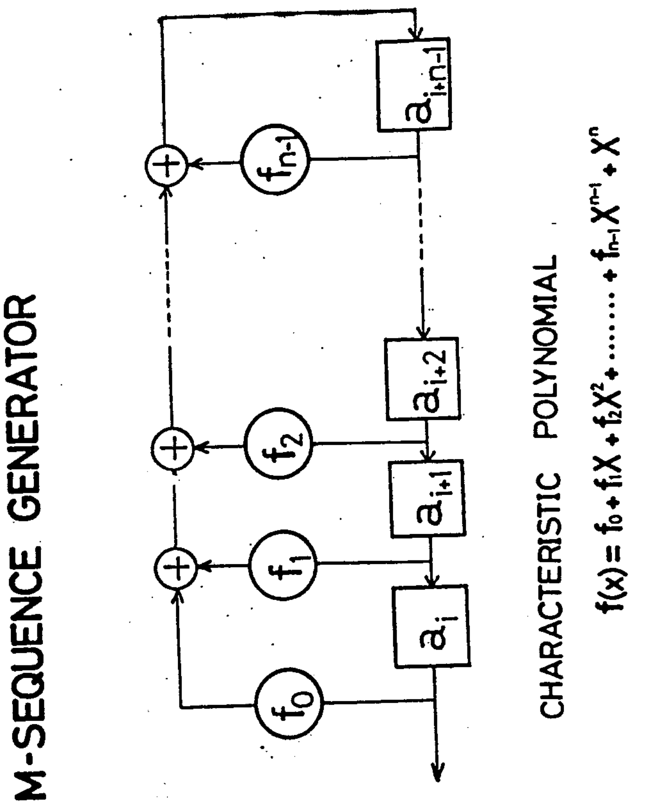
(1)

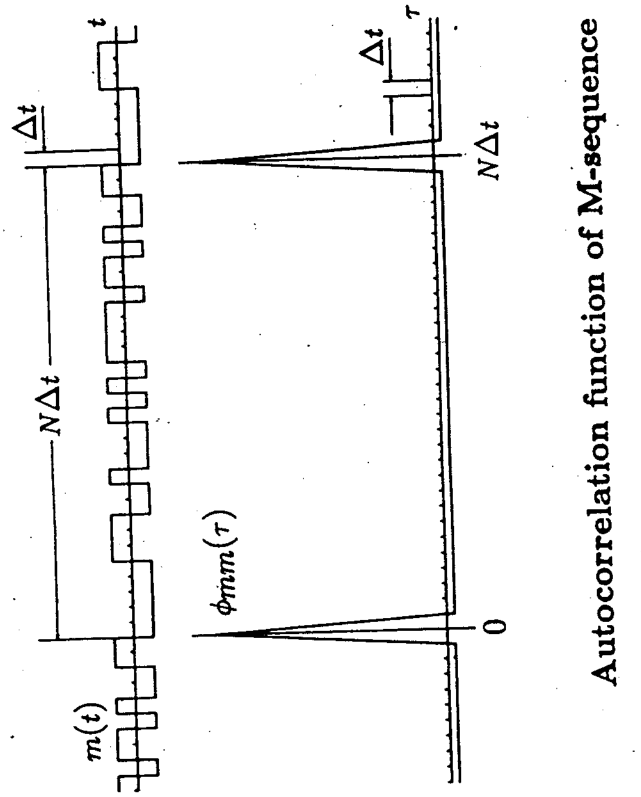
Applications of M-sequence

1. Delay time measurement
2. Random number generation
3. Information transmission
4. 2D Positioning
5. Fault detection of logical circuit
6. Fault detection of RAM
7. Linear system identification
8. Nonlinear system idenfication
9. M-transform
   9.1 Application to signal processing
   9.2 Applicaionts to image processing

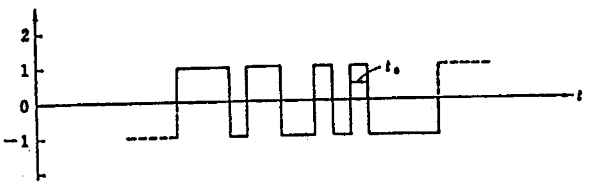
(2)



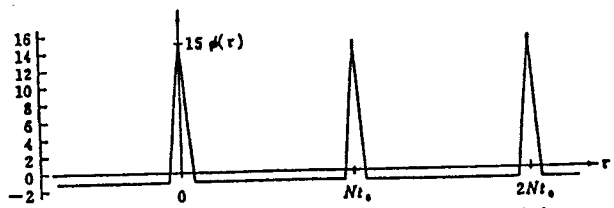
(3)



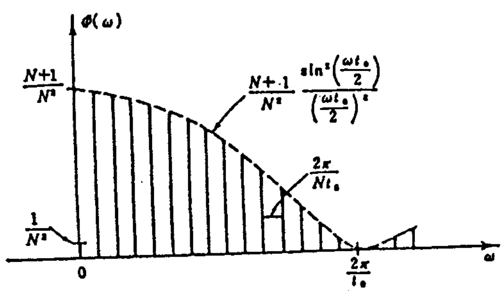
(4)



