# CHBE320 LECTURE VIII
# DYNAMIC BEHAVIORS OF CLOSED-LOOP CONTOL SYSTEMS

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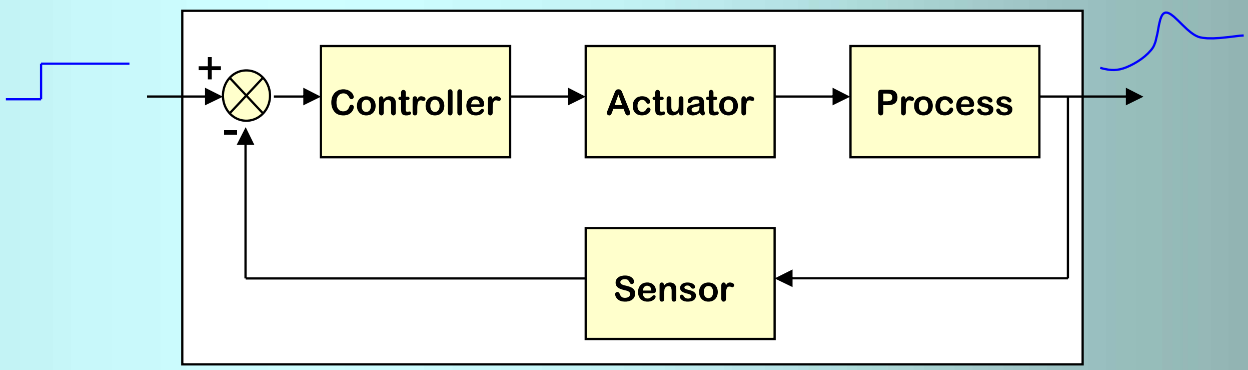
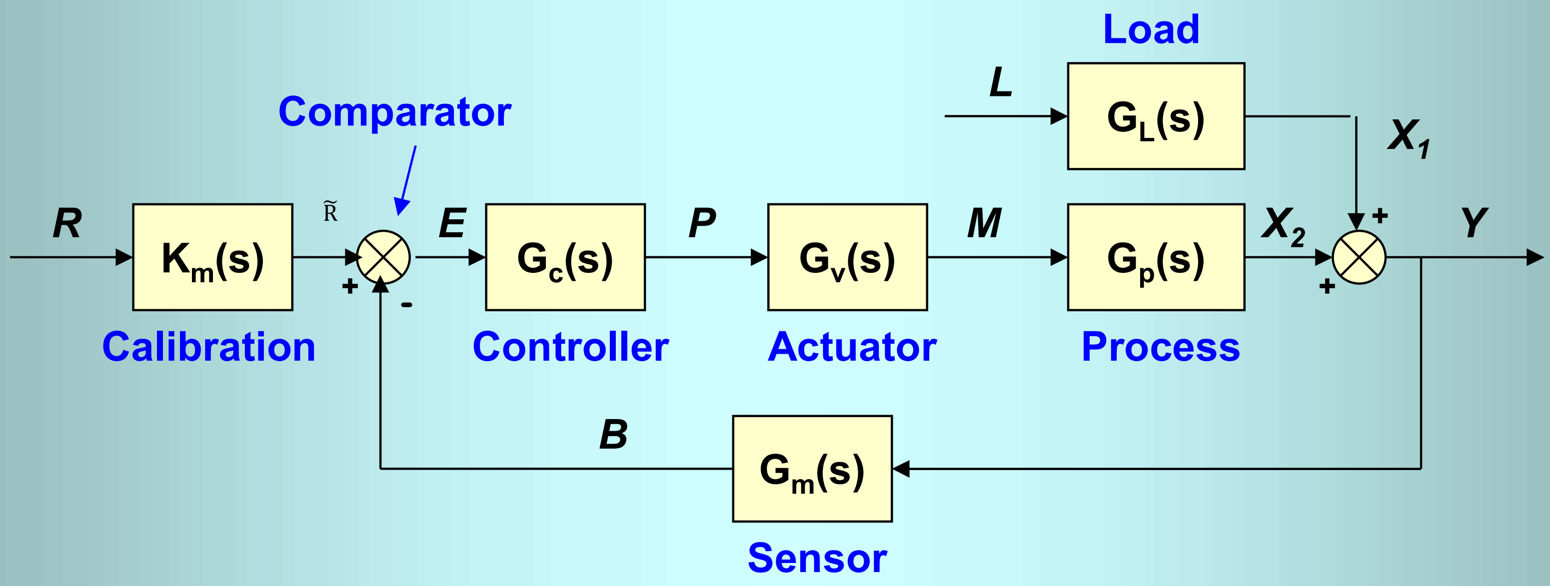
# Road Map of the Lecture VIII

- **Dynamic Behavior of Closed-loop Control System**
  - Closed-loop: controller is connected and working
  - Closed-loop transfer function
    - Response of output for set point change
    - Response of output for load/disturbance change
  - Effects of each block on closed-loop system
    - Effect of controller tuning parameters



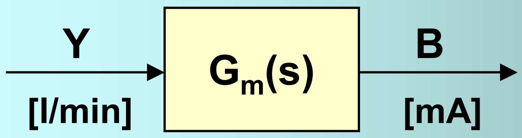
# BLOCK DIAGRAM REPRESENTATION

- Standard block diagram of a feedback control system




  - Process TF: MV (M) effect on CV ($X_2$, part of Y)
  - Load TF: DV (L) effect on CV ($X_1$, part of Y)
  - Sensor TF: CV (Y) is transferred to measurement (B)
  - Actuator TF: Controller output (P) is transferred to MV (M)
  - Controller TF: Controller output (P) is calculated based on error (E)
  - Calibration TF: Gain of sensor TF, used to match the actual var.

- **Individual TF of the standard block diagram**
  - TF of each block between input and output of that block
  - Each gain will have different unit.
    - [Example] Sensor TF
    - Input range: 0-50 l/min
    - Output range: 4-20 mA

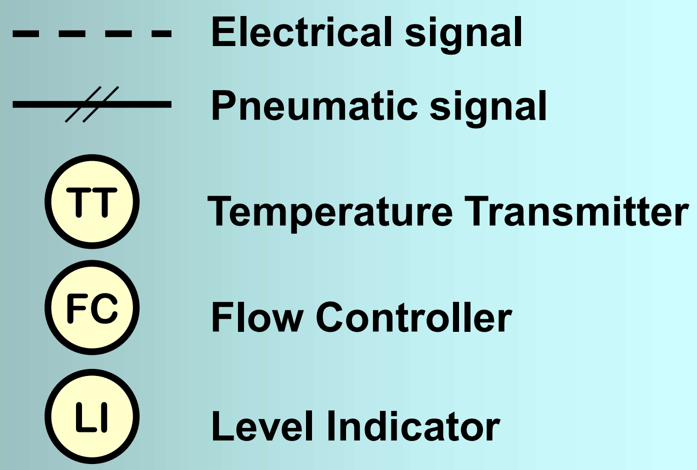


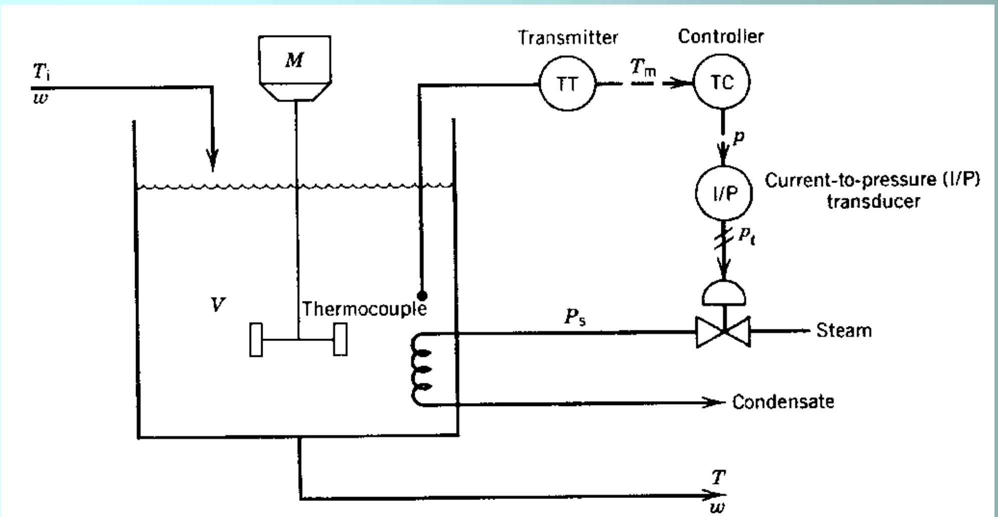

$$\text{Gain, } K_m = \frac{20-4}{50-0} = 0.32 \text{ [mA/(l/min)]}$$

    - Dynamics: usually 1st order with small time constant

$$G_m(s) = \frac{K_m}{\tau_m s + 1}$$

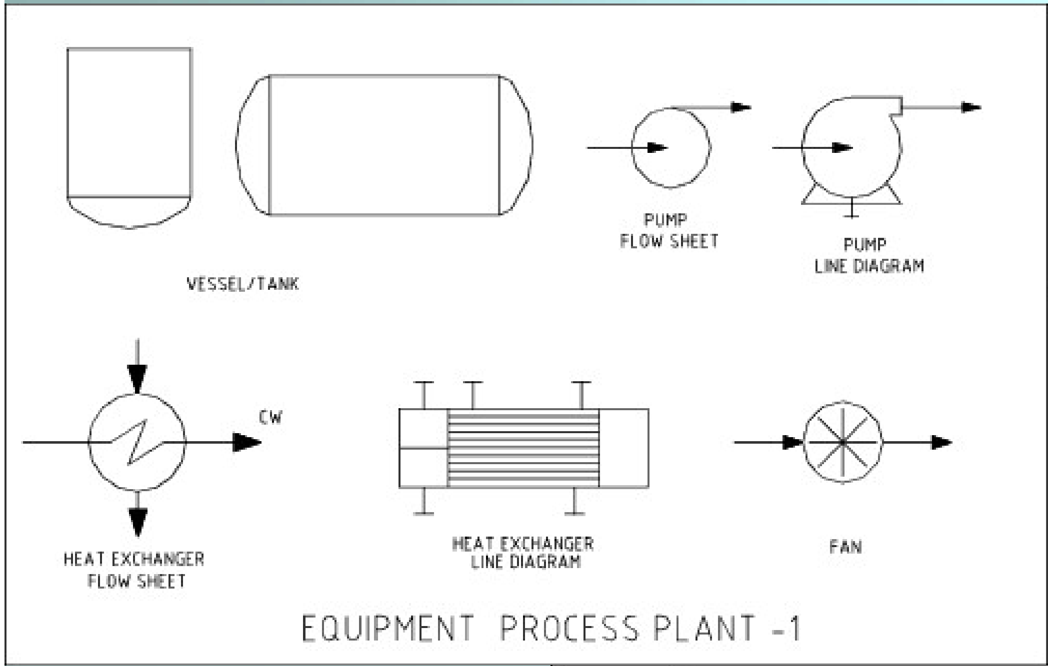
  - **Block diagram** shows the flow of signal and the connections
  - **Schematic diagram** shows the physical components connection

