

A Review on Equations of State Applicable to Polymers and Complex Systems

J.W.Kang, J.H.Lee, K.-P. Yoo and C.S.Lee

Department of Chemical Engineering
Korea Univeristy
Sogang University



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Motivation

- ❑ **Increasing demands of EOS models for various process conditions and complex systems**

- *Supercritical conditions and high pressure systems*
- *Phase equilibria of polymers*
- *Phase equilibria of associating mixtures (alcohols, acids)*
- *Simultaneous representation of VLE and H^E*

- ❑ **EOS approach provides integrated approach covering wider range of process conditions and various properties**

- ❑ **Characteristics and understanding of each EOS models are important for various applications.**

Three Classes of EOS Models

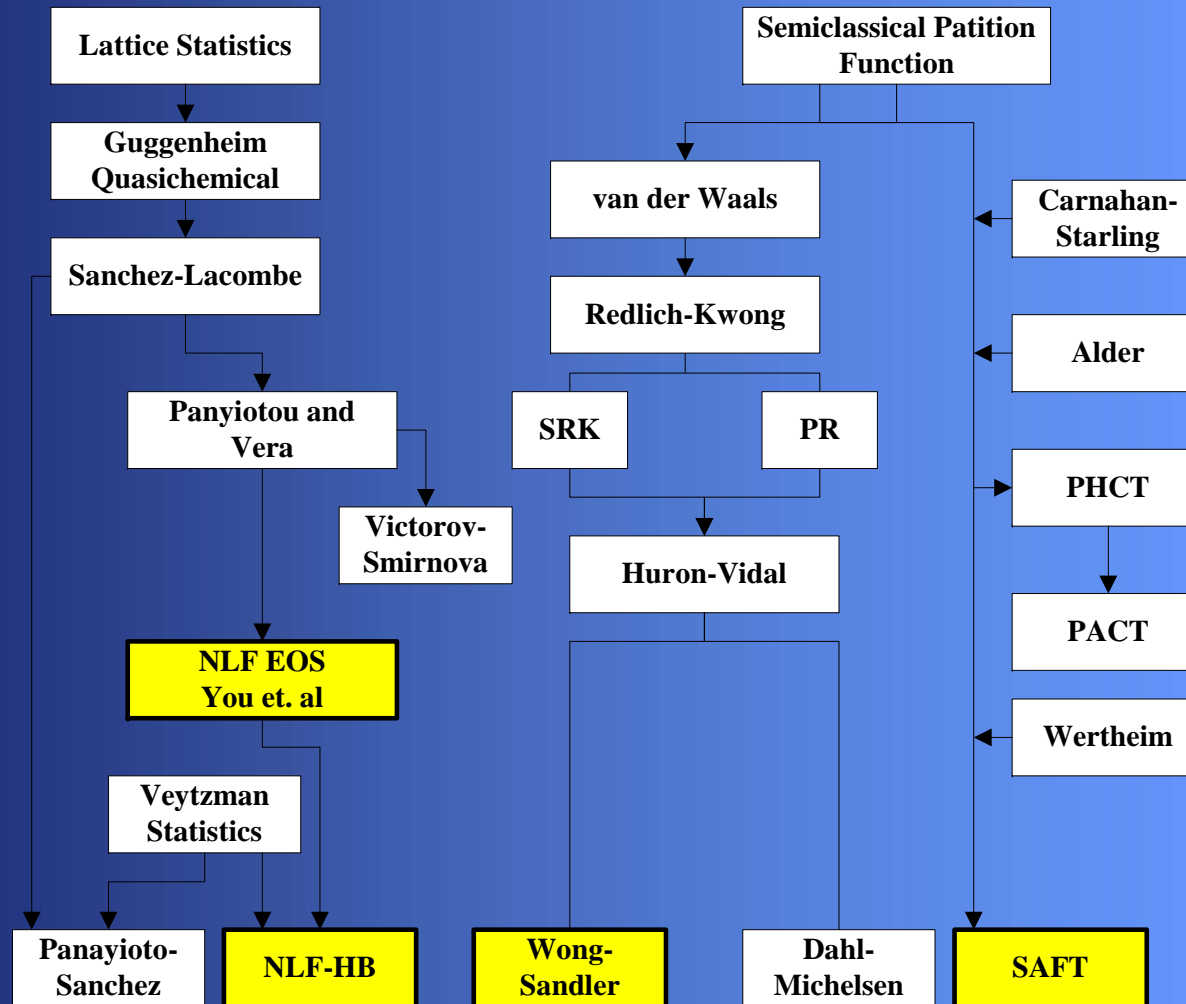
❑ Based on Semi-Classical Partition Function

- *Cubic Equation of State and Other van-der Waals based EOS*
 - *Soave-Redlich-Kwong (1972), Peng-Robinson EOS (1976)*
 - *Mixing Rules proposed by Huron-Vidal (1979), Michelsen (1990) and Wong-Sandler (1992)*
- *Radial Distribution Function Approaches*
 - *PHCT (1978), PACT (1985)*
 - *SAFT (Huang and Radosz, Chapman, 1990)*

❑ Based on Lattice Statistics

- *Sanchez-Lacomb (1976)*
- *Panayiotou-Vera (1981)*
- *NLF -EOS proposed by You, Yoo and Lee (1993)*
- *Hydrogen Bonding proposed by Veytzman (1990) + Lattice EOS*
 - *Panayiotou-Sanchez (1991), NLF-HB EOS (Yeom et. al., 1999)*

Three Classes of EOS Models – Historical Review



EOS Models Selected for Comparison

Peng-Robinson Equation of State with Wong-Sandler Mixing Rule (PR-WS)

- *Peng and Robinson (1976) : EOS*
- *Wong and Sandler (1992) : Mixing Rule*
- *Mathias and Copeman (1983) : $a(T)$ law*

Statistically Associated Fluid Model (SAFT)

- *Huang and Radosz, Chapman (1990)*

Nonrandom Lattice Fluid EOS with Hydrogen Bonding (NFL-HB)

- *You, Yoo and Lee (1993)*
- *Yeom, Yoo and Lee (1999)*

Comparison

- Number of pure and mixture parameters**
- Procedure to obtain pure component parameters and binary interaction parameters**

- Pure component properties (vapor pressure and liquid density)**
- Binary VLE calculation of various mixtures**
- Polymer solubility calculation**

- Computation time**
- Sensitivity of VLE calculation with respect to binary interaction parameters (robustness)**

Peng-Robinson EOS with Wong-Sanler Mixing Rule (PR-WS EOS)

EOS

$$Z = \frac{V}{V-b} - \frac{a(T)V}{RT(V+\epsilon b)(V+\sigma b)}$$

Generalized
form of Cubic
EOS

$a(T)$ law

$$a(T) = a_c \alpha(T)$$

$$\alpha = [1 + C_1(1 - T_R^{0.5}) + C_2(1 - T_R^{0.5})^2 + C_3(1 - T_R^{0.5})^3]^2$$

Mathias-
Copeman $a(T)$
law (1983)

Mixing Rule

$$G^{ex}(T, P = 1 \text{ bar}, x_i) \approx A^{ex}(T, P = 1 \text{ bar}, x_i) \approx A^{ex}(T, \text{high } P, x_i)$$

$$B(x_i, T) = \sum \sum x_i x_j B_{ij}(T) = \sum \sum x_i x_j \left(b_{ij} - \frac{a_{ij}}{RT} \right) = b - \frac{a}{RT}$$

$$b_{ij} - \frac{a_{ij}}{RT} = \frac{1}{2} \left[\left(b_{ii} - \frac{a_{ii}}{RT} \right) + \left(b_{jj} - \frac{a_{jj}}{RT} \right) \right] (1 - k_{ij})$$

Wong-Sandler
mixing rule
(1992)

Statistical Associating Fluid Theory (SAFT)

EOS

$$Z = 1 + Z_{hs} + Z_{chain} + Z_{disp} + Z_{assoc}$$

Carnahan and Starling (1969)

Hard Sphere Term

$$Z_{hs} = \frac{6}{\pi N_A \rho} \left[\frac{\xi_0 \xi_3}{1 - \xi_3} + \frac{3 \xi_1 \xi_2}{(1 - \xi_3)^2} + \frac{(3 - \xi_3) \xi_2^3}{(1 - \xi_3)^3} \right] \quad \xi_k = (\pi N_A \rho / 6) \sum_i x_i r_i (d_i)^k$$

Chain Term

$$Z_{chain} = \sum_i x_i (1 - r_i) L(d_i) \quad L(d_i) = \frac{2 \xi_3 + 3 d_i \xi_2 - 4 \xi_2^3 + 2 d_i \xi_2^2 + 2 \xi_3^3 + d_i^2 \xi_2^2 \xi_3 - 3 d_i \xi_2 \xi_3^2}{(1 - \xi_3)(2 - 4 \xi_3 + 3 d_i \xi_2 + 2 \xi_2^2 + d_i^2 \xi_2^3 - 3 d_i \xi_2 \xi_3)}$$

Chapman (1990)

Dispersion Term

$$Z_{disp} = r \sum_n \sum_m m D_{nm} (u / kT)^n (\xi_3 / \xi_{3cp})^m$$

Based on Alder (1972)
Chen and Kreglewski (1977)