(a) Waveform of M-sequence



(b) Autocorrelation function $\phi(\tau)$

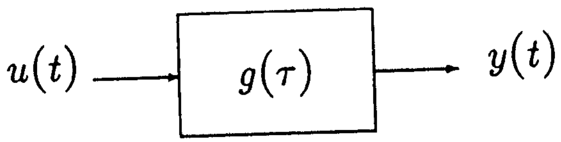


(c) Power Spectrum $\Phi(\omega)$

outline

- Identification of Volterra Kernel of nonlinear processes
- Model Predictive Control
- Model Predictive Control of nonlinear processes by use of Volterra Kernel model
- Simulation

Nonlinear Dynamical System



$$y(t) = \int_0^\infty \underline{g_1(\tau_1)}u(t-\tau_1)d\tau_1 + \int_0^\infty\int_0^\infty \underline{g_2(\tau_1,\tau_2)}u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2 + \int_0^\infty\int_0^\infty\int_0^\infty \underline{g_3(\tau_1,\tau_2,\tau_3)}u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \cdots$$

(c) When a k-shifted version of $\{a_i\}$ is denoted by $\{a_{i+k}\}$, there exists a unique j (mod N) such that

$$\{a_i + a_{i+k}\} = \{a_{i+j}\}$$

This property is called the shift and add property of M-sequences. In general, there exists a unique v such that

$$s_1 a_{i-1} + s_2 a_{i-2} + \cdots + s_n a_{i-n} = a_{i+v}$$

where $s_1, s_2, \cdots, s_n \in$ GF(2).

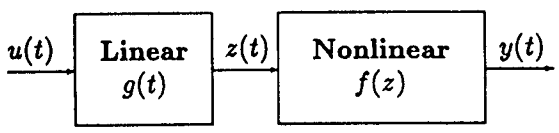
0, 1

0+0=0 0·0=0
0+1=1 0·1=0
1+0=1 1·0=0
1+1=0 1·1=1

Abstract

This paper proposes a new method of Model Predictive Control (MPC) of nonlinear process by using the measured Volterra kernels as the nonlinear model. A pseudo-random M-sequence is applied to the nonlinear process, and its output is measured. Taking the crosscorrelation between the input and output, we obtain the Volterra kernels up to 3rd order. By using the measured Volterra kernels, we construct the nonlinear model for MPC.The result of computer simulation show a good result for nonlinear chemical process.

An example of nonlinear system



When $f(z) = z + z^2 + z^3 + \cdots,$

$$g_1(\tau_1) = g(\tau_1)$$
$$g_2(\tau_1, \tau_2) = g(\tau_1)g(\tau_2)$$
$$g_3(\tau_1, \tau_2, \tau_3) = g(\tau_1)g(\tau_2)g(\tau_3)$$

......

Volterra kernel estimation

The crosscorrelation function between the input and output are written as

$$\begin{aligned}\phi_{uy}(\tau) &= \overline{u(t-\tau)y(t)} \\ &= \sum_{i=1}^{\infty}\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} g_i(\tau_1, \tau_2, \cdots, \tau_i) \\ &\times \overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)}d\tau_1 d\tau_2 \cdots d\tau_i\end{aligned}$$

Where $\overline{\quad\quad}$ denotes time average

When the input $u(t)$ is a two valued M-sequence (+1 or −1) of degree n,

$$\overline{u(t-\tau)u(t-\tau_1)u(t-\tau_2)\cdots u(t-\tau_i)} = \begin{cases} 1 & \text{(for certain } \tau) \\ -1/N & \text{(otherwise)} \end{cases} \quad (1)$$

Shift and add property of the M-sequence:
For any integer $k_{i1}, k_{i2}, \cdots k_{i,i-1}$ (suppose $k_{i1} < k_{i2} < \cdots, k_{ii}$), there exists a unique $k_{ii}(modN)$ such that

$$u(t)u(t+k_{i1})u(t+k_{i2})\cdots u(t+k_{i,i-1}) = u(t+k_{ii}) \quad (2)$$

Therefore Eq.(1) becomes unity when

$$\tau_1 = \tau - k_{i1}, \tau_2 = \tau - k_{i2}, \cdots \tau_i = \tau - k_{ii} \quad (3)$$

Therefore we have

$$\begin{aligned}\phi_{uy}(\tau) &= \Delta t g_1(\tau) + (\Delta t)^3 g_3(\tau, \tau, \tau) \\ &+3(\Delta t)^3 \sum_{q=1}^{m_1} g_3(\tau, q, q) \\ &+2(\Delta t)^2 \sum_{j=1}^{m_2} g_2(\tau - k_{21}^{(j)}, \tau - k_{22}^{(j)}) \\ &+6(\Delta t)^3 \sum_{j=1}^{m_3} g_3(\tau - k_{31}^{(j)}, \tau - k_{32}^{(j)}, \tau - k_{33}^{(j)})\end{aligned}$$

