s) = \frac{Ke^{-\theta s}}{\tau s + 1}$

$$K = \frac{2\pi}{K_{CU} P_U}$$

$$\tau = \frac{P_U}{2\pi} \tan \frac{\pi(P_U - 2\theta)}{P_U} \quad \text{or} \quad \tau = \frac{P_U}{2\pi}\sqrt{(KK_{CU})^2 - 1}$$

    - The θ is decided by visual inspection and K can be calculated using two equations of τ above.

DESIGN RELATIONS FOR PID CONTROLLERS

- **Cohen-Coon controller design relations**
  - Empirical relation for ¼ decay ratio for FOPDT model

Table 12.2 Cohen and Coon Controller Design Relations

Controller	*Settings*	*Cohen–Coon*
P	K_c	$\frac{1}{K}\frac{\tau}{\theta}[1 + \theta/3\tau]$
PI	K_c	$\frac{1}{K}\frac{\tau}{\theta}[0.9 + \theta/12\tau]$
	τ_I	$\frac{\theta[30 + 3(\theta/\tau)]}{9 + 20(\theta/\tau)}$
PID	K_c	$\frac{1}{K}\frac{\tau}{\theta}\left[\frac{16\tau + 3\theta}{12\tau}\right]$
	τ_I	$\frac{\theta[32 + 6(\theta/\tau)]}{13 + 8(\theta/\tau)}$
	τ_D	$\frac{4\theta}{11 + 2(\theta/\tau)}$

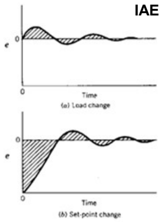
- **Design relations based on integral error criteria**
  - **¼ decay ratio is too oscillatory**
  - **Decay ratio concerns only two peak points of the response**
  - **IAE: Integral of the Absolute Error**

$$\text{IAE} = \int_0^\infty |e(t)|dt$$

  - **ISE: Integral of the Square Error**

$$\text{ISE} = \int_0^\infty [e(t)]^2 dt$$

    - **Large error contributes more**
    - **Small error contributes less**
    - **Large penalty for large overshoot**
    - **Small penalty for small persisting oscillation**



  - **ITAE: Integral of the Time-weighted Absolute Error**

$$\text{ITAE} = \int_0^\infty t|e(t)|dt$$

    - **Large penalty for persisting oscillation**
    - **Small penalty for initial transient response**

- **Controller design relation based on ITAE for FOPDT model**

Table 12.3 Controller Design Relations Based on the ITAE Performance Index and a First-Order plus Time-Delay Model [6–8][a]

Type of Input	*Type of Controller*	*Mode*	*A*	*B*
Load	PI	P	0.859	−0.977
		I	0.674	−0.680
Load	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a]Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.
[b]For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$. [8]

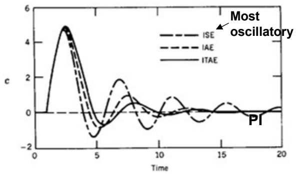
- **Similar design relations based on IAE and ISE for other types of models can be found in literatures.**

- **Example1**

$$G(s) = \frac{10e^{-s}}{2s+1}$$

$KK_c = (0.859)(1/2)^{-0.977} = 1.69$
$\Rightarrow K_c = 0.169$

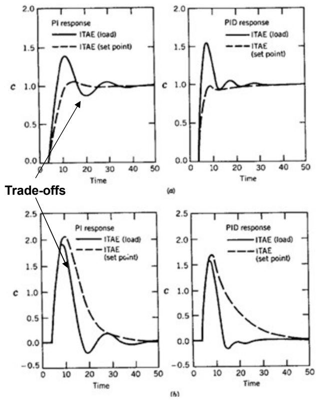
$\tau/\tau_I = (0.674)(1/2)^{-0.680} = 1.08$
$\Rightarrow \tau_I = 1.85$



Method	K_c	τ_I
IAE	0.195	2.02
ISE	0.245	2.44
ITAE	0.169	1.85

Example2

$$G(s) = \frac{4e^{-3.5s}}{7s+1}$$



- **Design relations based on process reaction curve**
  - **For the processes who have sigmoidal shape step responses (Not for underdamped processes)**
  - **Fit the curve with FOPDT model**

$$G(s) = \frac{Ke^{-\theta s}}{(\tau s+1)} \qquad S = K\Delta u/\tau \qquad S^* = S/\Delta u = K/\tau$$