Association Term

$$Z_{assoc} = \rho \sum x_i \left[\sum (1 / X^{S_i} - 1/2) (\partial X^{S_i} / \partial \rho) \right]$$

$$X^{S_i} = \left(1 + N_A \sum_j \sum_{Y_j} x_j \rho X^{Y_j} W_{ij} \right)^{-1}$$

Based on Wertheim (1984)
Chapman (1990)

Nonrandom Lattice Fluid EOS with Hydrogen Bonding (NLF-HB EOS)

EOS

$$Z = 1 + Z_{ath} + Z_{res} + Z_{assoc}$$

You, Yoo and Lee (1993)

Athermal Contribution

$$Z_{ath} = (zN_r / 2N_A) \ln[1 + (q_M / r_M - 1)\rho]$$

Residual Contribution

$$Z_{res} = (zN_r / 2N_A) \theta^2 \beta \epsilon_M$$

$$\epsilon_M = \frac{1}{\theta^2} \left[\sum \sum \theta_i \theta_j \epsilon_{ij} + \left(\frac{\beta}{2} \right) \sum \sum \sum \sum \theta_i \theta_j \theta_k \theta_l \epsilon_{ij} (\epsilon_{ij} + 3\epsilon_{kl} - 2\epsilon_{ik} - 2\epsilon_{jk}) \right]$$

You, Yoo and Lee (1993)

Association Contribution

$$Z_{assoc} = -(N_r / N_A) v_{HB} \rho$$

Fraction of hydrogen bond

$$v_{HB} = \sum_{i=1}^m \sum_{j=1}^n N_{ij}^{HB} / \sum_{i=1}^c N_i r_i$$

Yeom, Yoo, Park, Lee (1999)

$$N_{ij}^{HB} N_r = N_{io}^{HB} N_{oj}^{HB} \exp(-\beta A_{ij}^{HB}) = (N_d^i - \sum_{k=1}^n N_{ik}^{HB}) (N_a^j - \sum_{k=1}^m N_{kj}^{HB}) \exp(-\beta A_{ij}^{HB})$$

Balance equation
Simultaneous nonlinear eqn.

Comparison – pure component parameters

EOS Models	Parameters	T Dependency	Procedure To Get Parameters
* Non-specific interaction			
PR-WS	a_i, b_i	a_i Dependent b_i Independent	Using critical const. and vapor pressure
SAFT	$r_i, \epsilon_i, \sigma_i, e$	Independent	Using vapor pressure and liquid density
NLF-HB	r_i, ϵ_{ii}	Dependent	Using vapor pressure and liquid density
* Specific interaction (Hydrogen bonding)			
PR-WS	-	-	-
SAFT	$\epsilon^{S_i Y_j}, \kappa^{S_i Y_j}$	Independent	Required for individual species and additional bond formation
NLF-HB	U^{HB}, S^{HB}	Independent	Depends on the type of hydrogen bond

Comparison – mixing rule and binary interaction parameters

	<i>PR-WS</i>	<i>SAFT</i>	<i>NLF-HB</i>
Basis of Mixing Rule	Based on quadratic composition dependence of B_{ij}	Vdw / VF mixing rule Results are affected by the choice of mixing rule	Do not require empirical mixing rule
No. of Binary Parameters	2 (UNIQUAC) 3 (NRTL) k_{ij} calculated	1 (k_{ij})	1 (k_{ij})
Procedure	Fit activity coeff. (obtain k_{ij})	Global Optimization of DP and DY	Global Optimization of DP and DY
Cross Association	0	Additional cross association parameters (2)	Can be assumed(2)

Procedures for Wong-Sandler Mixing Rule

□ Controlled Optimization

- *Using VLE data*
- *Using existing activity model parameters (DDB or UNIFAC, ...)*
- *Using infinite dilution activity coefficient*

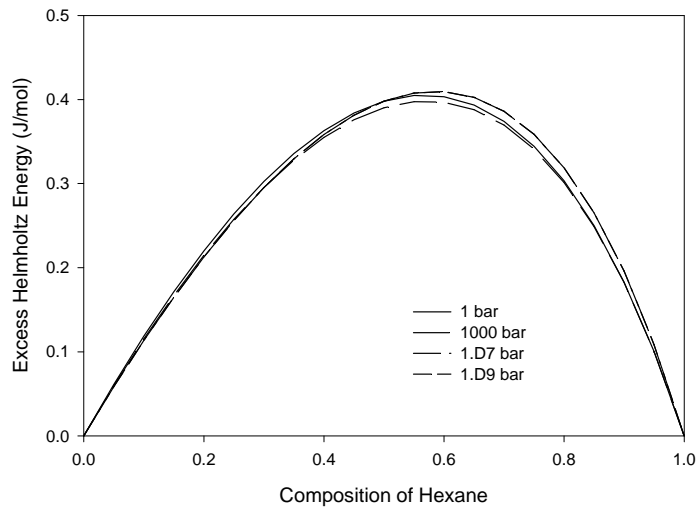
For given activity coefficient parameters , adjust k_{ij} to match following underlying assumption of Wong-Sandler mixing rule

$$G^{ex}(T, P = 1 \text{ bar}, x_i) \approx A^{ex}(T, P = 1 \text{ bar}, x_i) \approx A^{ex}(T, \text{high } P, x_i)$$

□ Global Optimization

- *Optimization of Gibbs free energy model parameter and k_{ij} value*
- *Can violate the basic assumption*

Sample Calculation – PR-WS can be inconsistent



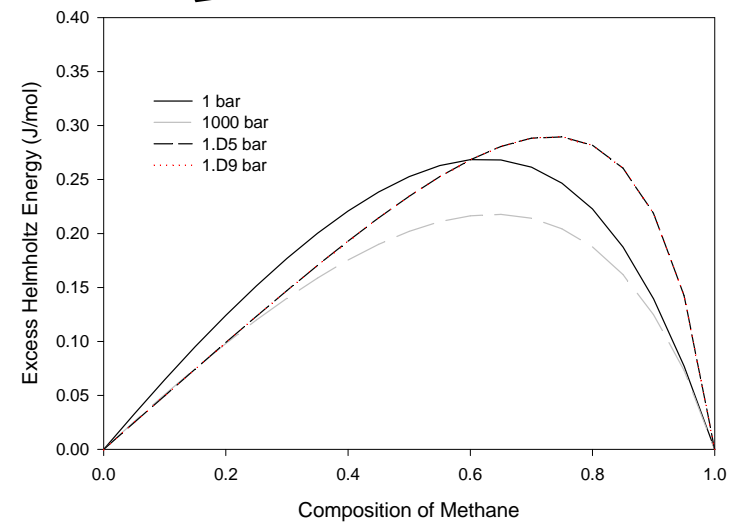
Plot of Excess Helmholtz energy for hexane + hexanol system

Controlled optimization

Fit A_{12} and A_{21}

Adjust k_{12} using $A^E = G^E$

G^E (at low P) are **unavailable** for mixtures of light gases at high P



Plot of Excess Helmholtz energy for methane + butane system

Global optimization

Fit A_{12} , A_{21} and k_{12}

Experimental Data for Comparison

	Source	No. of Data
Vapor pressure Liquid density	KDB (http://thermo.korea.ac.kr/kdb)	Total 58 Components
VLE data	Electronic version of DDB	34 systems 82 set 1321 points
Polymer Solubility	Hao et al. (DECHEMA)	4 systems 18 sets 205 points