Volterra kernel of every order from the crosscorrelation function can be obtained as follows

Volterra kernel of 2nd order

$$\sum_{i_a=1}^{m_a} g_2(\tau - h_{i_a}, \tau - k_{i_a}) = \frac{\phi_{uy}(\tau)}{2(\Delta t)^2}$$

Volterra kernel of 3rd order

$$\sum_{i_b=1}^{m_b} g_3(\tau - m_{i_b}, \tau - n_{i_b}, \tau - s_{i_b}) = \frac{\phi_{uy}(\tau)}{6(\Delta t)^3}$$

Volterra kernel of 1st order

$$g_1(\tau) = \frac{\phi_{uy}(\tau)}{\Delta t} - (\Delta t)^2 (3 \sum_{i_b=0}^{m_c} g_3(\tau, i_b, i_b) - 2 g_3(\tau, \tau, \tau))$$

Selection of M-sequence

$$\phi_{uy}(\tau) = g(\tau) + 2 \sum_{i=1}^{m} g_2(\tau - h_i, \tau - k_i)$$

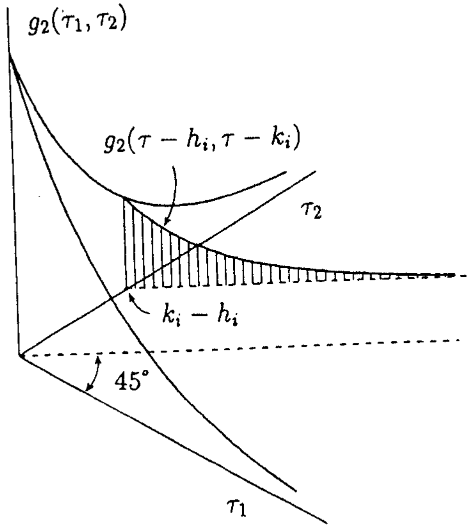
In order to obtain $g_2(\tau_1, \tau_2)$ from this equation, h_i and k_i must be sufficiently apart from each other.

Assumption: $g_2(\tau_1, \tau_2) \simeq 0$ for $\tau_1, \tau_2 > 50\Delta t$

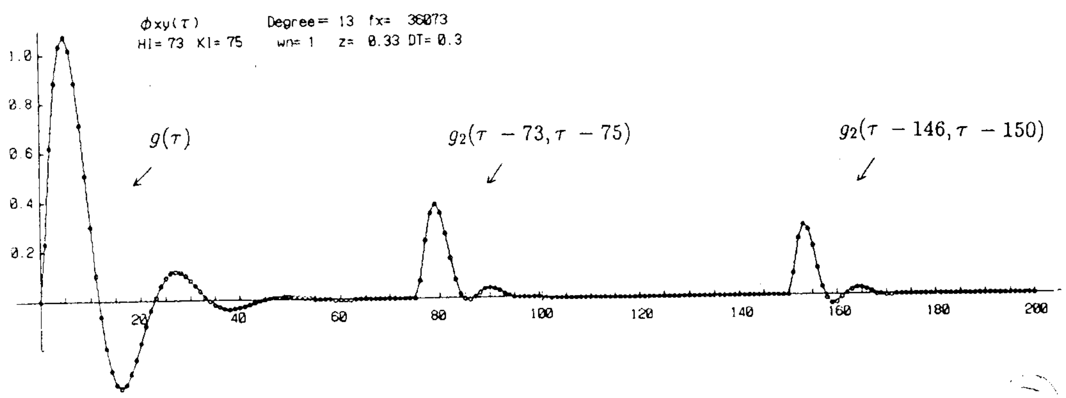
We have searched all primitive polynomials over GF(2) up to 15 degrees(total 3664 polynomials) to find those M-sequences for $k_i < 300$.

M-sequences suitable for identifying Volterra kerenels of nonlinear system having up to 3rd order Volterra kernels

