Guided Tutorial for Java Control Module VI

Lesson 1: *The poles of a transfer function indicate the dynamic behavior of the system. A system is stable if it has negative poles.*

1. The red circle represents location of poles of the system. For a system described by a transfer function

$$G(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s+a_1)(s+a_2)\cdots(s+a_n)}$$

roots of the denominator a_1, a_2, …, a_n are known as poles of the transfer function.

Start the Java applet VI. Currently the system has one pole at -1. The transfer function for this system is $G(s)=1/(s+1)$. Step response of this system shows that the output response increases and finally settles at 1. This is a first order system with time constant $\tau=1$. **A system is stable if the poles of the transfer function are negative.**

Lesson 2: *Closer the pole to zero, more sluggish is the response of the system*

2. Make sure that the radio button "Move Points" is clicked. Click on the radio button "Linear Move". Now move the pole to the left to value -2. The transfer function of the system is $G(s)=2/(s+2)= 1/(0.5s+1)$. Thus, the time constant of the system is $\tau=0.5$. The system is still stable. The output goes to final value of 1 faster, as the time constant is decreased.

3. Now, move the pole to -0.5. Again, it's a first order system with time constant $\tau=2$. The step response gets sluggish. The system is stable because the pole is negative. As you move the pole closer to zero, the response gets sluggish.

Lesson 3: *Step response of system with poles at zero is a ramp function. Systems with a positive pole are "unstable."*

4. Move the pole location to 0. The transfer function is $G(s)=1/s$. Step response of the system is a ramp function.

5. Move the pole location to 1. We now have a positive pole, which results in exponential growth with time. Positive pole results in an unstable system.

Lesson 4: *Complex roots for a system always exist as a* **complex conjugate pair**. *The real part of the complex root determines exponential growth or damping of the system, while the complex part results in sinusoidal response.*

6. Move the pole back to location -1. Click on the radio button “Circular move”. A black circle appears and the status bar mentions the time constant and damping factor. Move the pole around the circle to -0 5±*j*0.75. We now have two poles that are complex conjugates. The system shows slight oscillations. Negative real part of the root results in exponential decay in the amplitude of the sinusoid.

7. Move the pole to the imaginary axis (pole location would be 0±*j*0.95 or 0±*j*0.9). The poles are now purely imaginary, which are marginally stable. The response of the system is oscillatory with constant amplitude. As the real part of the poles is 0, there is neither attenuation nor increase in the amplitude.

8. Now move the poles to the right hand side of the Y-axis (usually referred to as “right half plane”). Place the poles at 0.25±*j*0.8. The response is sinusoidal with exponentially growing amplitude. As we move pole towards the real axis, the exponential increase becomes faster and faster and the oscillations become less and less.

Thus, in general, a pole in right half plane results in an unstable system while a pole in left half plane represents a stable system. Pole(s) in right half plane and left half plane has positive real part and negative real part, respectively.

Lesson 5: *The response of a second order system can also be determined by its time constant and damping coefficient. Damping coefficient determines the general shape of the dynamic behavior, while time constant indicates the time scale of the response.*

We will repeat steps 6 and 7 above. First, bring the pole back to location -1. To do so, click on the radio button “Free Move”. Move the poles to location -1. The two poles should merge into a single pole.

9. As in cases (6 and 7), click on “Circular move” and move the pole around the circle towards the imaginary axis (Y-axis). You will see that the damping coefficient decreases, and the response of the system becomes increasingly oscillatory.

10. For poles located at -1, the system is **critically damped**. The damping factor $\zeta=1$, and the system does not show oscillatory behavior. Thus, *real repeated roots correspond to a critically damped system*.

11. The transfer function of the system in case (6), i.e. poles at -0.5±*j*0.75 is

$$G(s)=\frac{1}{(s+0.5+j0.75)(s+0.5-j0.75)}=\frac{1}{(1.23s^2+1.23s+1)}$$

The time constant is τ=1.109 and damping factor is ζ=0.555. The system is underdamped. Thus, *complex roots correspond to an underdamped system.*

Lesson 6: *Distinct real roots correspond to an overdamped system.*

12. Click on the "Free move" radio button. Move the pole back to -1. Now click on "Add pole" radio button. On the chart, click once near point (-2, 0) on the X axis. This will introduce a pole at -2. If the pole is not at -2, click on "Move points" radio button. With the "Free move" radio button still checked, move the pole to -2. This corresponds to an overdamped system, given by

$$G(s)=\frac{2}{(s+1)(s+2)}=\frac{1}{(0.5s^2+1.5s+1)}$$

Thus, we have a second order system with τ=0.707 and ζ=1.06. This is an overdamped system.

Lesson 7: *Right half plane zero results in an inverse response. An overshoot is observed if we have a left half plane zero that is closer to the origin than all the poles of the transfer function.*

13. At this point, we should have two poles, one at -1 and the other at -2. Let us now introduce a zero and see its effect on the dynamics of the system. Click on the radio button "Add zero." Click on the chart at (1, 0) to introduce a zero at 1. The zero will be shown by a blue rectangle. We have introduced a zero in right half plane. The system shows inverse response, i.e. on introducing a positive step change, the output first decreases, and then increases to settle down at the steady state value of 1. **Inverse response will be observed whenever we have a zero in right half plane.**

14. Now click on the radio button "Move points." Click on the radio button "Linear move." Move the zero to location -0.5. As the zero is now in the left half plane, we do not see inverse response as before. However, the system shows an overshoot. This is because the zero is closer to the Y-axis than any of the poles. This is usually true in simple cases.

15. Move the zero to location -2.5. There is neither inverse response nor an overshoot observed.

You can now play with this module by introducing more poles and zeros (number of zeros should not exceed the number of poles), moving them around and seeing the response of higher order systems with various pole-zero locations.