Vapor Pressure Error

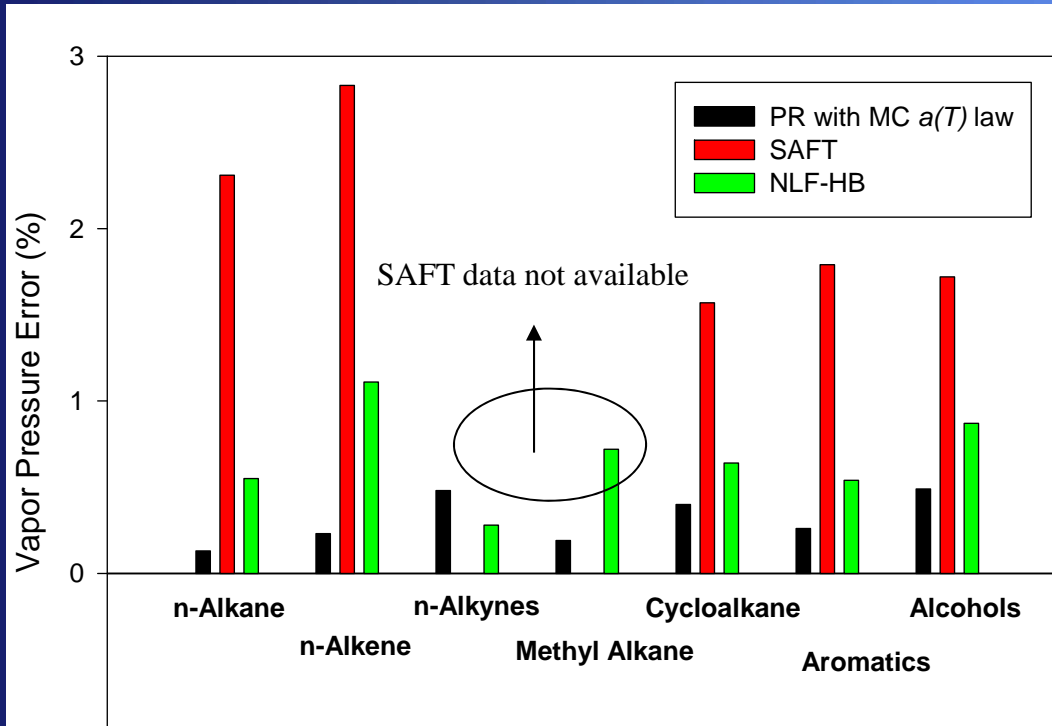


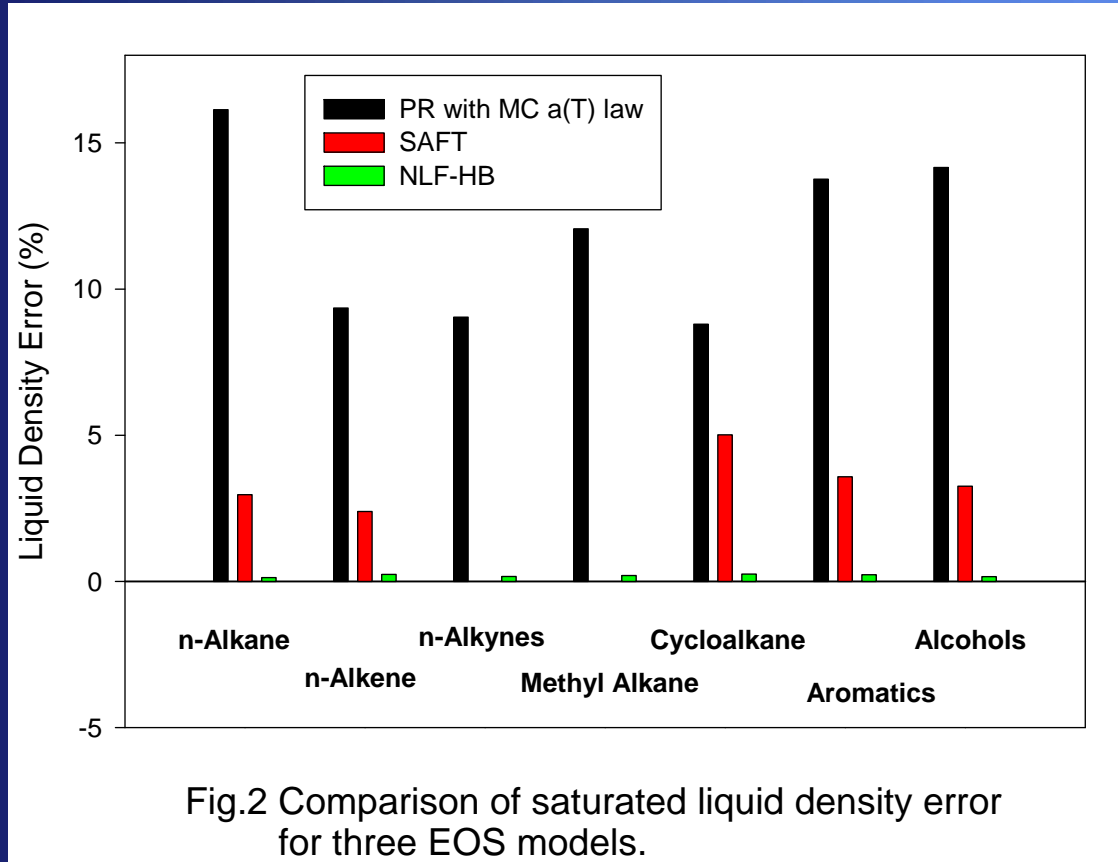
Fig.1 Comparison of vapor pressure error for three EOS models.

$$DP(\%) = \sum_i \sqrt{(P_i^{calc} - P_i^{exp})^2 / P_i^{exp2}}$$

- **PR-MC is most accurate with 3 T dependent parameters**
- **SAFT is most inaccurate due to T independent parameters**

	PR-MC	SAFT	NLF-HB
Average % Error	0.27	2.11	0.67

Liquid Density Error



$$D\rho(\%) = \sum_i \sqrt{(\rho_i^{calc} - \rho_i^{exp})^2 / \rho_i^{exp2}}$$

- **NLF-HB is most accurate with 3 T dependent size parameters**
- **PR-MC is most inaccurate because liquid density data were not fitted**

	PR-MC	SAFT	NLF-HB
Average % Error	5.27	3.19	0.19

Classification of components and objective function for VLE comparison

- ❑ **NONPOLAR COMPONENTS (NP)**
 - *Non-hydrogen bonding, low dipole moments*
 - *Methane, ethane, ...*
- ❑ **ASSOCIATING COMPONENTS (AS)**
 - *Self hydrogen bonding components*
 - *Alcohols, acid, amines, ...*
- ❑ **NON-ASSOCIATING COMPONENTS (NA)**
 - *Do not form self-association, weakly polar*
 - *Can form cross-association*
 - *Ketone, ether, ester, aldehyde,...*

$$OBJF = \sum_i \sqrt{(P_i^{calc} - P_i^{exp})^2 / P_i^{exp^2} + (y_i^{calc} - y_i^{exp})^2 / y_i^{exp^2}}$$

$$DP(\%) = \frac{1}{N} \sum_i |(P_i^{calc} - P_i^{exp})| / P_i^{exp} \times 100$$

$$DY = \sum_i |y_i^{calc} - y_i^{exp}| / N$$

System Index : VLE for NP + NP System at low P

□ NP + NP : Hexane + n-Alkane Systems

- *C7 : Hexane + Heptane*
- *C8 : Hexane + Octane*
- *C10 : Hexane + Decane*
- *C12 : Hexane + Dodecane*
- *C16 : Hexane + Hexadecane*

VLE of NP + NP System at low P (hexane + n-alkane)

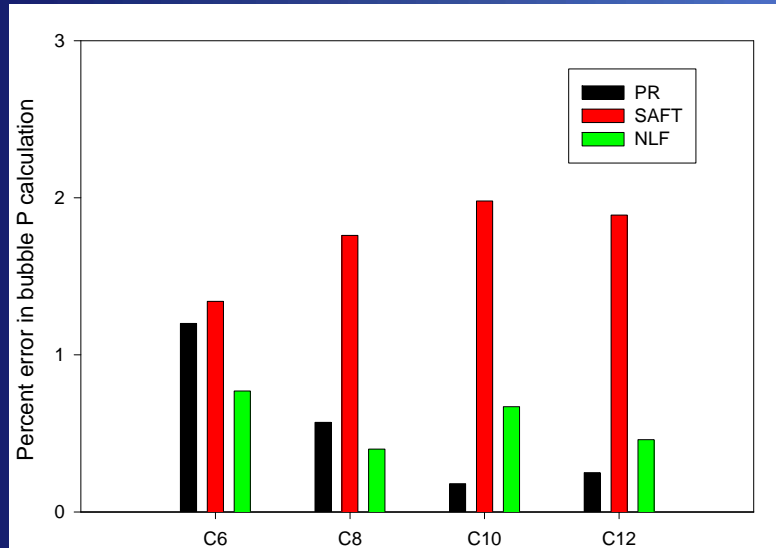


Fig.3 Comparison of error in bubble pressure calculation for NP + NP systems at low P (hexane + n-alkane)

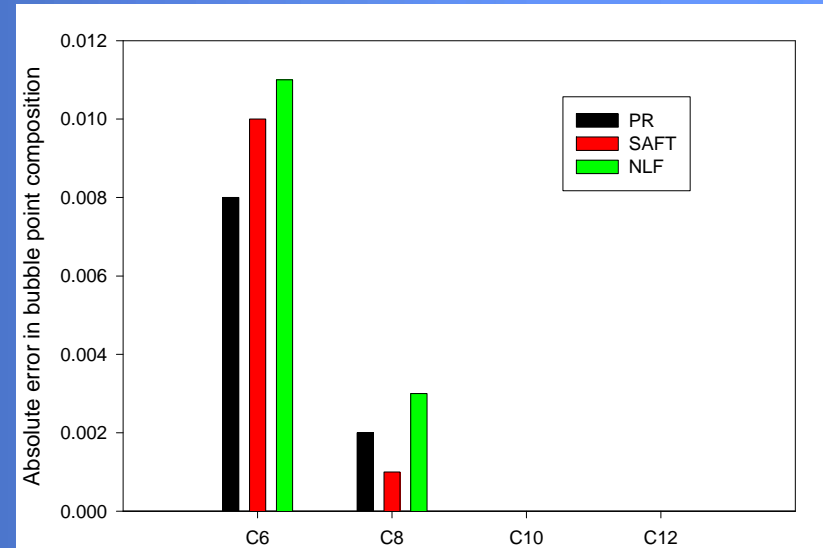


Fig.4 Comparison of error in bubble point composition calculation for NP + NP systems at low P (hexane + n-alkane)

	PR		SAFT		NLF	
	DP(%)	DY	DP(%)	DY	DP(%)	DY
Average	0.52	0.005	1.69	0.006	0.55	0.007