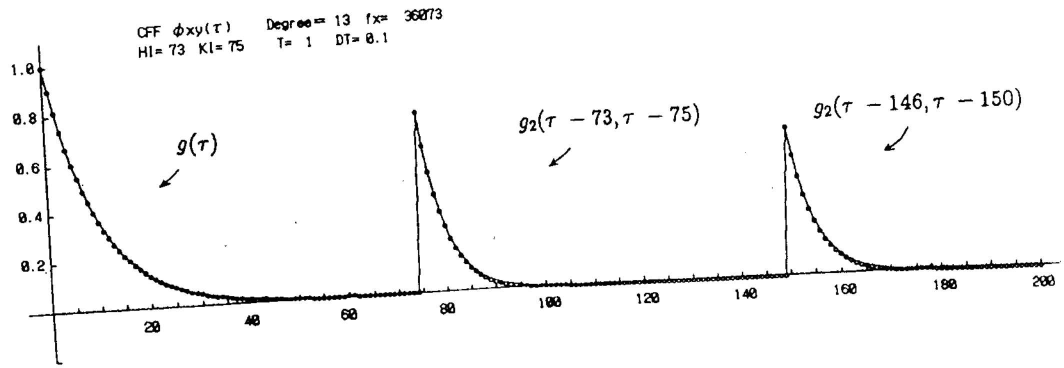
n	f(x) (in octal notation)
18	1002623
18	1013471
18	1057043
18	1116535
18	1146461
18	1174075
18	1207077
18	1215757
18	1237423
18	1244153
18	1320543
18	1327245
18	1334575
18	1343105
18	1403675
18	1407521
18	1430733
18	1435155
18	1444777
18	1503071
18	1515155
18	1530225
19	2227023
19	2001711
19	2766447
21	10040315
21	10000635
21	10103075
21	10002135
22	20401207
22	20430607



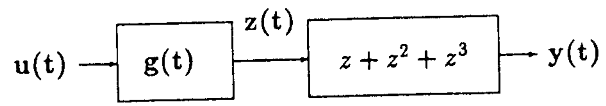
Simulation result on 2nd order Volterra kernal measurement when $g(\tau)$ is of second order



Simulation result on 2nd order Volterra kernel measurement when $g(\tau)$ is of first order



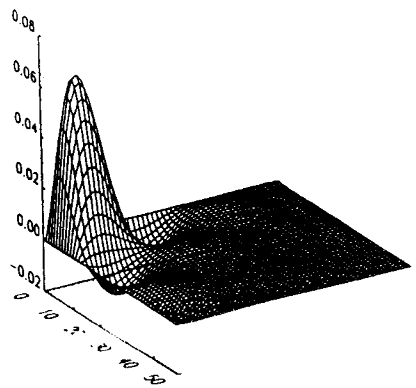
Measurement of Volterra Kernels up to 3rd Order



$$y(t) = z(t) + z^2(t) + z^3(t)$$
$$= \int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1 + \{\int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1\}^2 + \{\int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1\}^3$$
$$= \int_0^\infty g(\tau_1)u(t-\tau_1)d\tau_1 + \int_0^\infty \int_0^\infty g(\tau_1)g(\tau_2)u(t-\tau_1)u(t-\tau_2)d\tau_1 d\tau_2$$
$$+ \int_0^\infty \int_0^\infty \int_0^\infty g(\tau_1)g(\tau_2)g(\tau_3)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)d\tau_1 d\tau_2 d\tau_3$$

Volterra kernels are

$$g_1(\tau_1) = g(\tau_1)$$
$$g_2(\tau_1, \tau_2) = g(\tau_1)g(\tau_2)$$
$$g_3(\tau_1, \tau_2, \tau_3) = g(\tau_1)g(\tau_2)g(\tau_3)$$



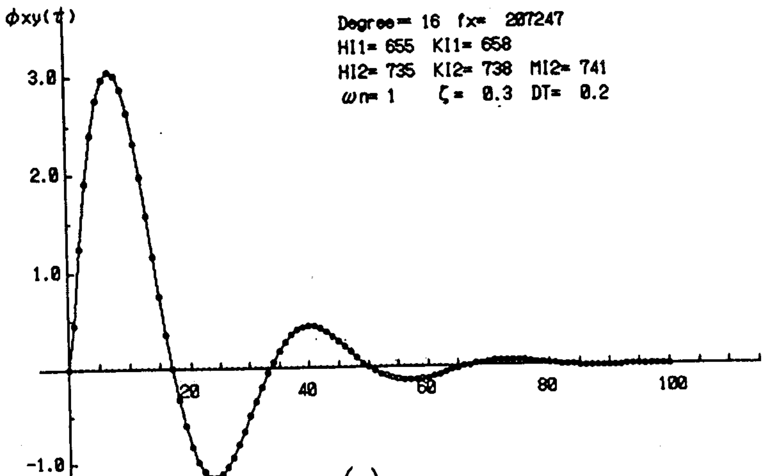
(21)

When we use the M-sequence having the characteristic polynomial of $f(x) = 207247$ (16th degree, in octal notation)

$$k_{21} = 655, \; k_{22} = 658$$
$$k_{31} = 735, \; k_{32} = 738, \; k_{33} = 741$$