Table 13.3 Ziegler–Nichols Tuning Relations (Process Reaction Curve Method)

Controller Type	K_c	τ_I	τ_D
P	$\frac{1}{\theta S^*}$	—	—
PI	$\frac{0.9}{\theta S^*}$	3.33θ	—
PID	$\frac{1.2}{\theta S^*}$	2θ	0.5θ

  - **Very simple**
  - **Inherits all the problems of FOPDT model fitting**

MISCELLANEOUS TUNING RELATIONS

- **Hägglund and Åström (2002)**

Table 12.4 PI Controller Settings: Hägglund and Åström (2002)

$G(s)$	K_c	τ_I
$\frac{Ke^{-\theta s}}{s}$	$\frac{0.35}{K\theta}$	7θ
$\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{0.14}{K}+\frac{0.28\tau}{\theta K}$	$0.33\theta+\frac{6.8\theta\tau}{10\theta+\tau}$

- **Skogestad (2003)**

Table 12.5 Controller Settings for $G(s)=Ke^{-\theta s}/(\tau_1 s+1)(\tau_2 s+1)$: Skogestad (2003)

Conditions	K_c	τ_I	τ_D
$\tau_1 \le 8\theta$	$\frac{0.5(\tau_1+\tau_2)}{K\theta}$	$\tau_1+\tau_2$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$
$\tau_1 \ge 8\theta$	$\frac{0.5\tau_1}{K\theta}\left(\frac{8\theta+\tau_2}{8\theta}\right)$	$8\theta+\tau_2$	$\frac{8\theta\tau_2}{8\theta+\tau_2}$

- **Ziegler-Nichols (1942) and Cohen-Coon (1953) are not recommended since their relations are base on 1/4-decay ratio.**

CONTROLLERS WITH TWO DEGREES OF FREEDOM

- **Trade-off between set-point tracking and disturbance rejection**
- **Tuning for disturbance rejection is more aggressive.**
- **In general, disturbance rejection is more important. Thus, tune the controller for satisfactory disturbance rejection.**
- **Controllers with two degrees of freedom (Goodwin et al., 2001)**
  – **Strategies to adjust set-point tracking and disturbance rejection independently**

1. **Gradual change in set point (ramp or filtered)**

$$\frac{Y_{sp}^*}{Y_{sp}}=\frac{1}{\tau_f s+1} \quad \text{(filtered as first order)}$$

2. **Modification of PID control law**

$$p(t)=\bar{p}+K_c(\beta y_{sp}-y_m)+K_c\left(\frac{1}{\tau_I}\int_0^t e(t^*)dt^*-\tau_D\frac{dy_m}{dt}\right) \quad (0<\beta<1)$$

- **As b increase, the set-point response becomes faster but more overshoot.**

DIRECT SYNTHESIS METHOD

- **Analysis: Given $G_c(s)$, what is $y(t)$?**
- **Design: Given $y_d(t)$, what should $G_c(s)$ be?**
- **Derivation**

Let $G_{OL}=K_m G_c G_v G_p \triangleq G_c G$

$$\frac{Y(s)}{R(s)}=\frac{G_{OL}}{1+G_{OL}}=\frac{G_c G}{1+G_c G} \Rightarrow G_c=\frac{1}{G}\left(\frac{Y/R}{1-Y/R}\right)$$

Specify $(Y/R)_d \Rightarrow G_c=\frac{1}{G}\left(\frac{(Y/R)_d}{1-(Y/R)_d}\right)$

– **If $(Y/R)_d = 1$, then it implies perfect control. (infinite gain)**
– **The resulting controller may not be physically realizable**
– **Or, not in PID form and too complicated.**
– **Design with finite settling time:** $(Y/R)_d=\frac{1}{\tau_c s+1}$