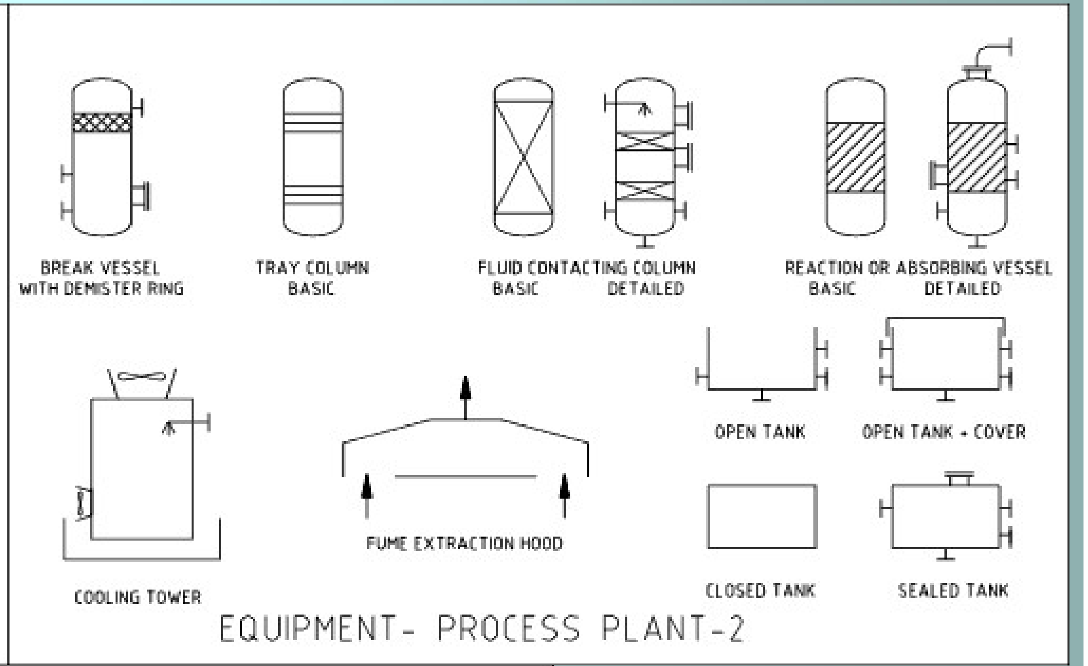


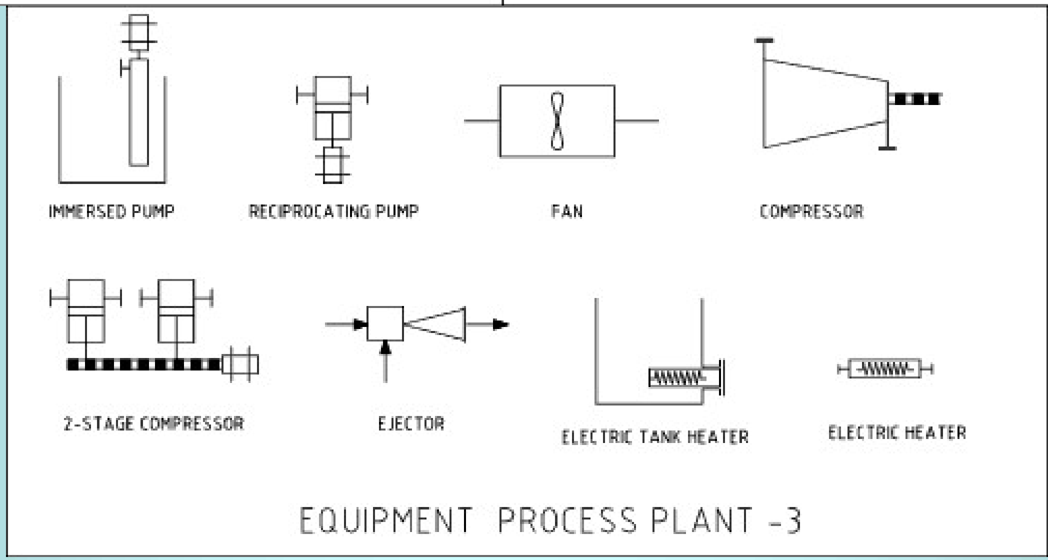



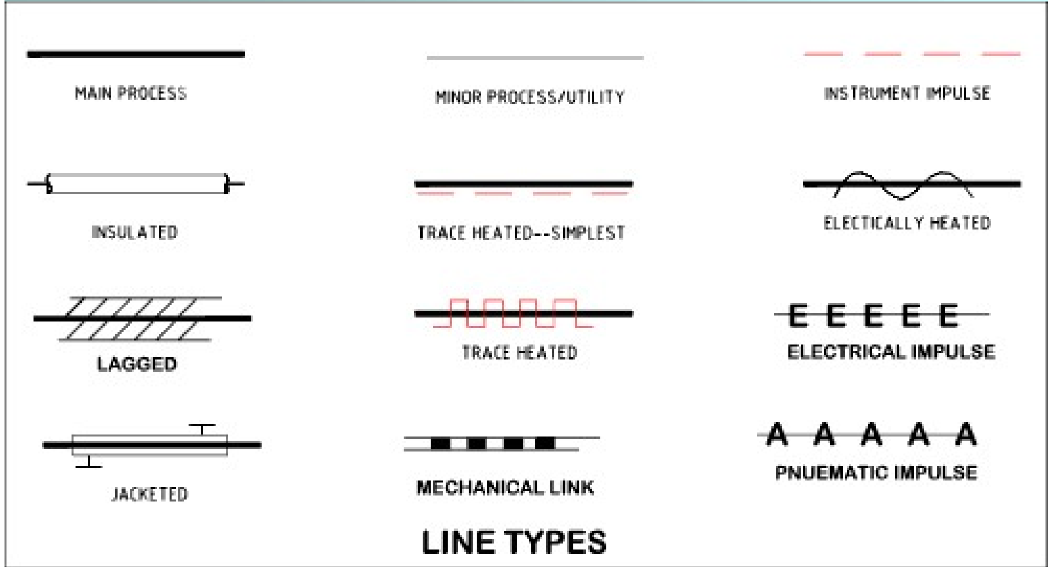
# P&ID

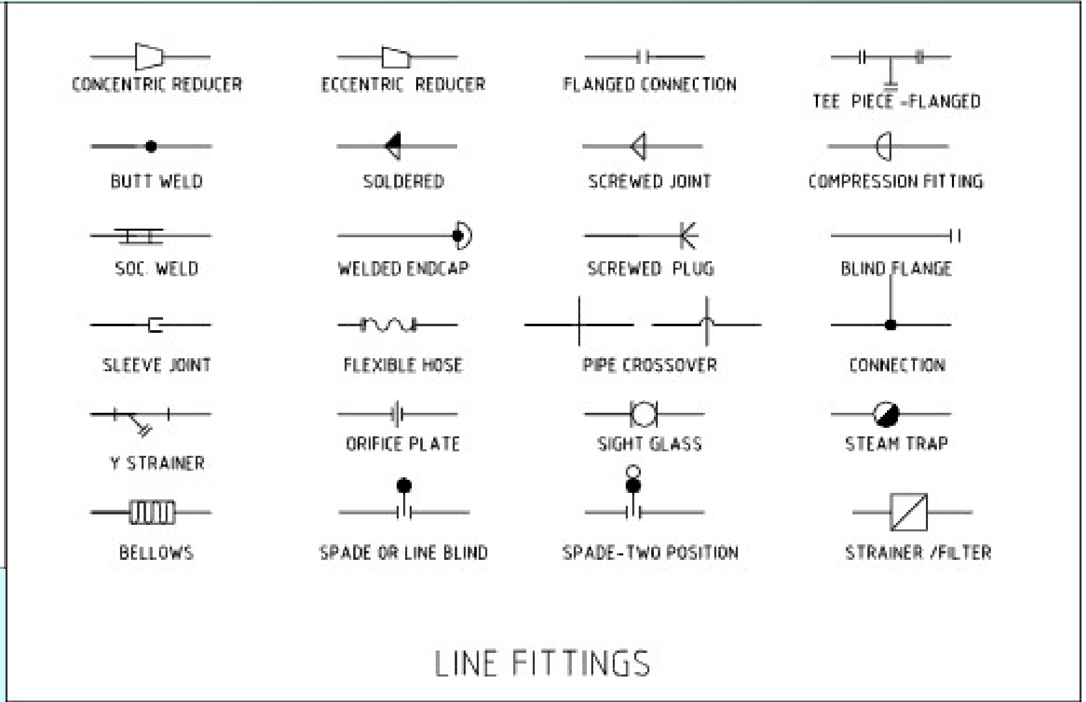
- **Piping and Instrumentation diagram**
  - **A P&ID is a blueprint, or map, of a process.**
  - **Technicians use P&IDs the same way an architect uses blueprints.**
  - **A P&ID shows each of the instruments in a process , their functions, their relationship to other components in the system.**
  - **Most diagrams use a standard format, such as the one developed by ISA (Instrumental Society of America) or SAMA (Scientific Apparatus Makers Association).**