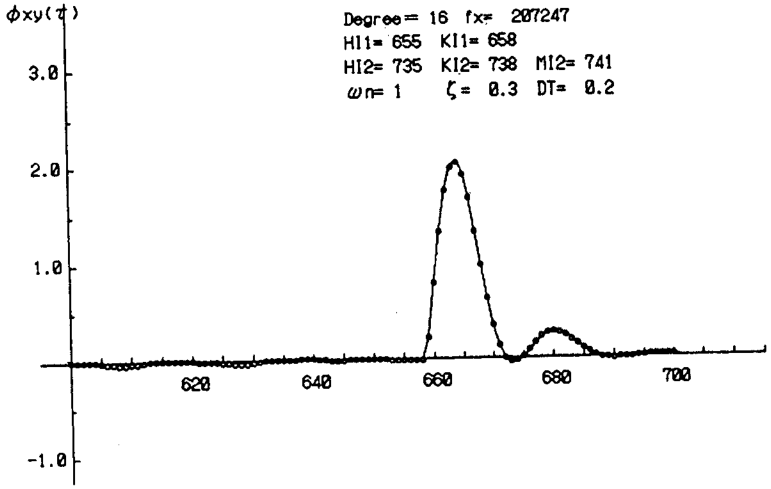
$$\phi_{uy}(\tau) = g(\tau) + g_2(\tau - 655, \tau - 658) + g_3(\tau - 735, \tau - 738, \tau - 741) + \cdots$$

(22)

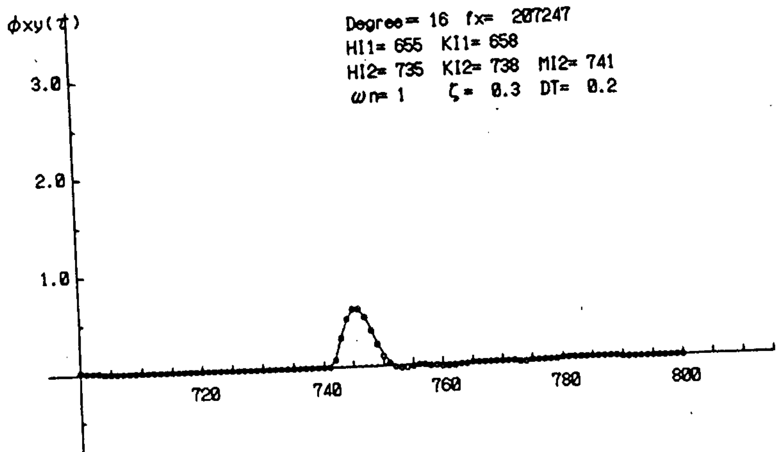


$g_1(\tau)$

(23)



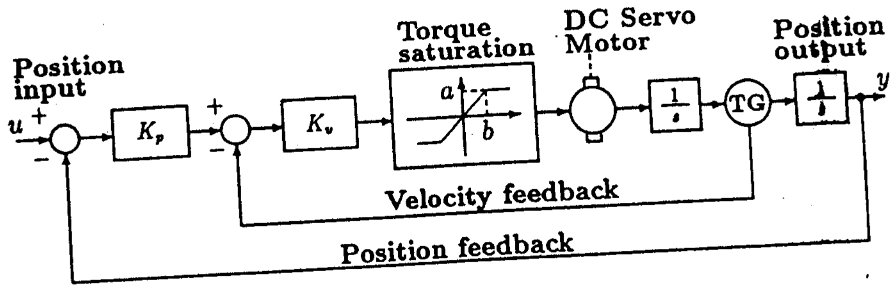
(24)



$g_3(\tau - 735, \tau - 738, \tau - 741)$

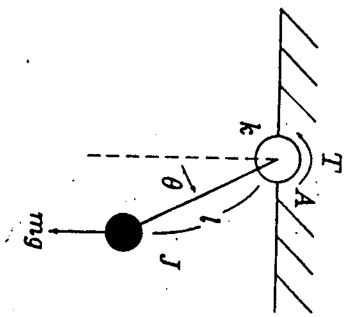
Applications

1. Mechatronic servo system
2. Mechanical pendulum system
3. Chemical process



A mechatronic servo system having a saturation-type nonlinear element

Mechanical pendulum system



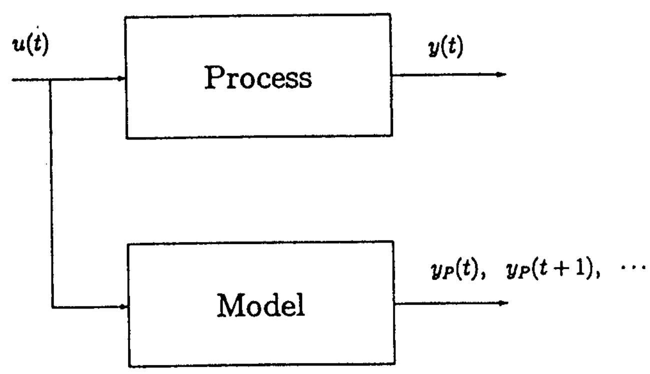
$$J\ddot{\theta} + k\dot{\theta} + mgl\sin\theta = T$$

where
- θ : angle of pendulum
- J : moment of inertia
- m : mass of the pendulum
- l : length of the pendulum
- g : accerelation of gravity
- T : applied torque
- k : damping coefficient