- **Examples**

1. Perfect control (K_c becomes infinite)

$$G(s)=\frac{K}{(\tau_1 s+1)(\tau_2 s+1)} \text{ and } (Y/R)_d=1$$

$$G_c(s)=\frac{1}{G(s)}\left(\frac{1}{1-1}\right)=\frac{\infty}{G(s)} \text{ (infinite gain, unrealizable)}$$

2. Finite settling time for 1st-order process

$$G(s)=\frac{K}{(\tau s+1)} \text{ and } (Y/R)_d=\frac{1}{\tau_c s+1}$$

$$G_c(s)=\frac{1}{G(s)}\left(\frac{1/(\tau_c s+1)}{1-1/(\tau_c s+1)}\right)=\frac{\tau s+1}{K\tau_c s}=\frac{\tau}{\tau_c K}\left(1+\frac{1}{\tau s}\right) \text{ (PI)}$$

3. Finite settling time for 2nd-order process

$$G(s)=\frac{K}{(\tau_1 s+1)(\tau_2 s+1)} \text{ and } (Y/R)_d=\frac{1}{\tau_c s+1}$$

$$G_c(s)=\frac{(\tau_1+\tau_2)}{\tau_c K}\left(1+\frac{1}{(\tau_1+\tau_2)s}+\frac{\tau_1\tau_2}{(\tau_1+\tau_2)}s\right) \text{ (PID)}$$

- **Process with time delay**
  - **If there is a time delay, any physically realizable controller cannot overcome the time delay. (Need time lead)**
  - **Given circumstance, a reasonable choice will be**

$$(Y/R)_d = \frac{e^{-\theta_c s}}{\tau_c s + 1}$$

  - **Examples**

1. $G(s) = \frac{Ke^{-\theta s}}{(\tau s + 1)}$ and $(Y/R)_d = \frac{e^{-\theta}}{\tau_c s + 1}$ $(\theta_c = \theta)$

$$G_c(s) = \frac{1}{G(s)}\left(\frac{e^{-\theta}/(\tau_c s + 1)}{1 - e^{-\theta s}/(\tau_c s + 1)}\right) = \frac{\tau s + 1}{K}\boxed{\frac{1}{\tau_c s + 1 - e^{-\theta s}}} \text{ (not a PID)}$$

Physically unrealizable

2. With 1st-order Taylor series approx. ($e^{-\theta} \approx 1 - \theta s$ **)**

$$G_c(s) = \frac{\tau s + 1}{K}\frac{1}{(\tau_c + \theta)s} = \frac{\tau}{K(\tau_c + \theta)}\left(1 + \frac{1}{\tau s}\right) \text{ (PI)}$$

3. $G(s) = \frac{Ke^{-\theta}}{(\tau_1 s + 1)(\tau_2 s + 1)}$ and $(Y/R)_d = \frac{e^{-\theta}}{\tau_c s + 1}$ $(\theta_c = \theta)$

$$G_c(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{K}\frac{1}{(\tau_c + \theta)s} = \frac{(\tau_1 + \tau_2)}{K(\tau_c + \theta)}\left(1 + \frac{1}{(\tau_1 + \tau_2)s} + \frac{\tau_1\tau_2}{(\tau_1 + \tau_2)}s\right) \text{ (PID)}$$

- **Observations on Direct Synthesis Method**
  - **Resulting controllers could be quite complex and may not even be physically realizable.**
  - **PID parameters will be decided by a user-specified parameter: The desired closed-loop time constant (** τ_c **)**
  - **The shorter** τ_c **makes the action more aggressive. (larger** K_c **)**
  - **The longer** τ_c **makes the action more conservative. (smaller** K_c **)**
  - **For a limited cases, it results PID form.**
    - **1st-order model without time delay: PI**
    - **FOPDT with 1st-order Taylor series approx.: PI**
    - **2nd-order model without time delay: PID**
    - **SOPDT with 1st-order Taylor series approx.: PID**
    - **Delay modifies the** K_c**.**

$$\frac{\tau}{K\tau_c} \rightarrow \frac{\tau}{K(\tau_c + \theta)} \text{ (1st order)} \qquad \frac{(\tau_1 + \tau_2)}{K\tau_c} \rightarrow \frac{(\tau_1 + \tau_2)}{K(\tau_c + \theta)} \text{ (2nd order)}$$