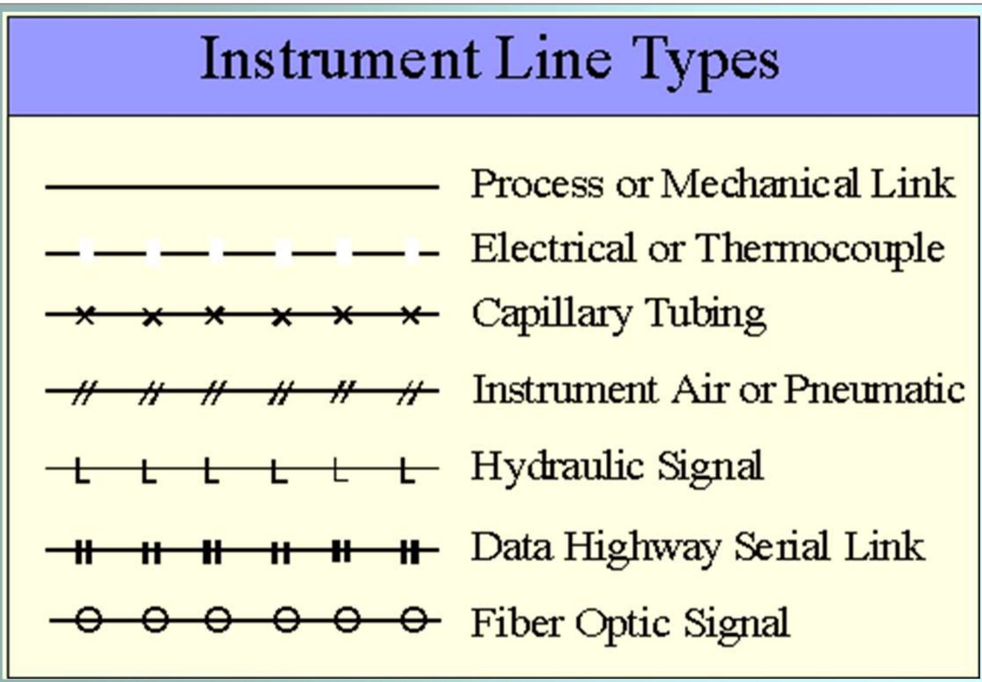

VESSEL/TANK
PUMP
FLOW SHEET
PUMP
LINE DIAGRAM
CW
HEAT EXCHANGER
FLOW SHEET
HEAT EXCHANGER
LINE DIAGRAM
FAN
EQUIPMENT PROCESS PLANT -1

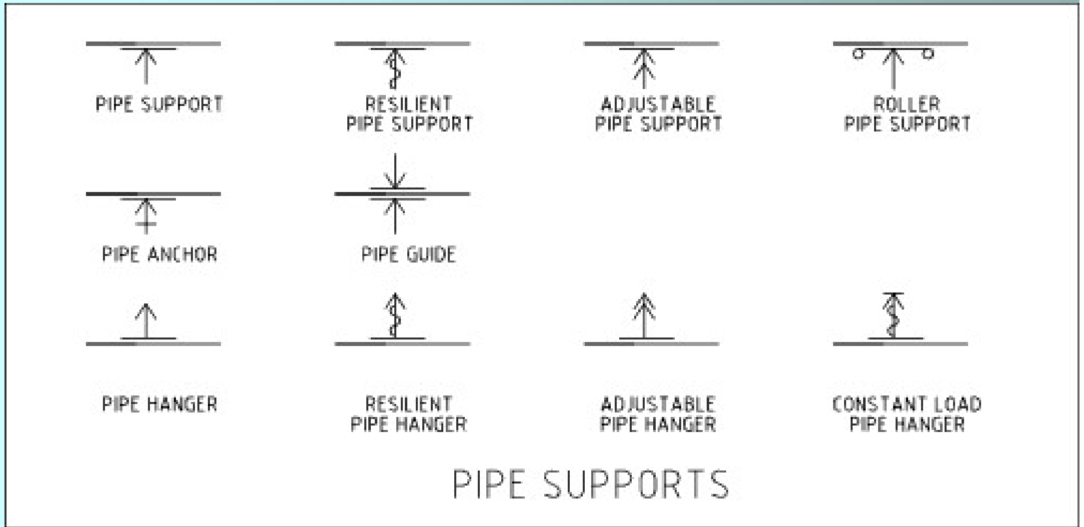

BREAK VESSEL
WITH DEMISTER RING
TRAY COLUMN
BASIC
FLUID CONTACTING COLUMN
BASIC
DETAILED
REACTION OR ABSORBING VESSEL
BASIC
DETAILED
OPEN TANK
OPEN TANK + COVER
COOLING TOWER
FUME EXTRACTION HOOD
CLOSED TANK
SEALED TANK
EQUIPMENT- PROCESS PLANT-2

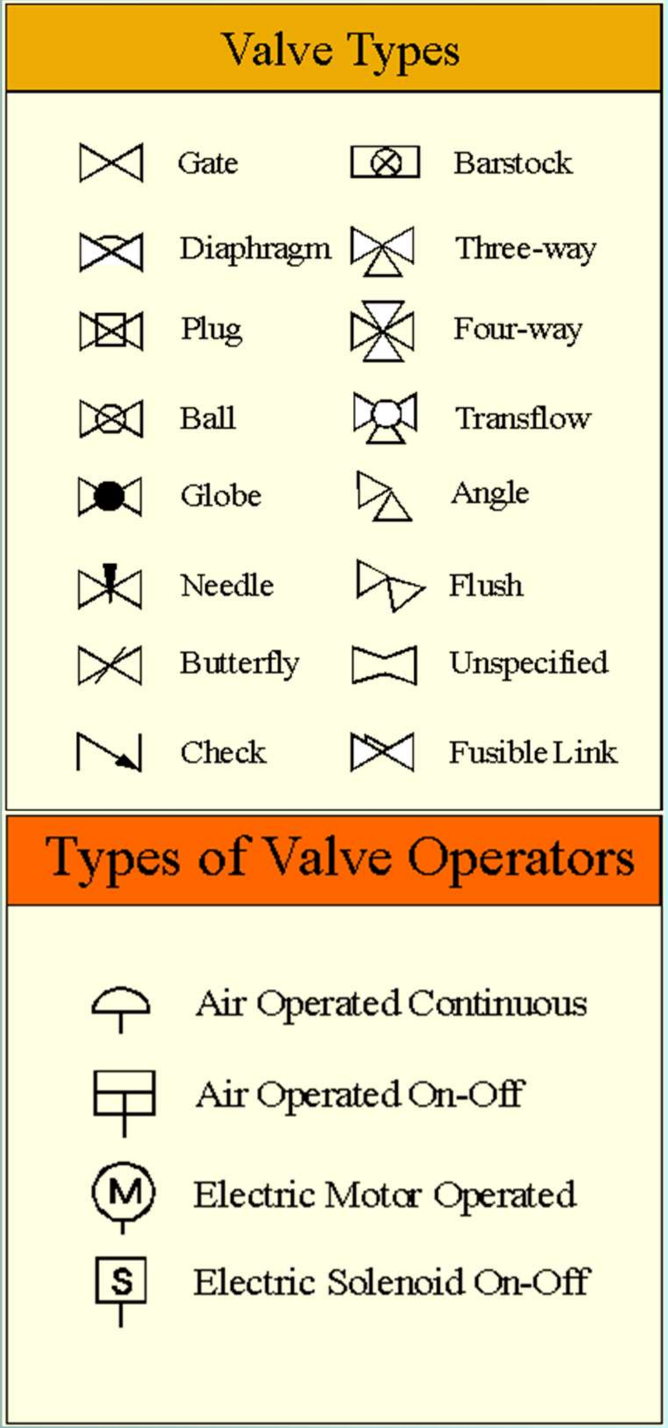

IMMERSED PUMP
RECIPROCATING PUMP
FAN
COMPRESSOR
2-STAGE COMPRESSOR
EJECTOR
ELECTRIC TANK HEATER
ELECTRIC HEATER
EQUIPMENT PROCESS PLANT -3

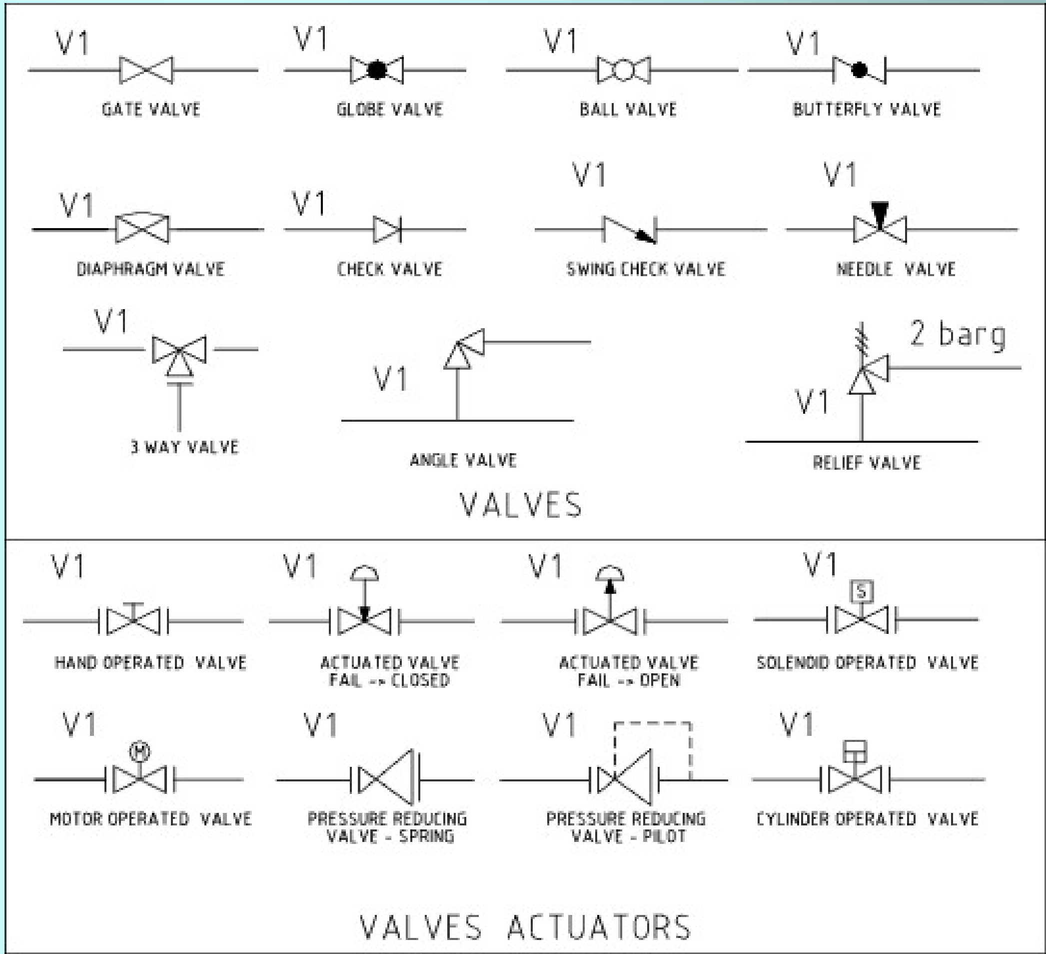

MAIN PROCESS
MINOR PROCESS/UTILITY
INSTRUMENT IMPULSE
INSULATED
TRACE HEATED--SIMPLEST
ELECTICALLY HEATED
LAGGED
TRACE HEATED
ELECTRICAL IMPULSE
JACKETED
MECHANICAL LINK
PNUEMATIC IMPULSE
LINE TYPES