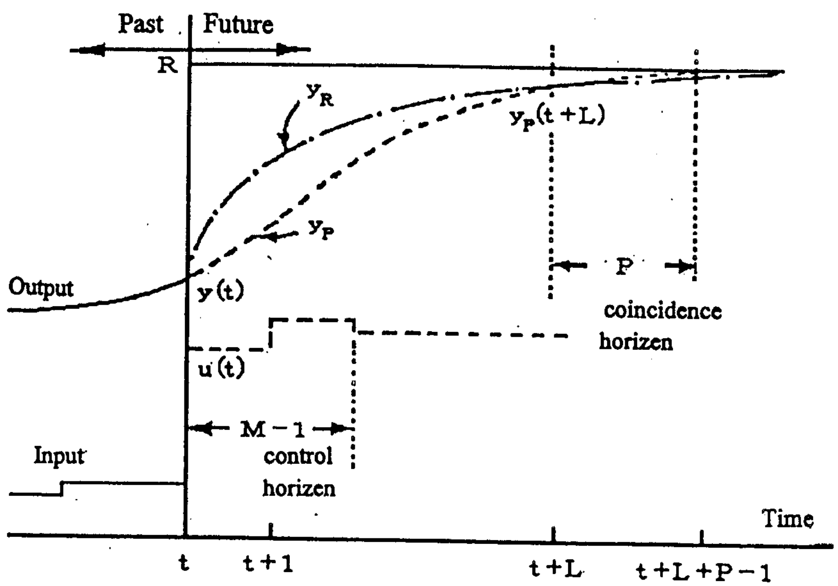
Nonlinear chemical process

$$\frac{dy(t)}{dt} = -Ky(t)^2 + \frac{1}{V}(d - y(t))u(t)$$

where
- $u(t)$: volumetric flow rate of feed stream(l/h)
- $y(t)$: output of the reactor indicating concentration of outlet stream(mol/l)
- K : rate of reaction($1/mol/l \cdot h$)
- V : reactor volume(l)
- d : concentration of inlet stream(mol/l)



Model Predictive Control of a process



Basic Idea of Model Predictive Control (1)

In case of using linear model

$$y_M(t+j) = \sum_{k=1}^{s} h_k u(t+j-k)$$

Predicted Output

$$y_P(t+j) = y(t) + y_M(t+j) - y_M(t)$$

Desired output trajectory

$$y_R(t+j) = \alpha^j y(t) + (1-\alpha^j)R$$

Evaluation function

$$J := \min_{u(t),\dots,u(t+M-1)} \sum_{j=L}^{L+P-1} \{y_P(t+j) - y_R(t+j)\}^2$$

Input

$$u_n = (H_M^T H_M)^{-1} H_M^T (Y_R - Y - H_0 u_0)$$

$$H_M = \begin{pmatrix} h_L, & h_{L-1}, & \dots, & h_{L-M+2}, & h_{L-M+1} + \Sigma_{k=M}^{L+P-2} h_{L-k} \\ h_{L+1}, & h_L, & \dots, & h_{L-M+3}, & h_{L-M+2} + \Sigma_{k=M-1}^{L+P-3} h_{L-k} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{L+P-1}, & h_{L+P-2}, & \dots & \dots, & h_{L+P-M} + \Sigma_{k=M-P+1}^{L-1} h_{L-k} \end{pmatrix}$$

$$H_0 = \begin{pmatrix} h_{L+1} - h_1, & h_{L+2} - h_2, & \dots\dots, & h_{L+s} - h_s \\ h_{L+2} - h_1, & h_{L+3} - h_2, & \dots\dots, & h_{L+s+1} - h_s \\ \vdots & \vdots & \ddots & \vdots \\ h_{L+P} - h_1, & h_{L+P+1} - h_2, & \dots\dots, & h_{L+P+s} - h_s \end{pmatrix}$$

$$Y_R = \begin{pmatrix} y_R(t+L) \\ y_R(t+L+1) \\ \vdots \\ y_R(t+L+P-1) \end{pmatrix} \qquad Y = \begin{pmatrix} y(t) \\ y(t) \\ \vdots \\ y(t) \end{pmatrix} \qquad u_0 = \begin{pmatrix} u(t-1) \\ u(t-2) \\ \vdots \\ u(t-s) \end{pmatrix}$$

In case of using nonlinear model

$$
\begin{aligned}
y_M(t+j) = & \int_0^\infty g_1(\tau_1)u(t+j-\tau_1)d\tau_1 \\
& + \int_0^\infty \int_0^\infty g_2(\tau_1,\tau_2)u(t+j-\tau_1)u(t+j-\tau_2)d\tau_1 d\tau_2 \\
& + \int_0^\infty \int_0^\infty \int_0^\infty g_3(\tau_1,\tau_2,\tau_3)u(t+j-\tau_1)u(t+j-\tau_2)u(t+j-\tau_3)d\tau_1 d\tau_2 d\tau_3
\end{aligned}
$$