- All three EOS model shows good agreements with data ($DP < 1-2\%$, $DY < 0.01$)
- Although SAFT seems most inaccurate, most error is due to inaccuracy in the pure component vapor pressure

System Index : VLE for NP + AS System at low P

□ NP + AS : hexane + 1-alkanol

- *OL1 : hexane + methanol*
- *OL2 : hexane + ethanol*
- *OL3 : hexane + 1-propanol*
- *OL4 : hexane + 1-butanol*
- *OL5 : hexane + 1-pentanol*
- *OL6 : hexane + 1-hexanol*
- *OL7 : hexane + 1-heptanol*

VLE of NP + AS System at low P (hexane + 1-alkanol)

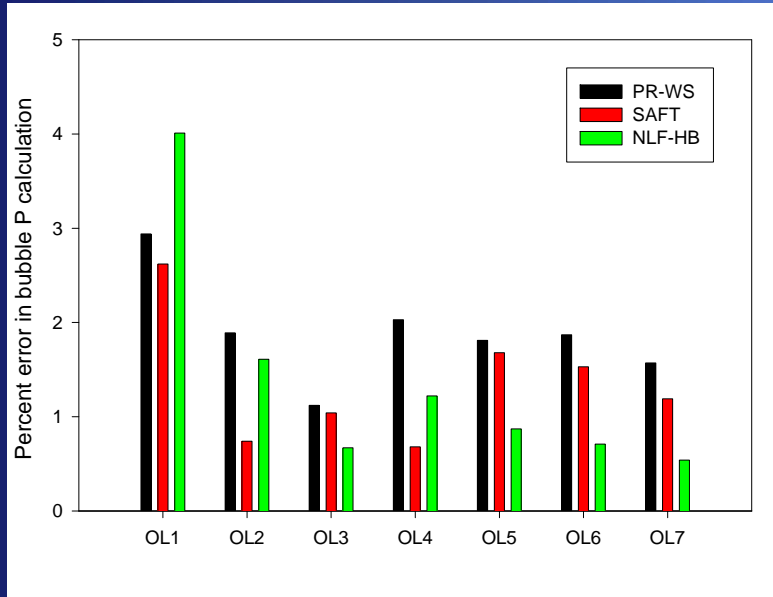


Fig.5 Comparison of error in bubble pressure calculation for NP + AS systems at low P (hexane + 1-alkanol)

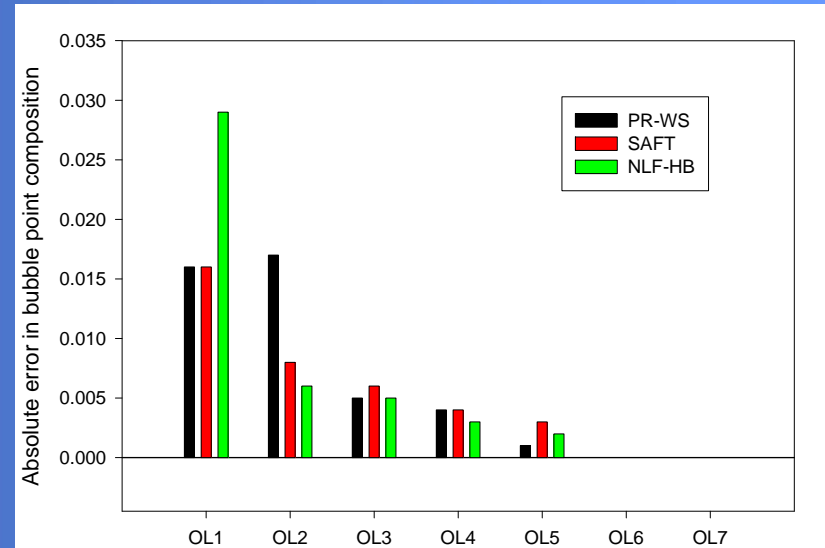


Fig.6 Comparison of error in bubble point composition calculation for NP + AS systems at low P (hexane + 1-alkanol)

	PR-WS		SAFT		NLF-HB	
	DP(%)	DY	DP(%)	DY	DP(%)	DY
Average	1.89	0.009	1.36	0.008	1.37	0.009

- All three EOS model shows good agreements with data ($DP < 1-2\%$, $DY < 0.01$)
- NLF-HB EOS shows better performance as the molecular weight of 1-alkanol becomes larger

System Index : VLE of NA + NA System at low P

□ NA + NA : Mixtures of ketone, ether, ester

- *DEE293 : acetone + diethyl ether at 293.15 K*
- *DEE303 : acetone + diethyl ether at 303.15 K*
- *MA308 : acetone + methyl acetate at 308.15 K*
- *MA328 : acetone + methyl acetate at 328.15 K*

VLE of NA + NA System at low P (ether, ketone, ester)

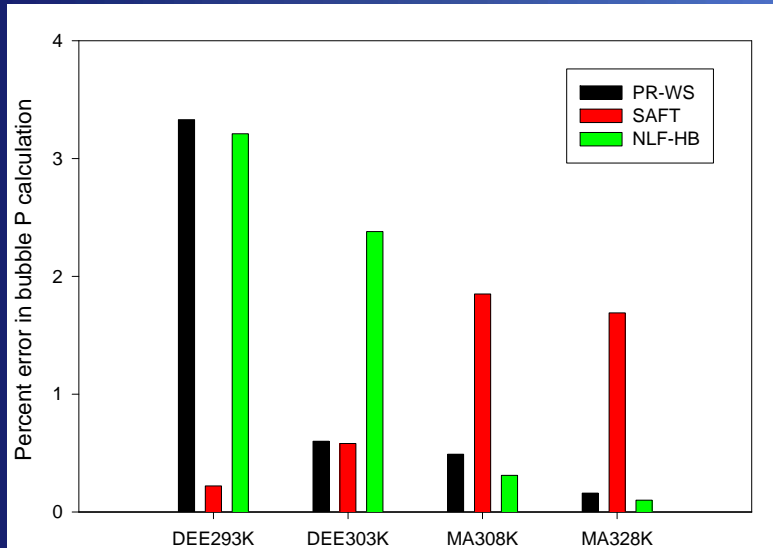


Fig.7 Comparison of error in bubble pressure calculation for NA + NA systems at low P (mixtures with acetone) < DEE : Diethyl ketone , MA : methyl acetate >

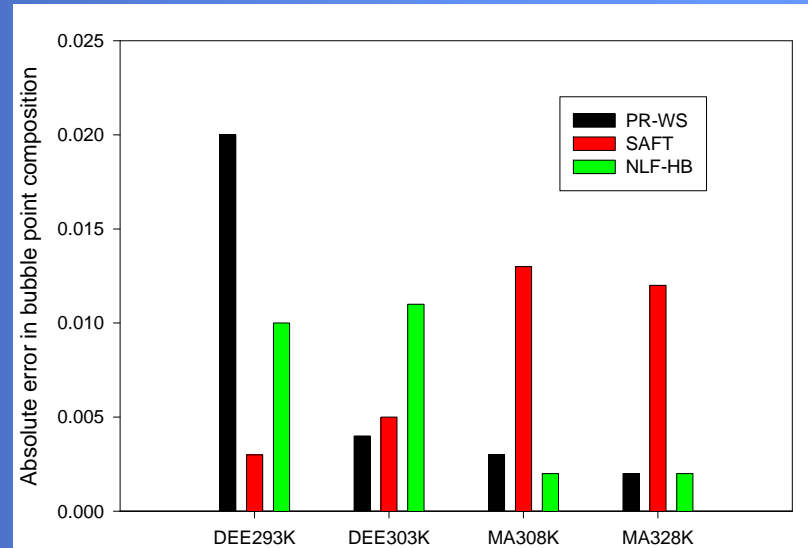


Fig.8 Comparison of error in bubble point composition calculation for NA + NA systems at low P (mixtures with acetone) < DEE : Diethyl ketone , MA : methyl acetate >

	PR-WS		SAFT		NLF-HB	
	DP(%)	DY	DP(%)	DY	DP(%)	DY
Average	1.15	0.007	1.08	0.008	1.50	0.006

- All three EOS model shows good agreements with data (DP < 1-2 %, DY < 0.01)
- VLE error is mostly determined from pure component property error (*very low vapor pressure*)

System Index : VLE of NA + AS System at low P

□ NA + AS : ketone, ether, ester + 1-alkanol

- 1 : acetone + methanol at 298.15 K
- 2 : acetone + methanol at 328.15 K
- 3 : acetone + ethanol at 305.15 K
- 4 : acetone + ethanol at 328.15 K
- 5 : diethyl ether + methanol at 303.15 K
- 6 : diethyl ether + ethanol at 298.15 K
- 7 : diethyl ether + ethanol at 313.15 K
- 8 : methyl acetate + methanol at 298.15 K
- 9 : methyl acetate + methanol at 308.15 K
- 10 : methyl acetate + ethanol at 323.15 K
- 11 : methyl acetate + ethanol at 333.15 K

VLE of NA+AS System at low P (1-alcohol + ketone, ether, esters)