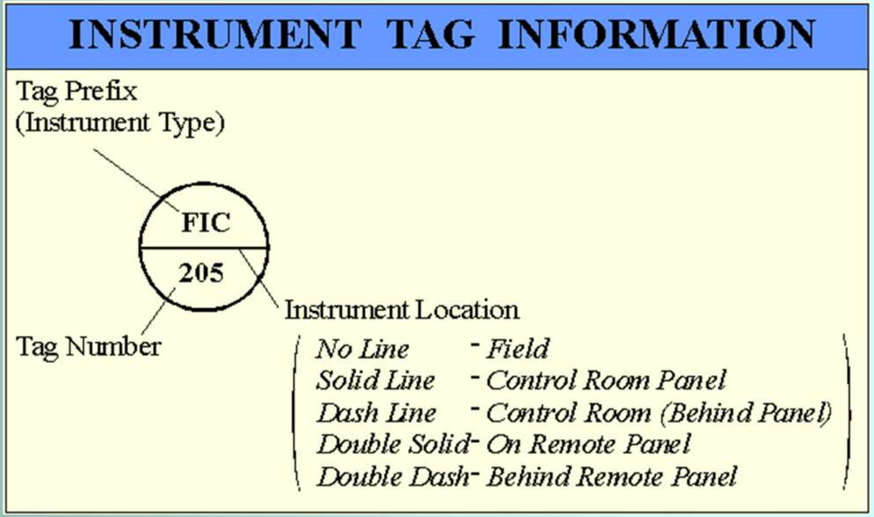

CONCENTRIC REDUCER
ECCENTRIC REDUCER
FLANGED CONNECTION
TEE PIECE -FLANGED
BUTT WELD
SOLDERED
SCREWED JOINT
COMPRESSION FITTING
SOC. WELD
WELDED ENDCAP
SCREWED PLUG
BLIND FLANGE
SLEEVE JOINT
FLEXIBLE HOSE
PIPE CROSSOVER
CONNECTION
Y STRAINER
ORIFICE PLATE
SIGHT GLASS
STEAM TRAP
BELLOWS
SPADE OR LINE BLIND
SPADE-TWO POSITION
STRAINER /FILTER
LINE FITTINGS

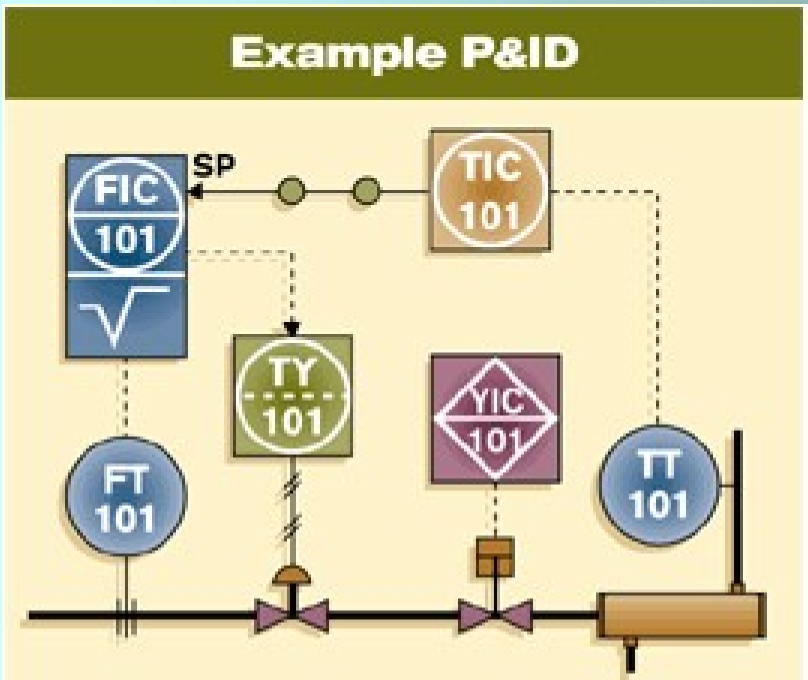

Instrument Line Types
Process or Mechanical Link
Electrical or Thermocouple
Capillary Tubing
Instrument Air or Pneumatic
Hydraulic Signal
Data Highway Serial Link
Fiber Optic Signal

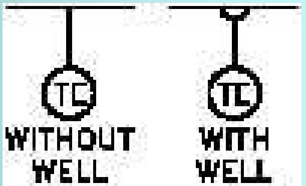

PIPE SUPPORT
RESILIENT PIPE SUPPORT
ADJUSTABLE PIPE SUPPORT
ROLLER PIPE SUPPORT
PIPE ANCHOR
PIPE GUIDE
PIPE HANGER
RESILIENT PIPE HANGER
ADJUSTABLE PIPE HANGER
CONSTANT LOAD PIPE HANGER
PIPE SUPPORTS

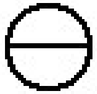
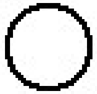
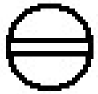
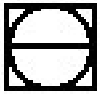
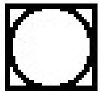
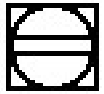
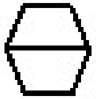
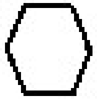
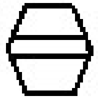
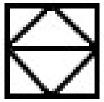
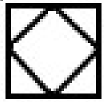
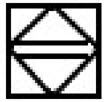

Valve Types
Gate
Barstock
Diaphragm
Three-way
Plug
Four-way
Ball
Transflow
Globe
Angle
Needle
Flush
Butterfly
Unspecified
Check
Fusible Link
Types of Valve Operators
Air Operated Continuous
Air Operated On-Off
M
Electric Motor Operated
S
Electric Solenoid On-Off


V1
GATE VALVE
GLOBE VALVE
BALL VALVE
BUTTERFLY VALVE
DIAPHRAGM VALVE
CHECK VALVE
SWING CHECK VALVE
NEEDLE VALVE
3 WAY VALVE
ANGLE VALVE
2 barg
RELIEF VALVE
VALVES
HAND OPERATED VALVE
ACTUATED VALVE FAIL -> CLOSED
ACTUATED VALVE FAIL -> OPEN
SOLENOID OPERATED VALVE
MOTOR OPERATED VALVE
PRESSURE REDUCING VALVE - SPRING
PRESSURE REDUCING VALVE - PILOT
CYLINDER OPERATED VALVE
VALVES ACTUATORS


INSTRUMENT TAG INFORMATION
Tag Prefix
(Instrument Type)
FIC
205
Tag Number
Instrument Location
No Line - Field
Solid Line - Control Room Panel
Dash Line - Control Room (Behind Panel)
Double Solid - On Remote Panel
Double Dash - Behind Remote Panel


Example P&ID
SP
FIC
101
TIC
101
TY
101
YIC
101
FT
101
TT
101


TC
WITHOUT
WELL
TC
WITH
WELL

| General instrument or function symbols | | | |
|---|---|---|---|
| | **Primary location accessible to operator** | **Field mounted** | **Auxiliary location accessible to operator** |
| **Discrete instruments** | 1  | 2  | 3  |
| **Shared display, shared control** | 4  | 5  | 6  |
| **Computer function** | 7  | 8  | 9  |
| **Programmable logic control** | 10  | 11  | 12  |

1. Symbol size may vary according to the user's needs and the type of document.
2. Abbreviations of the user's choice may be used when necessary to specify location.
3. Inaccessible (behind the panel) devices may be depicted using the same symbol but with a dashed horizontal bar.

Source: Control Engineering with data from ISA S5.1 standard

**Shared display: usually used to indicate video display in DCS**
**Auxiliary location: panel mounted—normally having an analog faceplate**