Find the input u(t) to minimize

$$J := \min_{u(t),\dots,u(t+M-1)} \sum_{j=L}^{L+P-1} \{y_P(t+j) - y_R(t+j)\}^2$$

(33)

(2) Nonlinear chemical process

$$
\begin{aligned}
\frac{dx_1}{dt} &= \frac{1}{Tp_1}(-x_1 + Kp_1 u_1) \\
\frac{dx_2}{dt} &= \frac{1}{Tp_2}(Kp_2 x_1 x_2 - x_2 + Kp_3 u_2)
\end{aligned}
$$

$$y = x_2$$

$u_1 = 0.05(kg/h)$ (Feed rate of catalyst) — $u_2 = 3195(kg/h)$ (supply quantity of polyethylene)

$Kp_1 = 0.4$ — $Kp_2 = -1648(1/kg/h)$

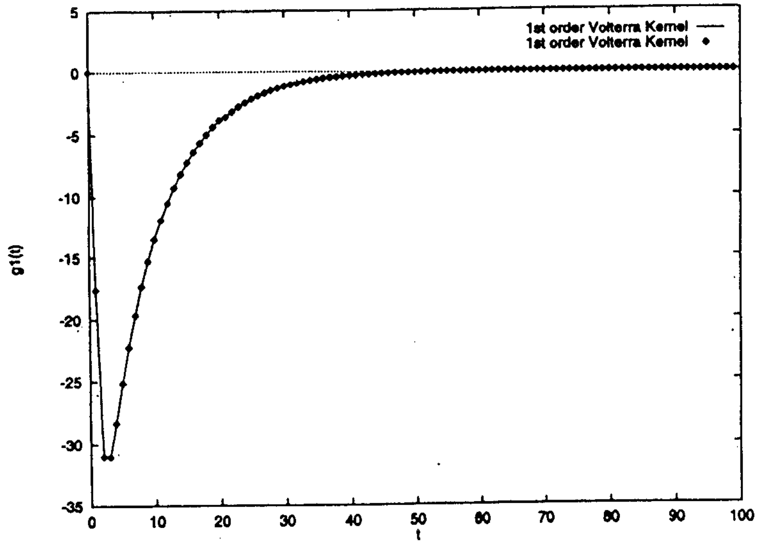
$Kp_3 = 0.05317(kg/cm^2/kg/h)$

$Tp_1 = 2.4(h)$ — $Tp_2 = 7.1(h)$

$x_1 = 0.02(kg/h)$ (Consumption velocity of catalyst) — $x_2 = 5.0(kg/cm^2)$ (gas density)

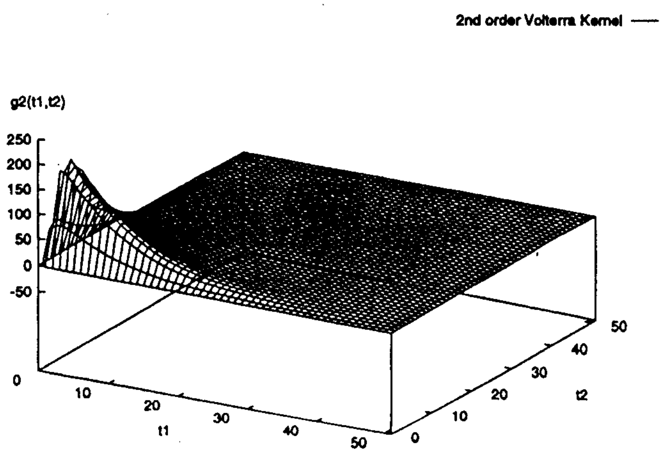
(34)

Result of Identification



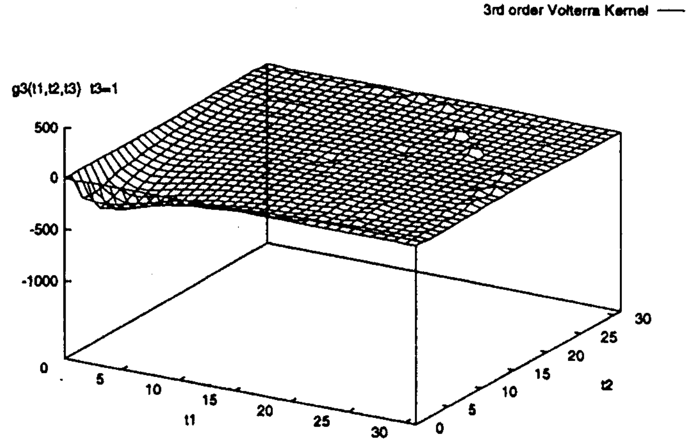
The 1st order Volterra Kernel

(35)