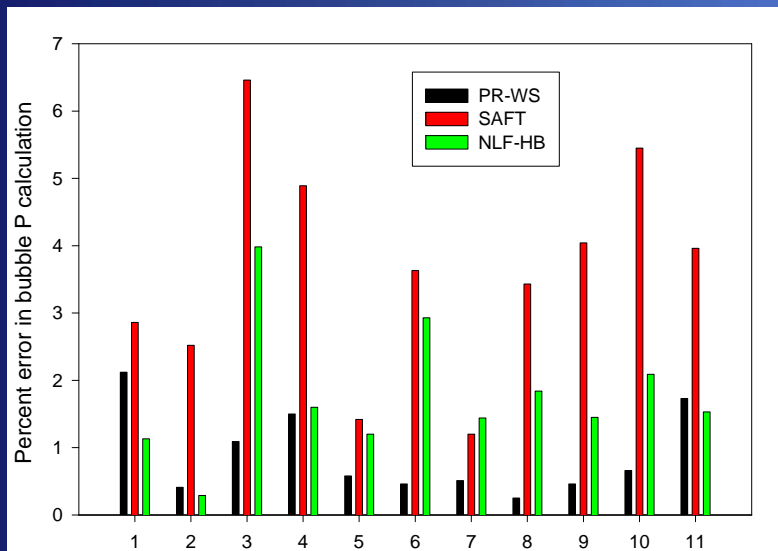


Fig.9 Comparison of error in bubble pressure calculation for AS + NA systems at low P

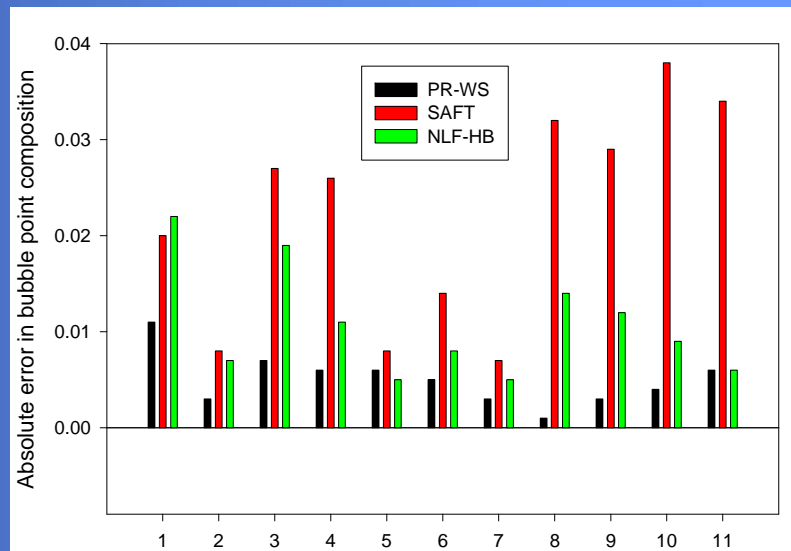


Fig.10 Comparison of error in bubble point composition calculation for AS + NA systems at low P

	PR-WS		SAFT		NLF-HB	
	DP(%)	DY	DP(%)	DY	DP(%)	DY
Average	0.89	0.005	3.62	0.022	1.77	0.011

- **SAFT and NLF-HB require additional parameters for cross-association**
- **While SAFT seems most inaccurate, the result can be further optimized using more refined cross-association parameters.**
- **NLF-HB shows good performance despite the simplification that cross association energies are the same as those of self association of alcohols**

System index : VLE of AS+AS systems at low P

□ AS + AS : mixtures of water, alcohol, acid

- *A : water + methanol at 298.15 K*
- *B : water + methanol at 308.15 K*
- *C : water + methanol at 323.15 K*
- *D : water + methanol at 373.15 K*
- *E : water + methanol at 423.15 K*
- *F : water + ethanol at 298.15 K*
- *G : water + ethanol at 312.91 K*
- *H : water + ethanol at 347.94 K*
- *I : water + ethanol at 413.15 K*
- *J : water + 1-propanol at 303.15 K*
- *K : water + 1-propanol at 333.15 K*
- *L : water + 1-propanol at 352.95 K*
- *M : water + acetic acid at 293.15 K*
- *N : water + acetic acid at 298.15 K*
- *O : water + acetic acid at 303.15 K*

VLE of AS + AS System at low P (mixtures of water, alcohol, acid)

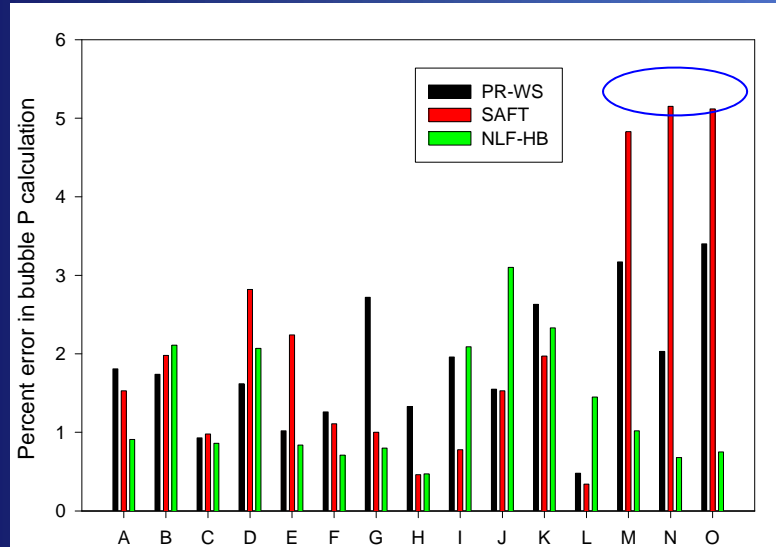


Fig.11 Comparison of error in bubble pressure calculation for AS + AS systems at low P (water, alcohol, acids)

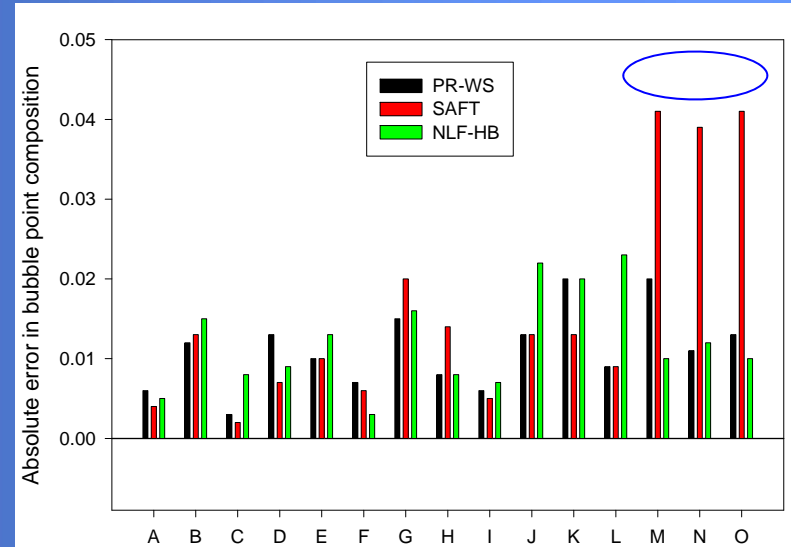


Fig.12 Comparison of error in bubble point composition calculation for AS + AS systems at low P (water, alcohol, acids)

	PR-WS		SAFT		NLF-HB	
	DP(%)	DY	DP(%)	DY	DP(%)	DY
Average	1.84	0.011	2.12	0.016	1.35	0.012

- All three EOS model shows good agreements with data ($DP < 1-2\%$, $DY < 0.01$)
- NLF-HB shows good result for water + acid systems using the **extended hydrogen bonding theory** (Park, Kang, Yoo and Lee, 2001)

Extended hydrogen bonding theory for dimers

- ❑ Extended lattice statistics considering dimerization for acids
- ❑ Presented at PPEPPD 2001 symposium (poster session)
- ❑ New partition function proposed for dimerization