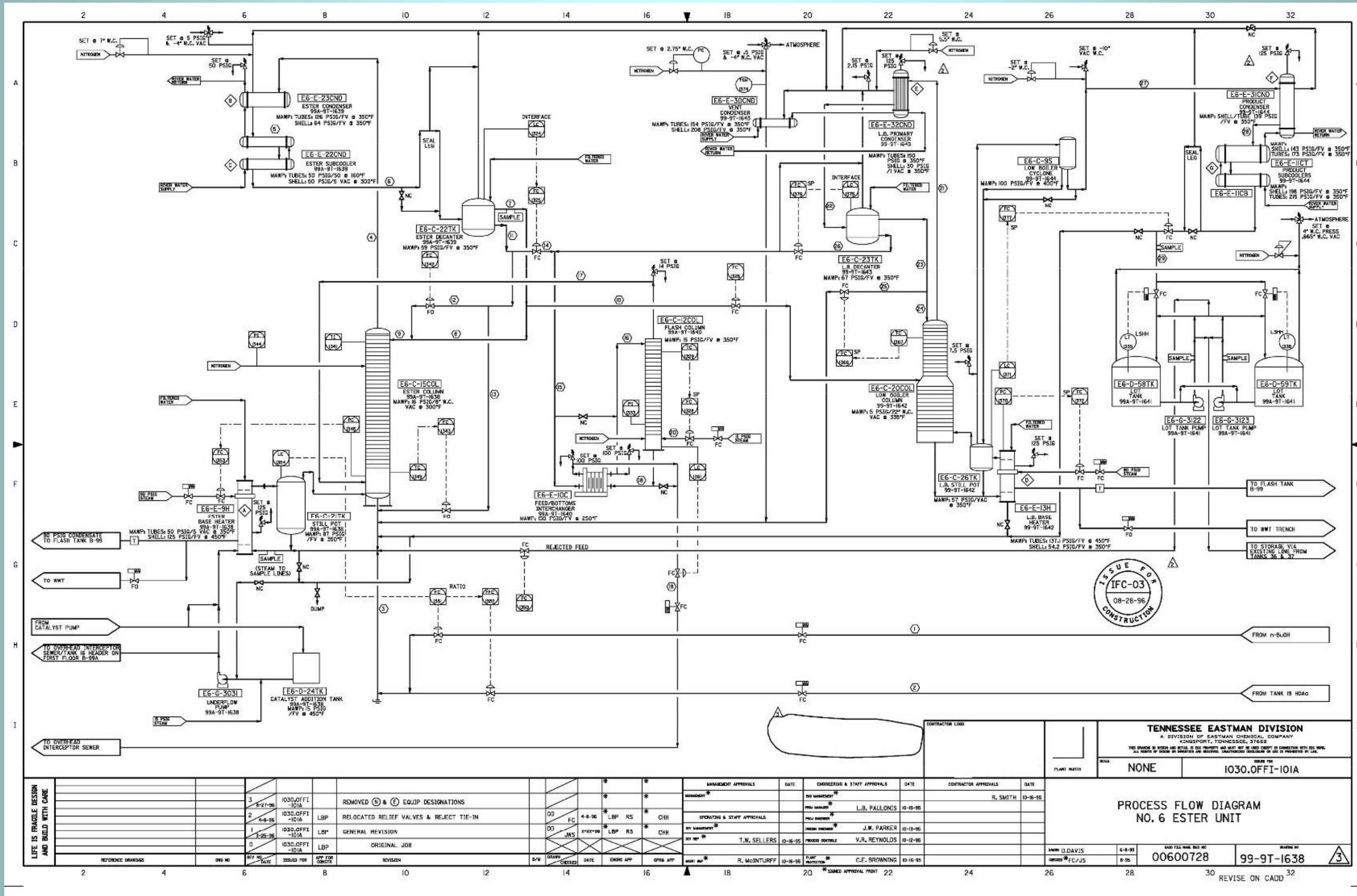
## Identification letters

| | First letter | | Succeeding letters | | |
|---|---|---|---|---|---|
| | **Measured or initiating var.** | **Modifier** | **Readout or passive func.** | **Output function** | **Modifier** |
| **A** | Analysis | | Alarm | | |
| **B** | Burner, combustion | | User's choice | User's choice | User's choice |
| **C** | User's choice | | | Control | |
| **D** | User's choice | Differential | | | |
| **E** | Voltage | | Sensor (primary element) | | |
| **F** | Flow rate | Ration (fraction) | | | |
| **G** | User's choice | | Glass, viewing device | | |
| **H** | Hand | | | | High |
| **I** | Current (electrical) | | Indication | | |
| **J** | Power | Scan | | | |
| **K** | Time, time schedule | Time rate of change | | Control station | |
| **L** | Level | | Light | | Low |
| **M** | User's choice | Momentary | | | Middle, interm. |
| **N** | User's choice | | User's choice | User's choice | User's choice |
| **O** | User's choice | | Orifice, restriction | | |
| **P** | Pressure, vacuum | | Point (test connection) | | |
| **Q** | Quantity | Integrate, totalizer | | | |
| **R** | Radiation | | Record | | |
| **S** | Speed, frequency | Safety | | Switch | |
| **T** | Temperature | | | Transmit | |
| **U** | Multivariable | | Multifunction | Multifunction | Multifunction |
| **V** | Vibration, mechanical analysis | | | Valve, damper, louver | |
| **W** | Weight, force | | Well | | |
| **X** | Unclassified | X axis | Unclassified | Unclassified | Unclassified |
| **Y** | Event, state, or presence | Y axis | | Relay,compute,convert | |
| **Z** | Position, dimension | Z axis | | Driver, actuator | |

Source: Control Engineering with data from ISA S5.1 standard

- **Instrument description examples**
  - **FIC-101: Flow Indicator and Controller, 0 to 50 $m^3$/Hr, (normal reading 30 T/Hr). This instrument controls the flow of cold feedstock entering the tube side of the heat exchanger by positioning a valve on the cold feedstock flow path.**
  - **FR-103: Flow Recorder, 0 to 10 Ton/Hr, (2.14 T/Hr). This instrument records the steam flow rate.**
  - **HS-101: Hand Switch, ON/OFF (ON). This switch turns on/off cold feedstock pump P-101. When the switch is in the ON condition, the pump is running. When the switch is in the OFF condition, the pump is not running.**
  - **HV-102: Hand Valve, OPEN/CLOSED, (OPEN). This switch opens/closes the steam block valve through which steam is routed from the header to the shell side of the heat exchanger. When the switch is in the OPEN condition the block valve is open. When the switch is in the CLOSED condition, the block valve is closed.**
  - **PAL-103: Pressure Alarm Low, (Normal). This alarm fires should the steam header pressure be less than 6 kg/$cm^2$.**

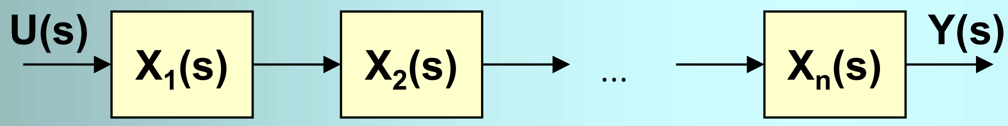
- **PI-100**: **Pressure Indicator, 0 to 15 kg/cm$^2$, (3.18 Kg/cm$^2$). This instrument displays the steam pressure at the shell side of the heat exchanger.**
- **PI-103**: **Pressure Indicator, 0 to 15 kg/cm$^2$, (10.55 Kg/cm$^2$). This instrument displays the steam header pressure.**
- **TAH/L-102**: **Temperature Alarm High/Low, (Normal). This alarm fires should the temperature of the feedstock at the exchanger outlet exceed 85°C or be less than 71°C.**
- **TI-103**: **Temperature Indicator, 0 to 200°C, (186°C). This instrument displays the temperature of the steam entering the shell side of the heat exchanger.**
- **TIRC-102** **:Temperature Indicator, Recorder, and Controller, 0 to 200°C, (80°C). This instrument controls the temperature of the feedstock at the exchanger outlet by positioning the valve that regulates the steam flow to the exchanger.**
- **TR-101**: **Temperature Recorder, 0 to 200°C, (38°C). This instrument displays the temperature of the feedstock entering the exchanger.**



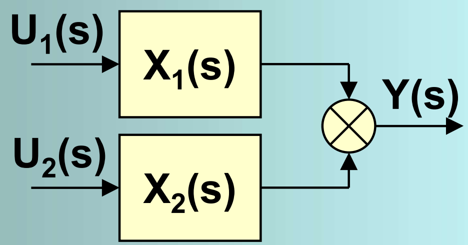
TENNESSEE EASTMAN DIVISION
A DIVISION OF EASTMAN CHEMICAL COMPANY
KINGSPORT, TENNESSEE, 37662
PROCESS FLOW DIAGRAM
NO. 6 ESTER UNIT
NONE
1030.0FFI-101A
00600728
99-9T-1638
LIFE IS FRAGILE DESIGN AND BUILD WITH CARE
ISSUE FOR CONSTRUCTION
IFC-03
08-28-96
E6-C-15COL
ESTER COLUMN
E6-C-20COL
LOW BOILER COLUMN
E6-C-12COL
FLASH COLUMN
E6-E-23CND
ESTER CONDENSER
E6-E-22CND
ESTER SUBCOOLER
E6-C-22TK
ESTER DECANTER
E6-C-23TK
L.B. DECANTER
E6-E-9H
ESTER BASE HEATER
E6-C-21TK
STILL POT
E6-D-24TK
CATALYST ADDITION TANK
E6-G-3031
UNDERFLOW PUMP
E6-E-10C
FEED/BOTTOMS INTERCHANGER
E6-E-30CND
VENT CONDENSER
E6-E-32CND
L.B. PRIMARY CONDENSER
E6-C-9S
LOW BOILER CYCLONE
E6-C-26TK
L.B. STILL POT
E6-E-13H
L.B. BASE HEATER
E6-E-31CND
PRODUCT CONDENSER
E6-E-11CT
PRODUCT SUBCOOLERS
E6-E-11CB
E6-D-58TK
LOT TANK
E6-D-59TK
LOT TANK
E6-G-3122
LOT TANK PUMP
E6-G-3123
LOT TANK PUMP
FROM CATALYST PUMP
TO WWT
FROM n-BuOH
FROM TANK 19 HOAc
TO OVERHEAD INTERCEPTOR SEWER
TO FLASH TANK B-99
TO WWT TRENCH
REJECTED FEED
ATMOSPHERE
NITROGEN
RIVER WATER SUPPLY
RIVER WATER RETURN
FILTERED WATER
REVISE ON CADD

# CLOSED LOOP TRANSFER FUNCTION

- **Block diagram algebra**




$$\frac{Y(s)}{U(s)} = X_n(s) \cdots X_2(s) X_1(s)$$




$$Y(s) = X_1(s)U_1(s) + X_2(s)U_2(s)$$

- **Transfer functions of closed-loop system**

$$X_2(s) = G_p(s)G_v(s)G_c(s)E(s) \qquad E(s) = K_m(s)R(s) - G_m(s)Y(s)$$

$$Y(s) = G_L(s)L(s) + X_2(s) \Longrightarrow Y(s) = G_L(s)L(s) + G_p(s)G_v(s)G_c(s)E(s)$$

$$\Longrightarrow Y(s) = G_L(s)L(s) + G_p(s)G_v(s)G_c(s)(K_m(s)R(s) - G_m(s)Y(s))$$

$$\Longrightarrow (1 + G_m(s)G_p(s)G_v(s)G_c(s))Y(s) = G_L(s)L(s) + K_mG_p(s)G_v(s)G_c(s)R(s)$$

- **For set-point change (L=0)**

$$\frac{Y(s)}{R(s)} = \frac{K_m G_p(s) G_v(s) G_c(s)}{1 + G_m(s) G_p(s) G_v(s) G_c(s)}$$

- **For load change (R=0)**

$$\frac{Y(s)}{L(s)} = \frac{G_L(s)}{1 + G_m(s) G_p(s) G_v(s) G_c(s)}$$

- **Open-loop transfer function ($G_{OL}$)**

$$G_{OL}(s) \triangleq G_m(s) G_p(s) G_v(s) G_c(s)$$

  - **Feedforward path**: **Path with no connection backward**
  - **Feedback path**: **Path with circular connection loop**
  - **$G_{OL}$: feedback loop is broken before the comparator**