    - **With time delay, the** K_c **will not become infinite even for the perfect control (** Y/R **=1).**

INTERNAL MODEL CONTROL (IMC)

- **Motivation**
  - **The resulting controller from direct synthesis method may not be physically unrealizable.**
  - **If there is RHP zero in the process, the resulting controller from direct synthesis method will be unstable.**
  - **Unmeasured disturbance and modeling error are not considered in direct synthesis method.**
- **Source of trouble**
  - **From direct synthesis method**

$$G_c = \boxed{\frac{1}{G}\left(\frac{(Y/R)_d}{1 - (Y/R)_d}\right)}$$

Resulting controller may have higher-order numerator than denominator

Direct inversion of process causes many problems

Process is unknown

- **IMC**
  - **Feedback the error between the process output and model output.**
  - **Equivalent conventional controller:** $G_c = \frac{G_c^*}{1 - G_c^*\tilde{G}}$
  - **Using block diagram algebra**

$C = GP + L \quad P = G_c^* E \quad E = R - (C - \tilde{C}) = R - C + \tilde{G}P$

$P = G_c^*(R - C + \tilde{G}P)$

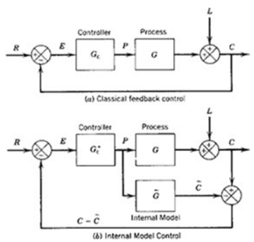
$\Rightarrow \quad P = G_c^*(R - C)/(1 - G_c^*\tilde{G})$

$C = GG_c^*(R - C)/(1 - G_c^*\tilde{G}) + L$

$(1 + GG_c^* - G_c^*\tilde{G})C = GG_c^*R + (1 - G_c^*\tilde{G})L$

$$C = \frac{G_c^* G}{1 + G_c^*(G - \tilde{G})}R + \frac{(1 - G_c^*\tilde{G})}{1 + G_c^*(G - \tilde{G})}L$$

If $\tilde{G} = G$, $C = G_c^* GR + (1 - G_c^* G)L$



- **IMC design strategy**
  - **Factor the process model as**

$$\tilde{G} = \tilde{G}_+ \tilde{G}_-$$

Uninvertibles

- $\tilde{G}_+$ **contains any time delays and RHP zeros and is specified so that the steady-state gain is one**
- $\tilde{G}_-$ **is the rest of** G.
  - **The controller is specified as**

$$G_c^* = \frac{1}{\tilde{G}_-} f$$

- **IMC filter** f **is a low-pass filter with steady-state gain of one**
- **Typical IMC filter:** $f = \dfrac{1}{(\tau_c s + 1)^r}$
- **The** τ_c **is the desired closed-loop time constant and parameter** r **is a positive integer that is selected so that the order of numerator of** G_c^* **is same as the order of denominator or exceeds the order of denominator by one.**

- **Example**
  - **FOPDT model with 1/1 Pade approximation**

$$\tilde{G} = \frac{K(1 - \theta s/2)}{(1 + \theta s/2)(\tau s + 1)}$$

$$\tilde{G}_+ = 1 - \theta s/2 \qquad \tilde{G}_- = \frac{K}{(1 + \theta s/2)(\tau s + 1)}$$

$$G_c^* = \frac{1}{\tilde{G}_-} f = \frac{(1 + \theta s/2)(\tau s + 1)}{K} \frac{1}{(\tau_c s + 1)}$$

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} = \frac{(1 + \theta s/2)(\tau s + 1)}{K(\tau_c + \theta/2)s} \quad \text{(PID)}$$

$$K_c = \frac{1}{K} \frac{(\tau + \theta/2)}{(\tau_c + \theta/2)} \qquad \tau_I = \tau + \theta/2 \qquad \tau_D = \frac{\tau\theta/2}{\tau + \theta/2}$$

IMC based PID controller settings

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ [4][a]

Case	*Model*	K_cK	τ_I	τ_D
A	$\dfrac{K}{\tau s + 1}$	$\dfrac{\tau}{\tau_c}$	τ	—
B	$\dfrac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\dfrac{\tau_1 + \tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\dfrac{\tau_1 \tau_2}{\tau_1 + \tau_2}$
C	$\dfrac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\dfrac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
D	$\dfrac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}, \beta > 0$	$\dfrac{2\zeta\tau}{\tau_c + \beta}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
E	$\dfrac{K}{s}$	$\dfrac{1}{\tau_c}$	—	—
F	$\dfrac{K}{s(\tau s + 1)}$	$\dfrac{1}{\tau_c}$	—	τ

[a]Based on Eq. 12-30 with $r = 1$.