$$g_D = \frac{N_D^1!}{N_{10}^D!(2!)^{N_{11}^H} N_{11}^H!}$$

- ❑ Generalized treatment of dimerization and normal association

VLE of water + acetic acid system

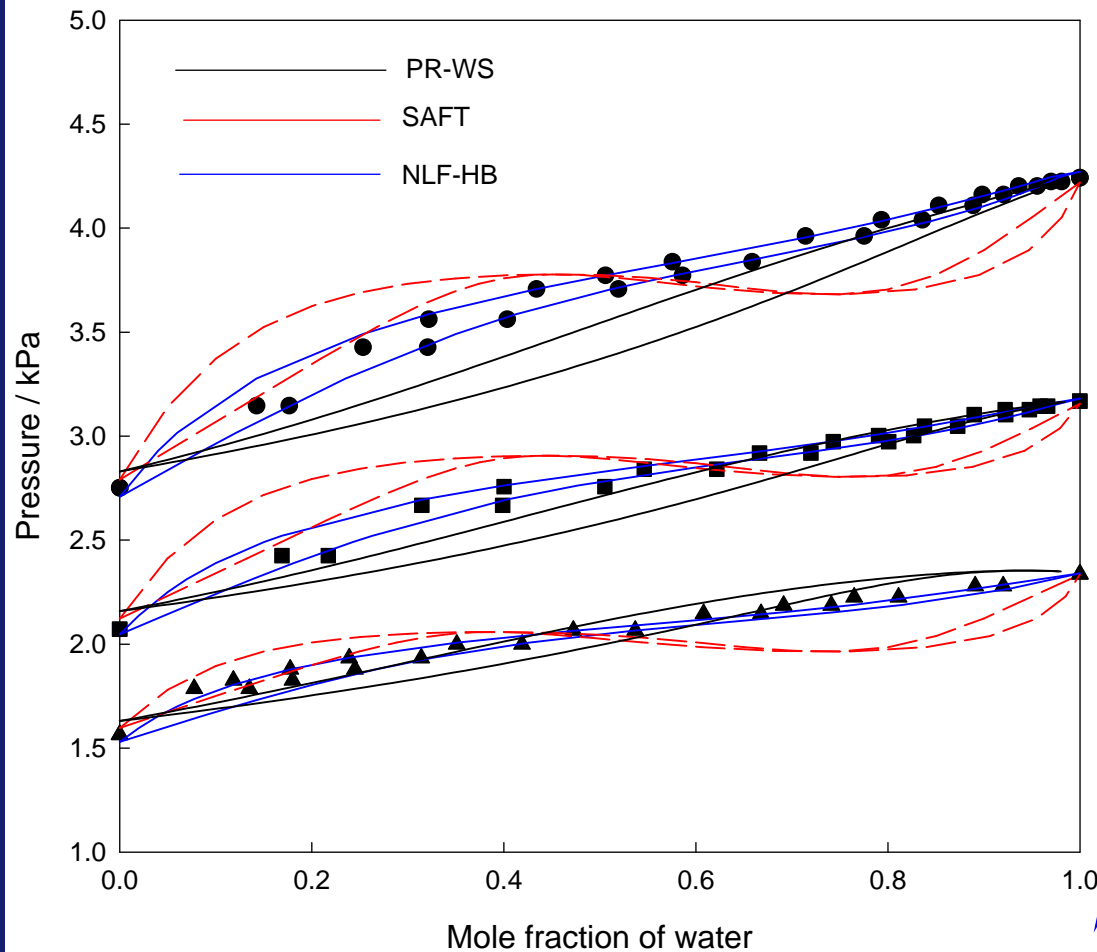


Fig.4 Comparison of Experimental Result with Calculation for Water + Acetic Acid System

• **Very complex behavior of system**

- **Self Association (water)**
- **Dimerization (acid)**
- **Cross Association (acid-water)**

• **NLF-HB shows most accurate result using new extended HB theory**

VLE of NP + NP systems at high P

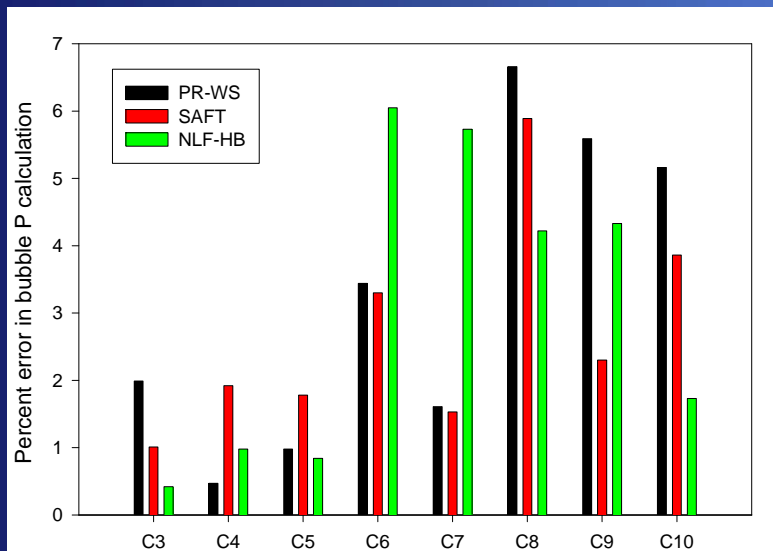


Fig.13 Comparison of error in bubble pressure calculation for NP + NP systems at high P (methane + n-alkane)

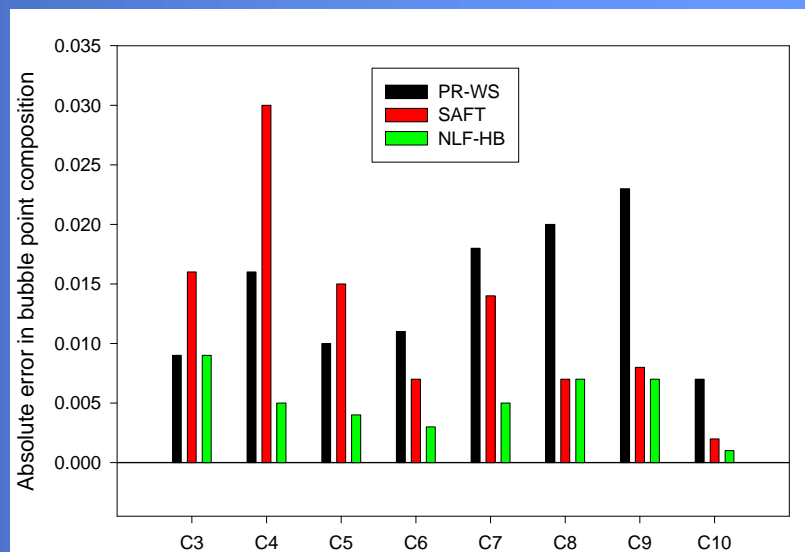


Fig.14 Comparison of error in bubble point composition calculation for NP + NP systems at high P (methane + n-alkane)

	PR-WS		SAFT		NLF-HB	
	DP(%)	DY	DP(%)	DY	DP(%)	DY
Average	3.24	0.014	2.70	0.012	3.04	0.005

- All the results were greatly influenced by the pure component parameters outside the normal range (supercritical condition)
- SAFT shows good result using the more parameter for light molecules (e)

NP + AS systems at high P (CO₂ + alcohol)

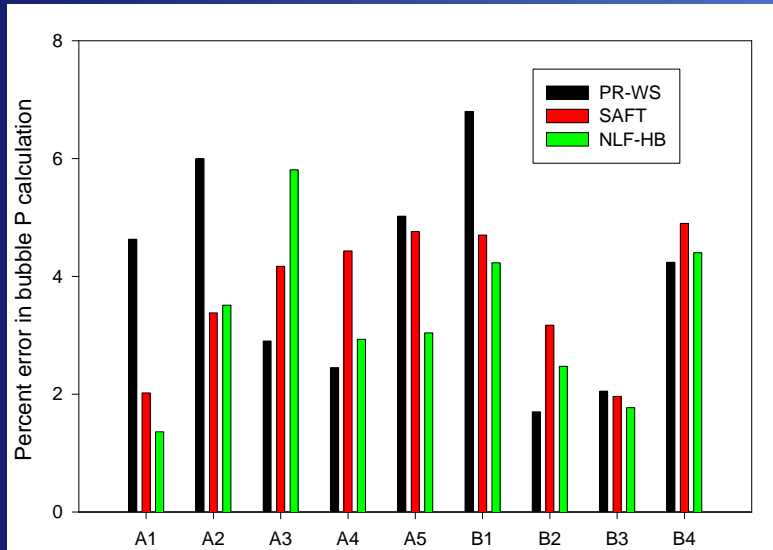


Fig.15 Comparison of error in bubble pressure calculation for NP + AS systems at high P (carbon dioxide + alcohol) < A: CO₂ + methanol / B : CO₂ + ethanol >