- **Simultaneous change of set point and load**

$$Y(s) = \frac{K_m G_p(s) G_v(s) G_c(s)}{1 + G_{OL}(s)} R(s) + \frac{G_L(s)}{1 + G_{OL}(s)} L(s)$$

# MASON'S RULE

- **General expression for feedback control systems**

$$\frac{Y}{X} = \frac{\pi_f}{1 + \pi_e}$$

$\pi_f \equiv$ product of the transfer functions in the path from X to Y
$\pi_e \equiv$ product of all transfer functions in the entire feedback loop

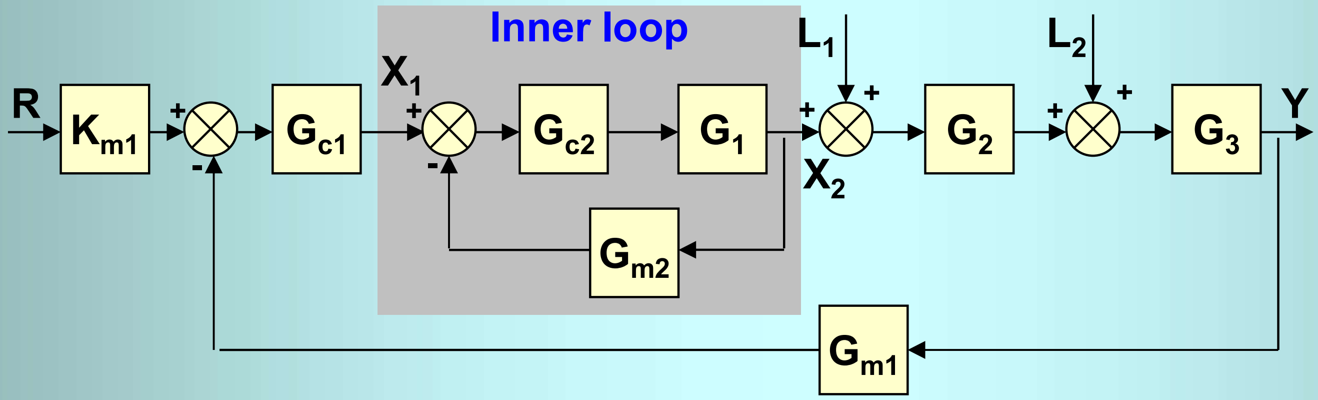
- **Assume feedback loop has negative feedback.**
- **If it has positive feedback,** $1 + \pi_e$ **should be** $1 - \pi_e$**.**
- **In the previous example, for set-point change**

$X = R \quad Y = Y \qquad \pi_f = K_m G_c(s) G_v(s) G_p(s) \qquad \pi_e = G_{OL}(s)$

$$\frac{Y(s)}{R(s)} = \frac{K_m G_p(s) G_v(s) G_c(s)}{1 + G_{OL}(s)}$$

- **For load change,** $X = L \quad Y = Y \qquad \pi_f = G_L(s) \qquad \pi_e = G_{OL}(s)$

- **Example 1**




– **Inner loop:** $X_2 = \dfrac{G_1 G_{c2}}{1 + G_{m2} G_1 G_{c2}} X_1$

– **TF between R and Y:**

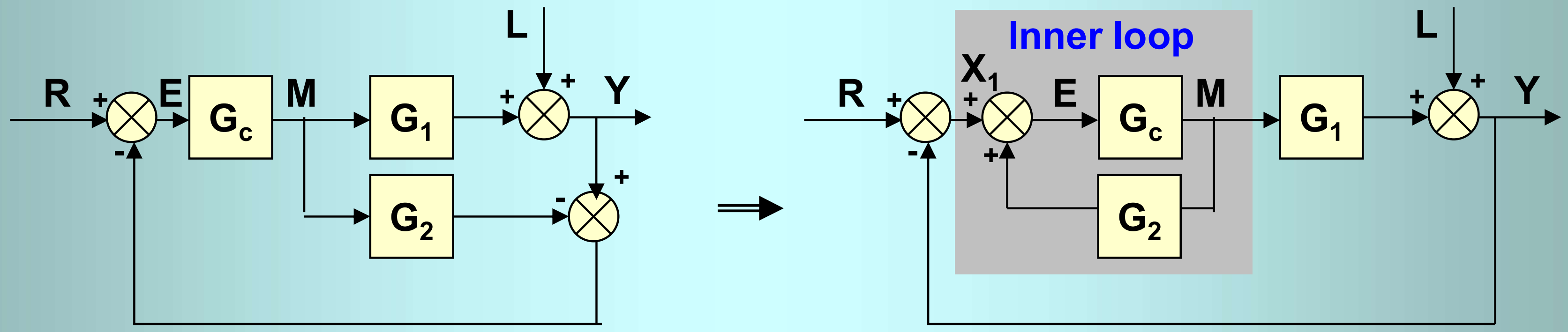
$$\pi_f = K_{m1} G_3 G_2 \frac{G_1 G_{c2}}{1 + G_{m2} G_1 G_{c2}} G_{c1} \qquad \pi_e = G_{m1} G_3 G_2 \frac{G_1 G_{c2}}{1 + G_{m2} G_1 G_{c2}} G_{c1}$$

$$\frac{Y}{R} = \frac{K_{m1} G_3 G_2 G_1 G_{c2} G_{c1}}{1 + G_{m2} G_1 G_{c2} + G_{m1} G_3 G_2 G_1 G_{c2} G_{c1}}$$

– **TF between $L_1$ and Y:**

$$\frac{Y}{L_1} = \frac{G_3 G_2 (1 + G_{m2} G_1 G_{c2})}{1 + G_{m2} G_1 G_{c2} + G_{m1} G_3 G_2 G_1 G_{c2} G_{c1}}$$

- **Example 2**




$$E = R - (G_1 - G_2)M = R - G_1 M + G_2 M$$

– **Inner loop:** $M = \dfrac{G_c}{1 - G_2 G_c} X_1$

– **TF between R and Y:**

$$\pi_f = \frac{G_c}{1 - G_2 G_c} G_1 \qquad \pi_e = \frac{G_c}{1 - G_2 G_c} G_1$$

$$\frac{Y}{R} = \frac{G_1 G_c}{1 - G_2 G_c + G_1 G_c} = \frac{G_1 G_c}{1 + (G_1 - G_2) G_c}$$

– **TF between L and Y:** $\dfrac{Y}{L} = \dfrac{1 - G_2 G_c}{1 - G_2 G_c + G_1 G_c} = \dfrac{1 - G_2 G_c}{1 + (G_1 - G_2) G_c}$

# PID CONTROLLER REVISITED

- **P control**

$$p(t) = \bar{p} + K_c e(t) \xrightarrow{\mathfrak{L}} \frac{P(s)}{E(s)} = K_c$$

- **PI control**

$$p(t) = \bar{p} + K_c \left\{ e(t) + \frac{1}{\tau_I} \int_0^t e(\tau) d\tau \right\} \xrightarrow{\mathfrak{L}} \frac{P(s)}{E(s)} = K_c (1 + \frac{1}{\tau_I s}) = K_c \frac{(\tau_I s + 1)}{\tau_I s}$$

- **PID control**

$$p(t) = \bar{p} + K_c \left\{ e(t) + \frac{1}{\tau_I} \int_0^t e(\tau) d\tau + \tau_D \frac{de}{dt} \right\} \xrightarrow{\mathfrak{L}}$$

$$\frac{P(s)}{E(s)} = K_c (1 + \frac{1}{\tau_I s} + \tau_D s) = K_c \frac{(\tau_I \tau_D s^2 + \tau_I s + 1)}{\tau_I s}$$

  – **Ideal PID controller: Physically unrealizable**
  – **Modified form has to be used.**

- **Nonideal PID controller**
  - **Interacting type**

$$G_c^*(s) = K_c^* \frac{(\tau_I^* s + 1)}{\tau_I^* s} \frac{(\tau_D^* s + 1)}{(\beta \tau_D^* s + 1)} \quad (0 < \beta \ll 1)$$

Filtering effect

  - **Comparison with ideal PID except filter**

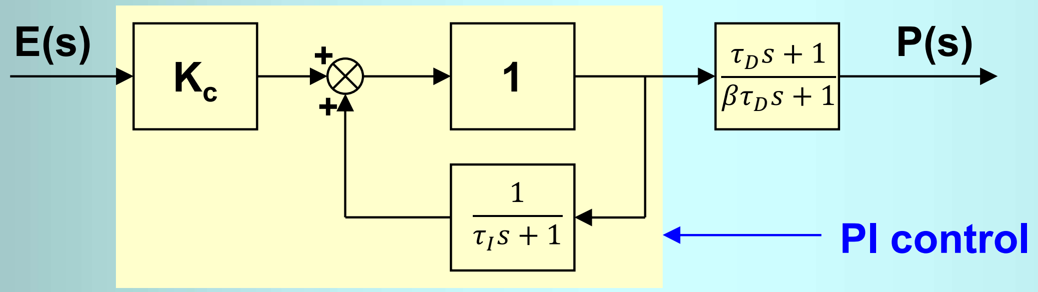
$$G_c(s) = K_c \frac{(\tau_I \tau_D s^2 + \tau_I s + 1)}{\tau_I s}$$

$$K_c^* \frac{(\tau_D^* \tau_I^* s^2 + (\tau_I^* + \tau_D^*) s + 1)}{\tau_I^* s} = \frac{K_c^* (\tau_I^* + \tau_D^*)}{\tau_I^*} \left( 1 + \frac{1}{(\tau_I^* + \tau_D^*)} \frac{1}{s} + \frac{\tau_D^* \tau_I^*}{(\tau_I^* + \tau_D^*)} s \right)$$

$$K_c = \frac{K_c^* (\tau_I^* + \tau_D^*)}{\tau_I^*}, \qquad \tau_I = \tau_I^* + \tau_D^*, \qquad \tau_D = \frac{\tau_D^* \tau_I^*}{(\tau_I^* + \tau_D^*)}$$

  - **These types are physically realizable and the modification provides the prefiltering of the error signal.**
  - **Generally,** $\tau_I \geq \tau_D$ **and typically** $\tau_I \approx 4\tau_D$ **.**
  - **In this form,** $\tau_I \geq \tau_D$ **is satisfied automatically since algebraic mean is not less than logarithm mean.**