IMC based PID controller settings

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

Case	Model	K_cK	τ_I	τ_D
A	$\dfrac{K}{\tau s + 1}$	$\dfrac{\tau}{\tau_c}$	τ	—
B	$\dfrac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\dfrac{\tau_1 + \tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\dfrac{\tau_1\tau_2}{\tau_1 + \tau_2}$
C	$\dfrac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\dfrac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
D	$\dfrac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}, \beta > 0$	$\dfrac{2\zeta\tau}{\tau_c + \beta}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
E	$\dfrac{K}{s}$	$\dfrac{2}{\tau_c}$	$2\tau_c$	—
F	$\dfrac{K}{s(\tau s + 1)}$	$\dfrac{2\tau_c + \tau}{\tau_c^2}$	$2\tau_c + \tau$	$\dfrac{2\tau_c\tau}{2\tau_c + \tau}$
G	$\dfrac{Ke^{-\theta s}}{\tau s + 1}$	$\dfrac{\tau}{\tau_c + \theta}$	τ	—
H	$\dfrac{Ke^{-\theta s}}{\tau s + 1}$	$\dfrac{\tau + \frac{\theta}{2}}{\tau_c + \frac{\theta}{2}}$	$\tau + \dfrac{\theta}{2}$	$\dfrac{\tau\theta}{2\tau + \theta}$
I	$\dfrac{K(\tau_3 s + 1)e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\dfrac{\tau_1 + \tau_2 - \tau_3}{\tau_c + \theta}$	$\tau_1 + \tau_2 - \tau_3$	$\dfrac{\tau_1\tau_2 - (\tau_1 + \tau_2 - \tau_3)\tau_3}{\tau_1 + \tau_2 - \tau_3}$
J	$\dfrac{K(\tau_3 s + 1)e^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\dfrac{2\zeta\tau - \tau_3}{\tau_c + \theta}$	$2\zeta\tau - \tau_3$	$\dfrac{\tau^2 - (2\zeta\tau - \tau_3)\tau_3}{2\zeta\tau - \tau_3}$
K	$\dfrac{K(-\tau_3 s + 1)e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\dfrac{\tau_1 + \tau_2 + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}{\tau_c + \tau_3 + \theta}$	$\tau_1 + \tau_2 + \dfrac{\tau_3\theta}{\tau_c + \tau_3 + \theta}$	$\dfrac{\tau_3\theta}{\tau_c + \tau_3 + \theta} + \dfrac{\tau_1\tau_2}{\tau_1 + \tau_2 + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}$
L	$\dfrac{K(-\tau_3 s + 1)e^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\dfrac{2\zeta\tau + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}{\tau_c + \tau_3 + \theta}$	$2\zeta\tau + \dfrac{\tau_3\theta}{\tau_c + \tau_3 + \theta}$	$\dfrac{\tau_3\theta}{\tau_c + \tau_3 + \theta} + \dfrac{\tau^2}{2\zeta\tau + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}$
M	$\dfrac{Ke^{-\theta s}}{s}$	$\dfrac{2\tau_c + \theta}{(\tau_c + \theta)^2}$	$2\tau_c + \theta$	—
N	$\dfrac{Ke^{-\theta s}}{s}$	$\dfrac{2\tau_c + \theta}{\left(\tau_c + \frac{\theta}{2}\right)^2}$	$2\tau_c + \theta$	$\dfrac{\tau_c\theta + \frac{\theta^2}{4}}{2\tau_c + \theta}$
O	$\dfrac{Ke^{-\theta s}}{s(\tau s + 1)}$	$\dfrac{2\tau_c + \tau + \theta}{(\tau_c + \theta)^2}$	$2\tau_c + \tau + \theta$	$\dfrac{(2\tau_c + \theta)\tau}{2\tau_c + \tau + \theta}$