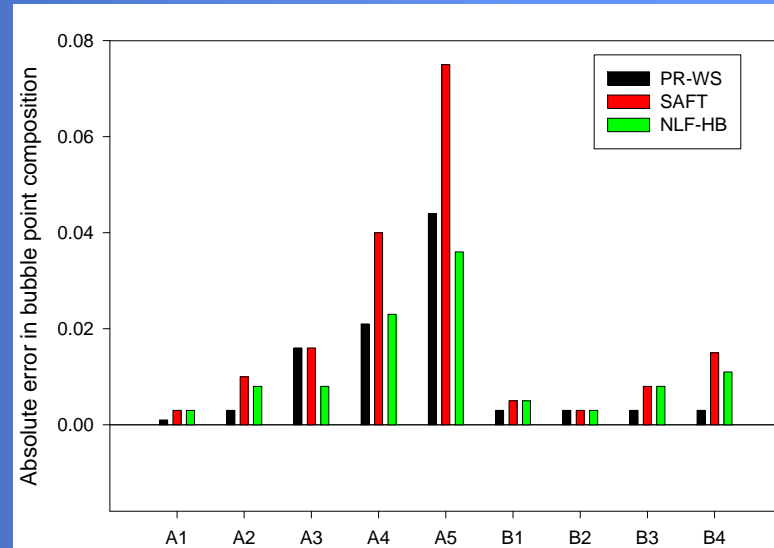
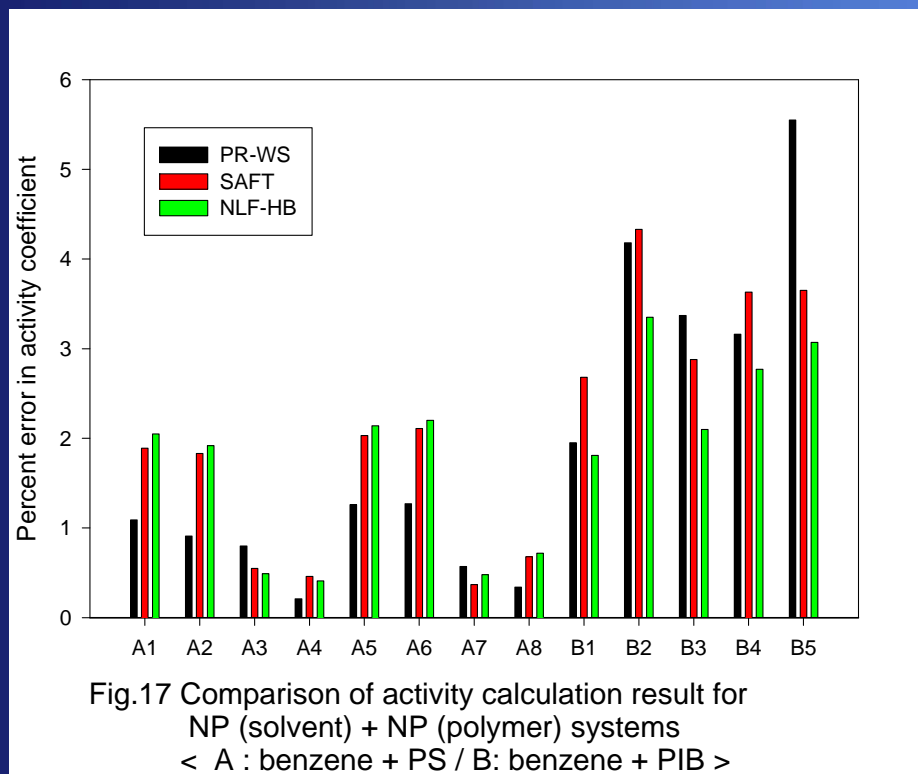


Fig.16 Comparison of error in bubble point composition calculation for NP + AS systems at high P (carbon dioxide + alcohol) < A: CO₂ + methanol / B : CO₂ + ethanol >

	PR-WS		SAFT		NLF-HB	
	DP(%)	DY	DP(%)	DY	DP(%)	DY
Average	3.98	0.011	3.72	0.019	3.28	0.012

- **Similar trend as the previous slide Calculation failed for PR-WS**
- **SAFT and NLF-HB shows inaccurate result**

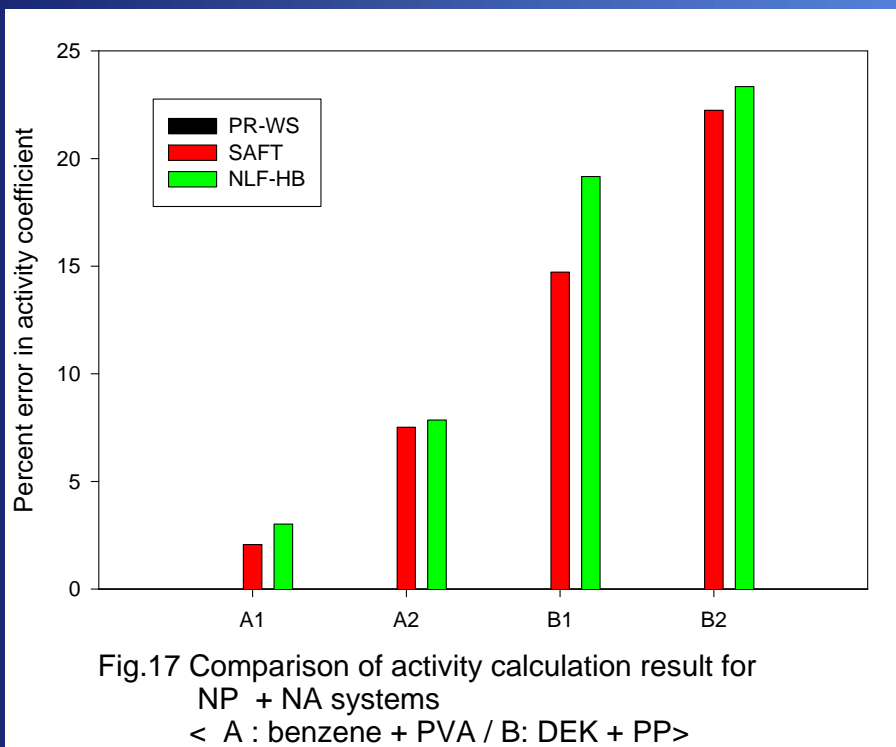
Polymer Solubility for NP + NP mixtures



• *All three EOS model shows good agreement with data*

	PR-WS	SAFT	NLF-HB
Average % Error	1.90	2.08	1.81

Polymer solubility for AS + NA mixtures



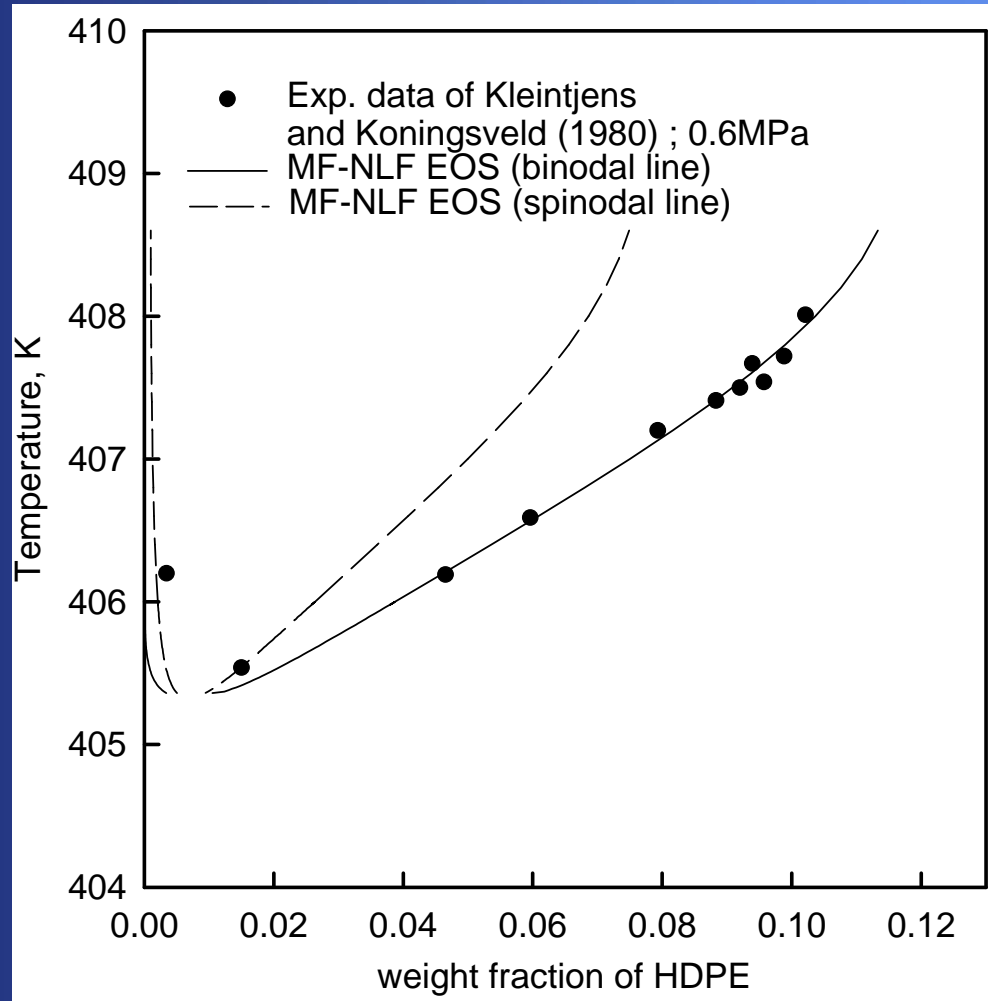
- *Calculation failed for PR-WS*
- *SAFT and NLF-HB shows inaccurate result*