- **Block diagram of PID controller**
  - **Nonideal interacting type PID**




$$\frac{1}{1-\frac{1}{\tau_I s+1}}=\frac{\tau_I s+1}{\tau_I s+1-1}=\frac{\tau_I s+1}{\tau_I s}$$

$$P(s)=K_c\frac{(\tau_I s+1)}{\tau_I s}\frac{(\tau_D s+1)}{(\beta\tau_D s+1)}E(s)$$

  - **Removal of derivative kick (PI-D controller)**

$$P(s)=K_c\left[\frac{(\tau_I s+1)}{\tau_I s}R(s)-\frac{(\tau_I s+1)}{\tau_I s}\frac{(\tau_D s+1)}{(\beta\tau_D s+1)}Y(s)\right]$$

  - **Removal of both P & D kicks (I-PD controller)**

$$P(s)=K_c\left[\frac{1}{\tau_I s}R(s)-\frac{(\tau_I s+1)}{\tau_I s}\frac{(\tau_D s+1)}{(\beta\tau_D s+1)}Y(s)\right]$$

- **Other variations of PID controller**
  - **Gain scheduling : modifying proportional gain**

    $K_c^{GS} = K_c K^{GS}$

    **where**

    $$1.\ K^{GS} = \begin{cases} K_{Gap} & \text{for (lower gap)} \le e(t) \le \text{(upper gap)} \\ 1 & \text{otherwise} \end{cases}$$

    $$2.\ K^{GS} = 1 + C_{GS}|e(t)|$$

    3. $K^{GS}$ is decided based on some strategy

  - **Nonlinear PID controller**
    - **Replace** $e(t)$ **with** $e(t)\,|\,e(t)\,|$**.**
    - **Sign of error will be preserved but small error gets smaller and larger error gets larger.**
    - **It imposes less action for a small error.**

# DIGITAL PID CONTROLLER

- **Discrete time system**
  - Measurements and actions are taken at every sampling interval.
  - An action will be hold during the sampling interval.
- **Digital PID controller**
  - using

$$\int_0^{t_n} e(\tau)d\tau = \Delta t \sum_{i=0}^{n} e(t_i) \quad \text{(Rectangular rule)}$$

$$\frac{de(t)}{dt} = \frac{e(t_n) - e(t_{n-1})}{\Delta t} \quad \text{(Backward difference approx.)}$$

$$p(t_n) = \bar{p} + K_c \left[ e(t_n) + \frac{\Delta t}{\tau_I} \sum_{i=0}^{n} e(t_i) + \tau_D \frac{e(t_n) - e(t_{n-1})}{\Delta t} \right] \quad \text{(Position form)}$$

$$\Delta p(t_n) = p(t_n) - p(t_{n-1})$$

$$= K_c \left[ e(t_n) - e(t_{n-1}) + \frac{\Delta t}{\tau_I} e(t_n) + \tau_D \frac{e(t_n) - 2e(t_{n-1}) + e(t_{n-2})}{\Delta t} \right] \quad \text{(Velocity form)}$$

- **Most modern PID controllers are manufactured in digital form with short sampling time.**
- **If the sampling time is small, there is not much difference between continuous and digital forms.**
- **Velocity form does not have reset windup problem because there is no summation (integration).**
- **Other approximation such as trapezoidal rule and etc. can be used to enhance the accuracy. But the improvement is not substantial.**

$$\int_0^{t_n} e(\tau)d\tau = \Delta t \sum_{i=1}^{n} \frac{e(t_i) + e(t_{i-1})}{2} \qquad \text{(Trapezoidal rule)}$$

$$\frac{de(t)}{dt} = \frac{e(t_n) + 3e(t_{n-1}) - 3e(t_{n-2}) - e(t_{n-3})}{\Delta t} \qquad \text{(Interpolation formula)}$$

- **For discrete time system, z-transform is the counterpart of Laplace transform. (out of scope of this lecture)**

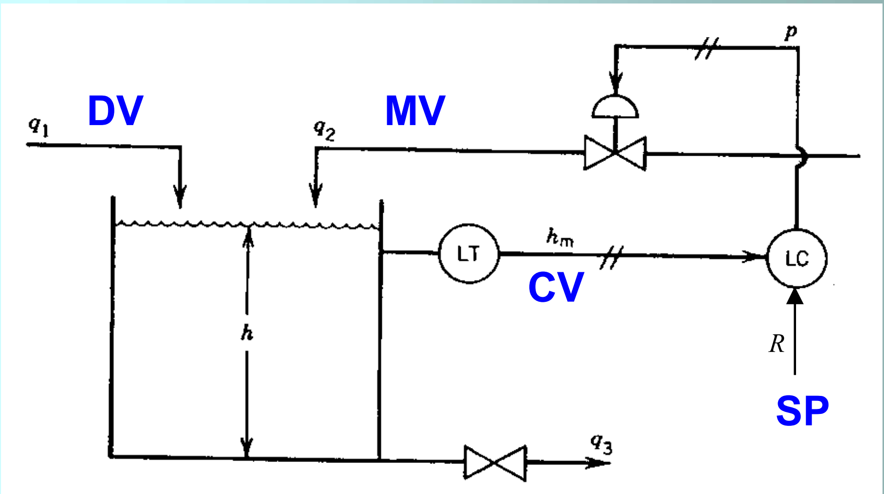
# CLOSED-LOOP RESPONSE OF 1ST ORDER SYSTEM

- **Process**

$$\rho A \frac{dh}{dt} = \rho q_1 + \rho q_2 - \rho \frac{h}{R}$$

$$G_p(s) = \frac{H(s)}{Q_2(s)} = \frac{R}{RAs + 1} = \frac{K_p}{\tau s + 1}$$

$$G_L(s) = \frac{H(s)}{Q_1(s)} = \frac{R}{RAs + 1} = \frac{K_p}{\tau s + 1}$$

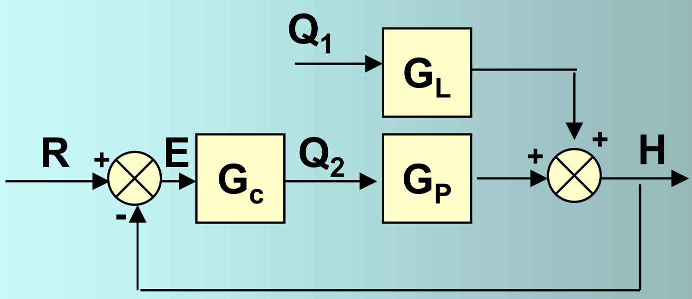



  – **Assume**

    **Sensor and actuator dynamics are fast enough to be ignored and gains are lumped in other TF.**

$$G_v(s) = G_m(s) = 1$$

$$H(s) = \frac{G_c G_p}{1 + G_c G_p} R(s) + \frac{G_L}{1 + G_c G_p} L(s)$$



# • P control for set-point change (L=0)

$$G_c(s) = K_c \quad (K_c > 0)$$

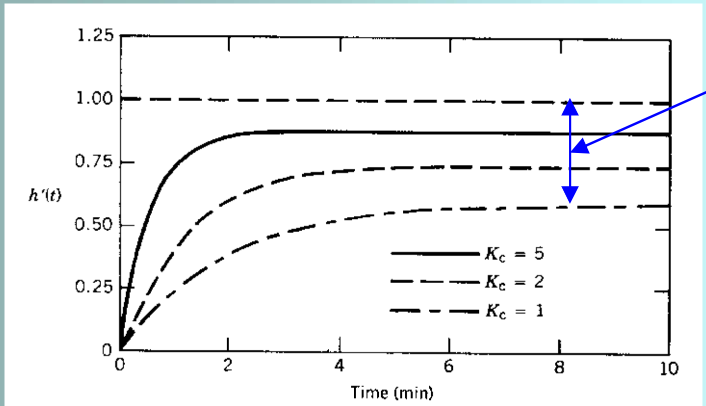
$$G_{CL}(s) = \frac{H(s)}{R(s)} = \frac{K_c K_p/(\tau s + 1)}{1 + K_c K_p/(\tau s + 1)} = \frac{K_c K_p/(1 + K_c K_p)}{(\tau/(1 + K_c K_p))s + 1} \quad \text{(closed−loop TF)}$$

– **Closed-loop gain and time constant**

$$K_{CL} = \frac{K_c K_p}{(1 + K_c K_p)}, \qquad \tau_{CL} = \frac{\tau}{(1 + K_c K_p)}$$

– **Steady-state behavior of closed-loop system**

$$K_{CL} = \frac{K_c K_p}{(1 + K_c K_p)} < 1, \qquad \lim_{K_c \to \infty} G_{CL} = 1 \quad (H(s) = R(s), \text{no offset})$$




$$\text{Steady−state offset} = r(\infty) - h(\infty) = 1 - K_{CL} = \frac{1}{1 + K_c K_p}$$

**Closed-loop response will not reach to set point (offset)**

**Infinite controller gain will eliminate the offset**

**Higher controller gain results faster closed-loop response: shorter time constant**

- **P control for load change (R=0)**

$$G_c(s) = K_c \quad (K_c>0)$$

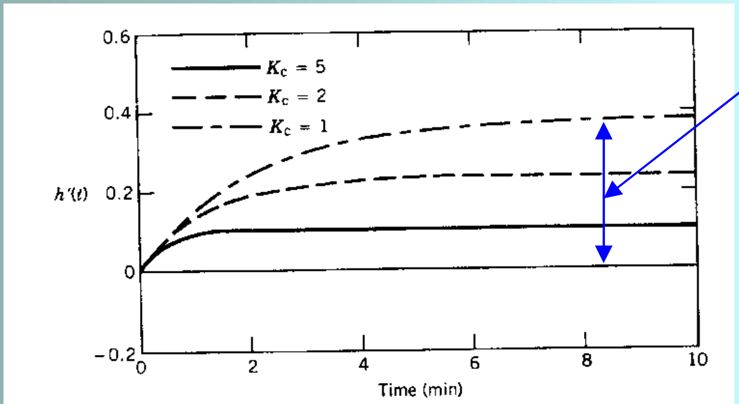
$$G_{CL}(s) = \frac{H(s)}{L(s)} = \frac{K_p/(\tau s+1)}{1+K_cK_p/(\tau s+1)} = \frac{K_p/(1+K_cK_p)}{(\tau/(1+K_cK_p))s+1} \quad \text{(closed−loop TF)}$$