- **Modification of IMC and DS methods**
  - **For lag dominant models (θ/τ<<1), IMC and DS methods provide satisfactory set-point response, but very slow disturbance responses because the value τ_I is very large.**
  - **Approximate the FOPDT with IPDT model and use IMC tuning relation for IPDT model**

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \Rightarrow G(s) = \frac{K^* e^{-\theta}}{s} \quad \text{where } K^* \triangleq K/\tau$$

  - **Limit the value of τ_I**

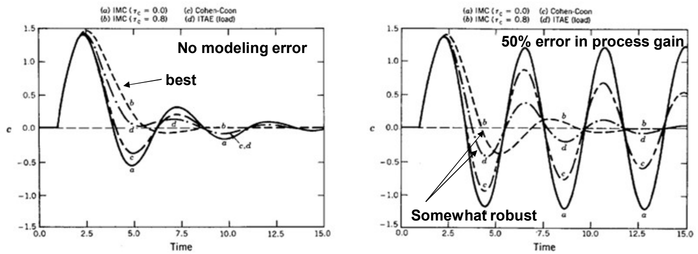
$$\tau_I = \min\{\tau_I, 4(\tau_c + \theta)\}$$

  - **Design the controller for disturbance rejection**

COMPARISON OF CONTROLLER DESIGN RELATIONS

- **PI controller settings for different methods**

$$G(s) = \frac{2e^{-s}}{s+1}$$



EFFECT OF MODELING ERROR

- **Actual plant**

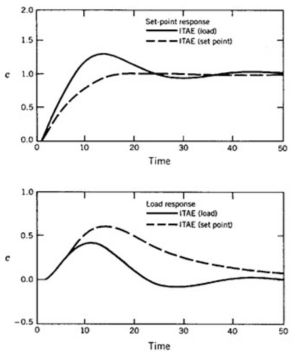
$$G(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}$$

- **Approx. model**

$$\tilde{G}(s) = \frac{2e^{-4.7s}}{12s+1}$$

  - **Satisfactory for this case**
  - **Use with care**

As the estimated time delay gets smaller, the performance degradation will be pronounced.



- **All kinds of tuning method should be used for initial setting and fine tuning should be done!!**

GENERAL CONCLUSION FOR PID TUNING

- **The controller gain should be inversely proportional to the products of the other gains in the feedback loop.**
- **The controller gain should decrease as the ratio of time delay to dominant time constant increases.**
- **The larger the ratio of time delay to dominant time constant is, the harder the system is to control.**
- **The reset time and the derivative time should increase as the ratio of time delay to dominant time constant increases.**
- **The ratio between derivative time and reset time is typically between 0.1 to 0.3.**
- **The ¼ decay ratio is too oscillatory for process control. If less oscillatory response is desired, the controller gain should decrease and reset time should increase.**
- **Among IAE, ISE and ITAE, ITAE is the most conservative and ISE is the least conservative setting.**

TROUBLESHOOTING CONTROL LOOPS

- **Causes of performance degradation of controller**
  - Changing process conditions, usually throughput rate
  - Sticking control valve stem
  - Plugged line in a pressure or DP transmitter
  - Fouled heat exchangers, especially reboilers for distillation
  - Cavitating pumps

- **Starting points of trouble shooting**
  - What is the process being controlled?
  - What is the controlled variable?
  - What are the control objectives?
  - Are closed-loop response data available?
  - Is the controller in the M/A mode? Is it reverse or direct acting?
  - If the pressure is cycling, what is the cycling frequency?
  - What control algorithm is used? What are the controller settings?
  - Is the process open-loop stable?
  - What additional documentation is available?

- **Checking points**
  - Components in the control loop (process, sensor, actuator, …)
    - Field instruments vs. instruments in central control room
    - Recent changes to the equipment or instrumentation (cleaning HX, catalyst replacement, transmitter span, …
    - Sensor lines (particles, bubbles)
    - Control valve sticking
    - Controller tuning parameters