	PR-MC	SAFT	NLF-HB
Average % Error	-	13.6	15.8

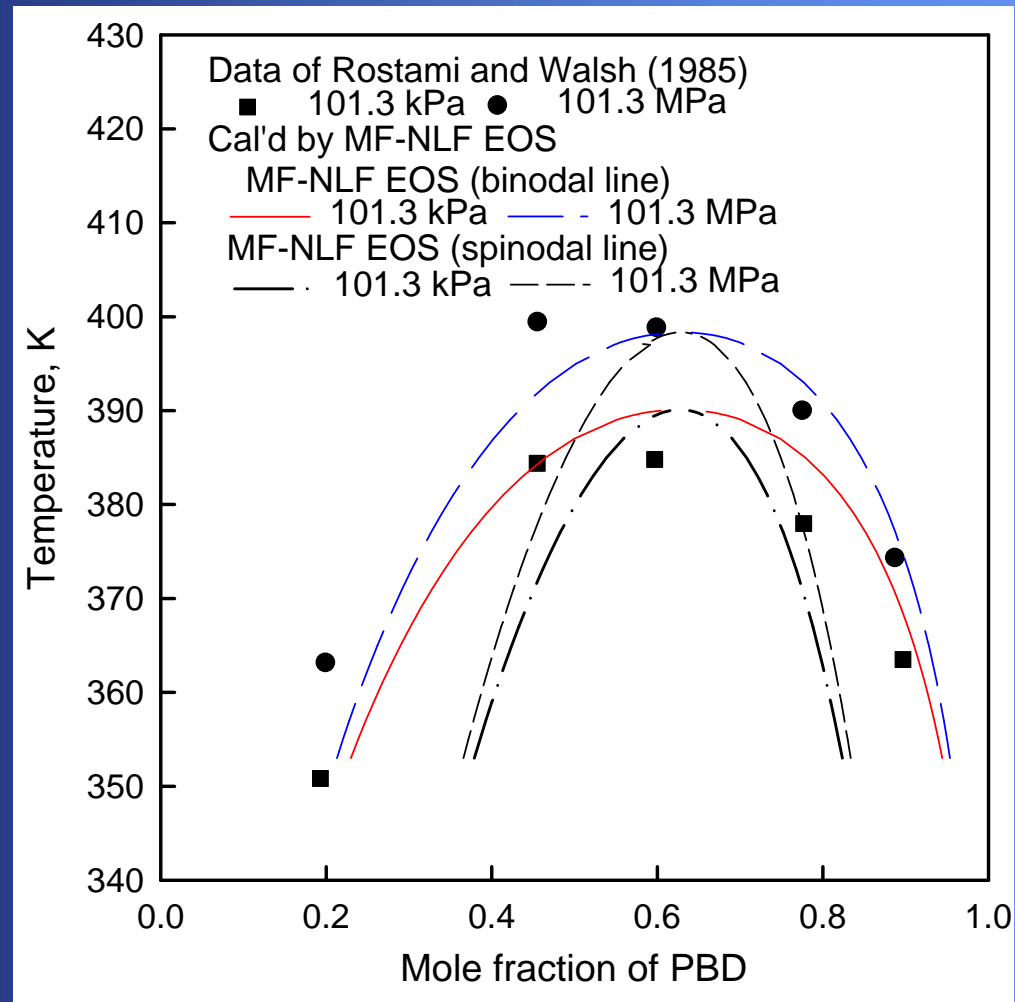
Polymer solubility calculations using PR-WS EOS

- **Result of Orbey and Sandler (AIChE J. 1994)**
 - *k_{ij} value range : 0.77 to 0.97 for NP + NP systems*
- **k_{ij} value failed to match underlying assumption within meaningful range (-1 to 1) of binary parameters for NP + AS polymer solubility**

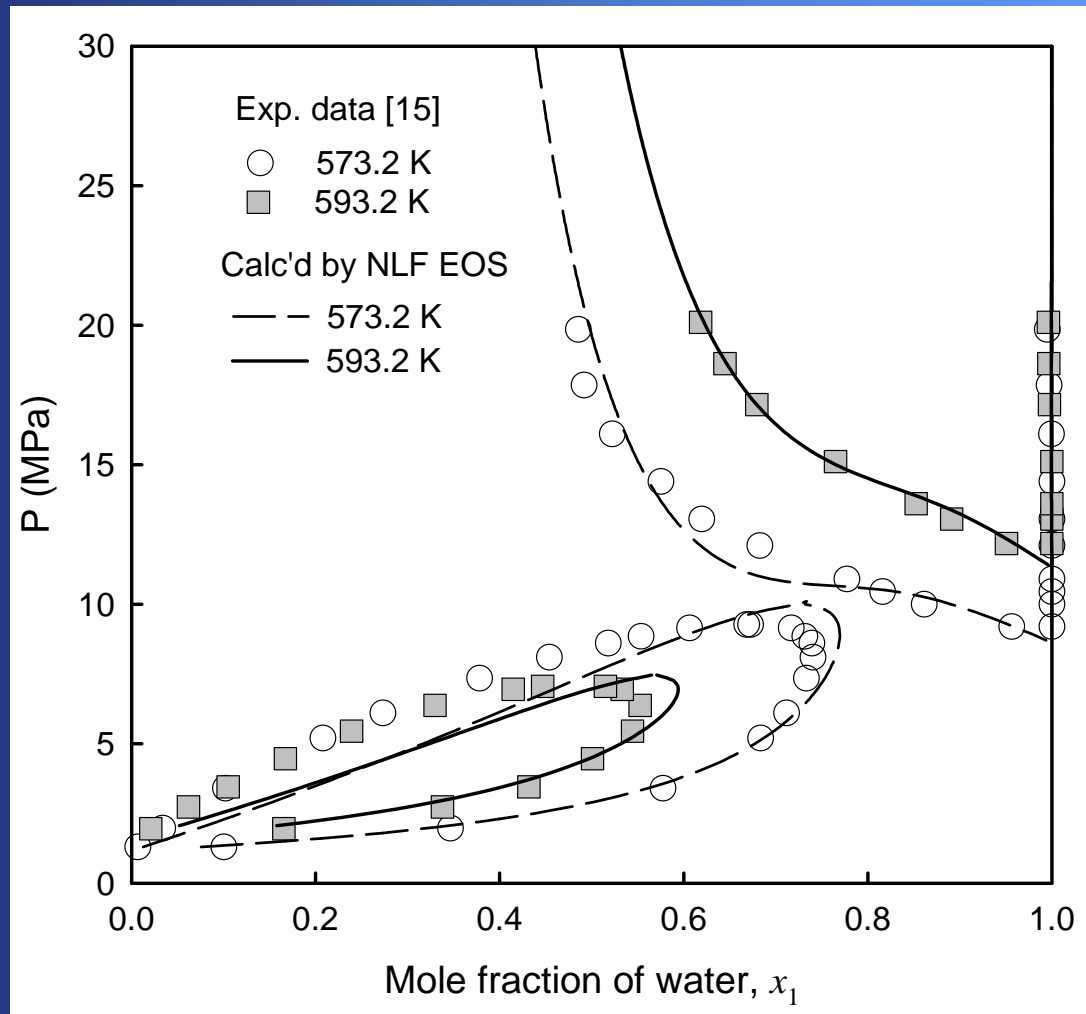
Complex phase behavior - LLE calculation for HDPE + hexane



Complex phase behavior – LLE of PB + PS blends



Phase behavior of water + decane system



Computational Aspects

	PVT Behavior	Computation speed	Remark
PR-WS	Simple	Fast	Can violate basic assumption using some parameter sets.
SAFT	Complex	Slow (10 times NLF-HB)	Extra computation time for balance eqn. solving when two or more types of association
NLF-HB	Relatively simple	Relatively fast (4 time PR-WS)	Extra computation time for balance eqn. solving when two or more types of association

Sensitivity of Binary Interaction Parameters

$$\frac{dP}{dk_{ij}} = \frac{P(1.01k_{ij}) - P(0.99k_{ij})}{0.02k_{ij}}$$

$$\frac{dY}{dk_{ij}} = \frac{y_1(1.01k_{ij}) - y_1(0.99k_{ij})}{0.02k_{ij}}$$

• **All the EOS model have small derivatives with respect to parameters -> robust behavior of binary interaction parameters**

System	EOS	Temperature	dP/dKij	dY/dKij
Hexane + Hexadecane	PR -WS	293.15	0.04	0.00003
	NLF -HB	293.15	0.62	0.00152
	SAFT	293.15	0.57	0.00092
Hexane + 1 -Hexanol	PR -WS	293.15	0.30	0.02600
	NLF -HB	293.15	0.69	0.08100
	SAFT	293.15	0.69	0.00699
Methanol + Water	PR -WS	298.15	0.22	1.52000
	NLF -HB	298.15	0.15	1.09000
	SAFT	298.15	0.26	1.91000

Error increase due to T difference in VLE sets

- System : *n*-Hexane + 1- Propanol

T (deg.C)	K _{ij}	PR-WS		SAFT		NLF-HB	
		DP	DY	DP	DY	DP	DY
25	Optimum	1.82	0.0044	1.33	0.002	1.14	0.003
75	Optimum	2.97	0.0177	1.74	0.010	2.92	0.012
75	Value at 25 deg.C	3.27	0.0151	3.85	0.023	2.99	0.019
Increase in error		0.30	-0.0026	2.11	0.013	0.07	0.007

- All three EOS models do not significantly affected by the temperature dependency of parameters
- Among 3 EOS models, SAFT EOS is most sensitive for this specific system

Conclusion

- ❑ Pure Properties (58 comp.s) and VLE (82 isotherms) were compared for PR-WS, SAFT and NLF-HB EOS.
- ❑ For pure components, NLF-HB gives best result due to temperature dependent pure component parameters.
- ❑ For VLE calculation most EOS considered showed similar degree of prediction.
- ❑ For AS + AS systems, new extended HB model shows better result than the other two EOS models
- ❑ For computational aspect, PR-WS EOS is fast and easy, while SAFT is most complex and time-consuming.
- ❑ All three EOS model showed robust result with respect to change in interaction parameters

Kyong Hoe Ru - Pavillion



Faces of Ancient Korean



*Korea-Japan 2002 World Cup
(who will be the winner ?)*



*Ancient statue
Bodhisattva
(A.D. 6 to 7)*