- **Closed-loop gain and time constant**

$$K_{CL} = \frac{K_p}{(1+K_cK_p)}, \qquad \tau_{CL} = \frac{\tau}{(1+K_cK_p)}$$

- **Steady-state behavior of closed-loop system**

$$K_{CL} = \frac{K_p}{(1+K_cK_p)} > 0, \qquad \lim_{K_c\to\infty} G_{CL} = 0 \ \text{(disturbance is compensated)}$$




$$\text{Steady−state offset} = 0 - h(\infty) = 0 - K_{CL} = -\frac{K_p}{1+K_cK_p}$$

**Disturbance effect will not be eliminated completely (offset)**

**Infinite controller gain will eliminate the offset**

**Higher controller gain results faster closed-loop response: shorter time constant**

- **PI control for load change (R=0)**

$$G_c(s) = K_c(\tau_I s + 1)/\tau_I s \quad (K_c > 0)$$

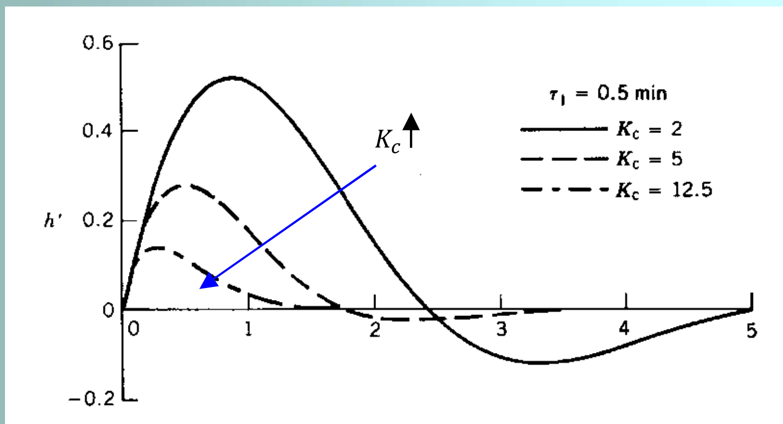
$$G_{CL}(s) = \frac{K_p/(\tau s + 1)}{1 + K_c K_p(\tau_I s + 1)/(\tau s + 1)/\tau_I s} = \frac{K_p \tau_I s}{\tau_I \tau s^2 + \tau_I(1 + K_c K_p)s + K_c K_p}$$

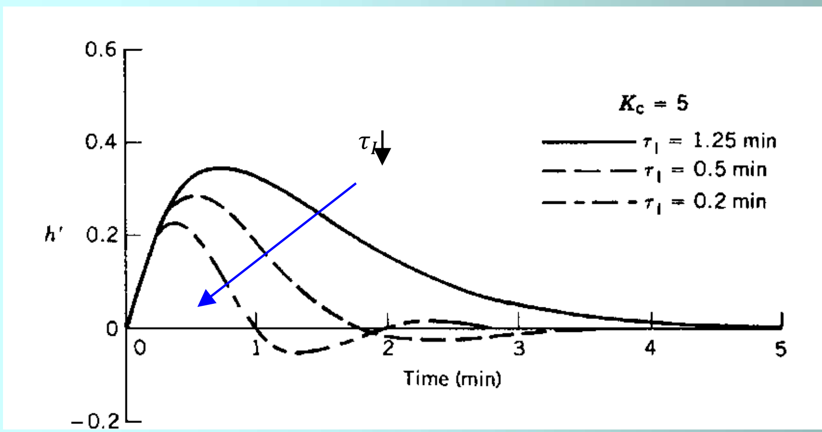
  - **Closed-loop gain, time constant, damping coefficient**

$$K_{num} = \frac{\tau_I}{K_c}, \qquad \tau_{CL} = \sqrt{\frac{\tau \tau_I}{K_c K_p}}, \qquad \zeta_{CL} = \frac{1}{2}\frac{(1 + K_c K_p)}{\sqrt{K_c K_p}}\sqrt{\tau_I/\tau}$$

  - **Steady-state behavior of closed-loop system**

$$\lim_{s \to 0} G_{CL}(s) = 0 \ \text{(disturbance is compensated for all cases)}$$



- **As $K_c$ increases, faster compensation of disturbance and less oscillatory response can be achieved.**
- **As $\tau_I$ decreases, faster compensation of disturbance and less overshooting response can be achieved.**
- **However, usually the response gets more oscillation as $K_c$ increases or $\tau_I$ decreases. => very unusual!!**
- **If there is small lag in sensor/actuator TF or time delay in process TF, the system becomes higher order and these anomalous results will not occur. These results is only possible for very simple process such as 1st order system.**
- **Usual effect of PID tuning parameters**
  - **As $K_c$ increases, the response will be faster, more oscillatory.**
  - **As $\tau_I$ decreases, the response will be faster, more oscillatory.**
  - **As $\tau_D$ increases, the response will be faster, less oscillatory when there is no noise.**

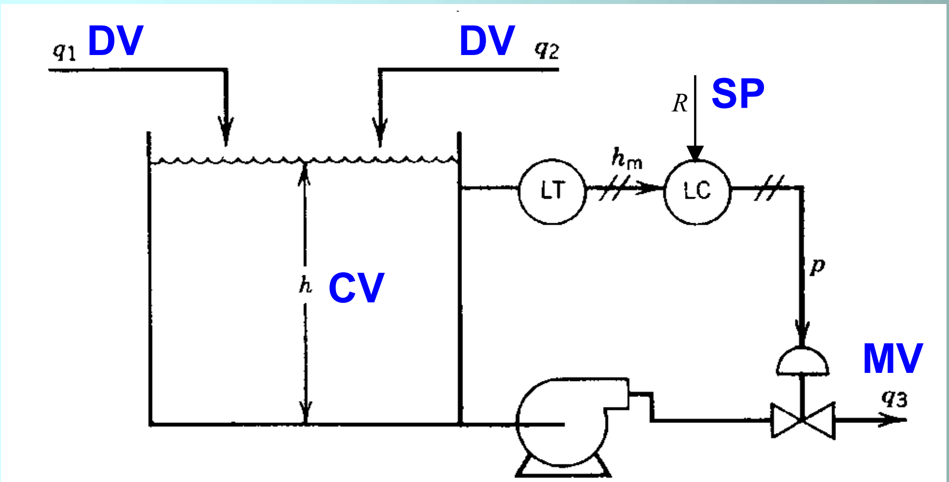
# CLOSED-LOOP RESPONSE OF INTEGRATING SYSTEM

- **Process**

$$\rho A \frac{dh}{dt} = \rho(q_1 + q_2) - \rho q_3$$

$$G_p(s) = \frac{H(s)}{Q_3(s)} = -\frac{1}{As}$$

$$G_L(s) = \frac{H(s)}{Q_1(s)} = \frac{1}{As}$$

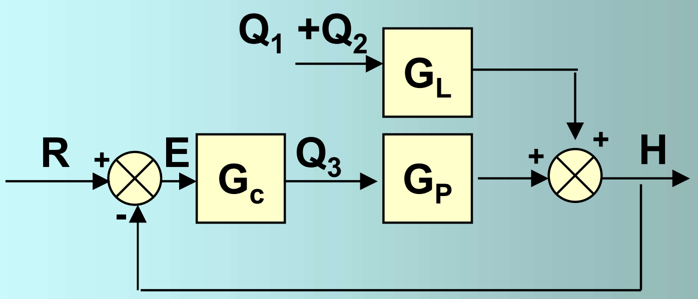



- Assume

Sensor and actuator dynamics are fast enough to be ignored and gains are lumped in other TF.

$$G_v(s) = G_m(s) = 1$$

$$H(s) = \frac{G_c G_p}{1 + G_c G_p} R(s) + \frac{G_L}{1 + G_c G_p} L(s)$$



- **P control for set-point change (L=0)**

$$G_c(s) = K_c \quad (K_c<0)$$

$$G_{CL}(s) = \frac{H(s)}{R(s)} = \frac{K_c/(-As)}{1+K_c/(-As)} = \frac{1}{(-A/K_c)s+1} \quad \text{(closed−loop TF)}$$

- **Closed-loop gain and time constant**

$$K_{CL} = 1, \qquad \tau_{CL} = -A/K_c$$

- **Steady-state behavior of closed-loop system**

$$K_{CL} = 1 \ \ (H(s) = R(s), \text{no offset even with p control})$$

- **It is very unique that the integrating system will not have offset even with P control for the set point change.**
- **Even though there are other dynamics in sensor or actuator, the offset will not be shown with P control for integrating systems.**
- **Higher controller gain results faster closed-loop response: shorter time constant**

- **P control for load change (R=0)**

$G_c(s) = K_c \quad (K_c<0)$

$$G_{CL}(s) = \frac{H(s)}{L(s)} = \frac{1/(As)}{1 + K_c/(-As)} = \frac{-1/K_c}{(-A/K_c)s + 1} \quad \text{(closed−loop TF)}$$

  - **Closed-loop gain and time constant**

$K_{CL} = (-1/K_c), \qquad \tau_{CL} = -A/K_c$

  - **Steady-state behavior of closed-loop system**

$$K_{CL} = \frac{1}{(-K_c)} > 0, \qquad \lim_{K_c \to \infty} G_{CL} = 0 \ \text{(disturbance is compensated)}$$

  - **Higher controller gain results faster closed-loop response: shorter time constant**

- **PI control for set-point change (L=0)**

$$G_c(s) = K_c(\tau_I s + 1)/\tau_I s \quad (K_c<0)$$

$$G_{CL}(s) = \frac{K_c(\tau_I s + 1)/(-As)/\tau_I s}{1 + K_c(\tau_I s + 1)/(-As)/\tau_I s} = \frac{(\tau_I s + 1)}{(-\tau_I A/K_c)s^2 + \tau_I s + 1}$$

  - **Closed-loop gain, time constant, damping coefficient**

$$K_{CL} = 1, \qquad \tau_{CL} = \sqrt{-\frac{\tau_I A}{K_c}}, \qquad \zeta_{CL} = \frac{1}{2}\sqrt{-\frac{\tau_I K_c}{A}}$$

  - **Steady-state behavior of closed-loop system**

$$K_{CL} = \lim_{s\to 0} G_{CL}(s) = 1 \quad (H(s) = R(s), \text{ no offset})$$

  - **As ($-K_c$) increases, closed-loop time constant gets smaller (faster response) and less oscillatory response can be achieved.**
  - **As $\tau_I$ decreases, closed-loop time constant gets smaller (faster response) and more oscillatory response can be achieved.**
  - **Partly anomalous results due to integrating nature**
  - **For integrating system, the effect of tuning parameters can be different. Thus, rules of thumb cannot be applied blindly.**