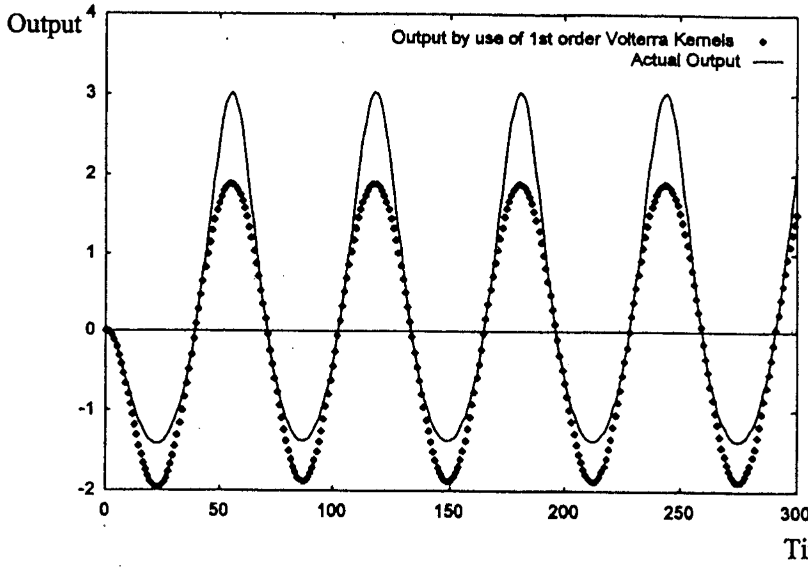
The 2nd order Volterra Kernel

(36)



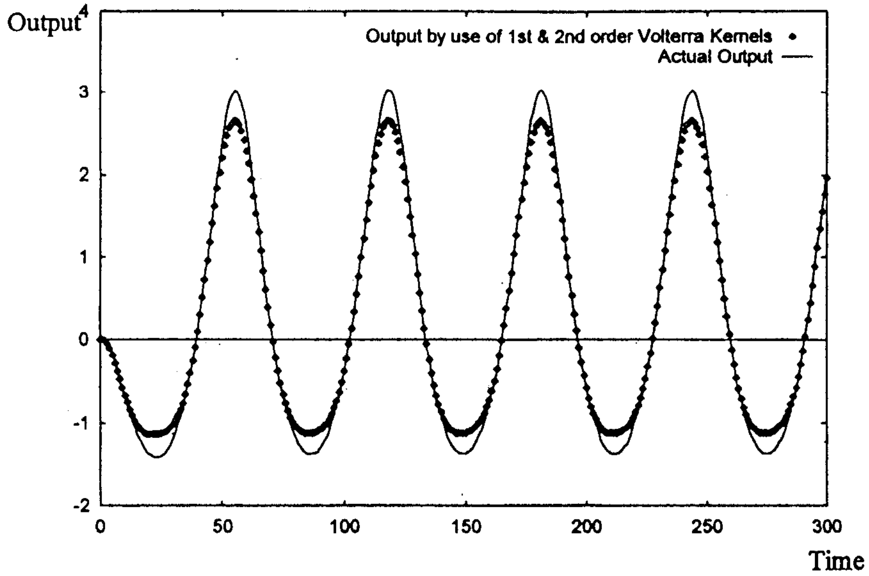
The 3rd order Volterra Kernel ($\tau_3 = 1$)

(37)



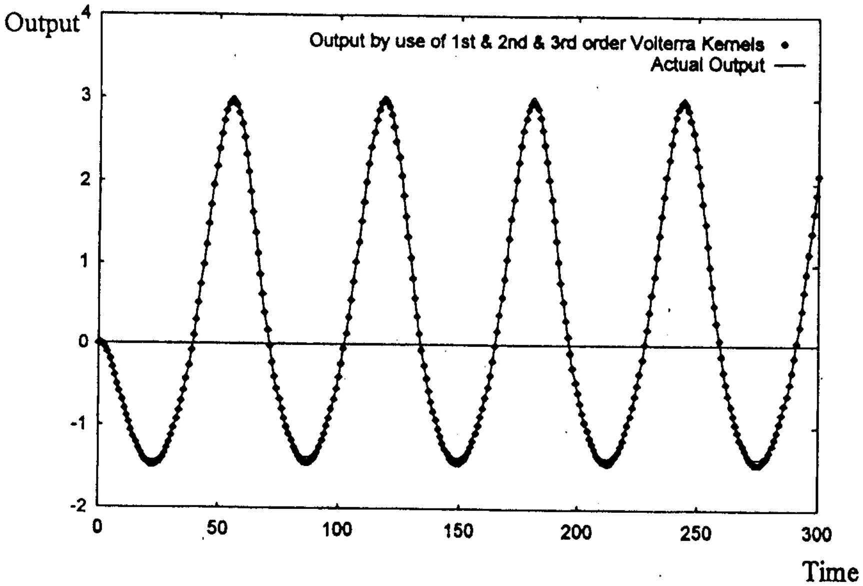
Comparison of output between actual and 1st order Volterra model

(38)



Comparison of output between actual and 2nd order Volterra model

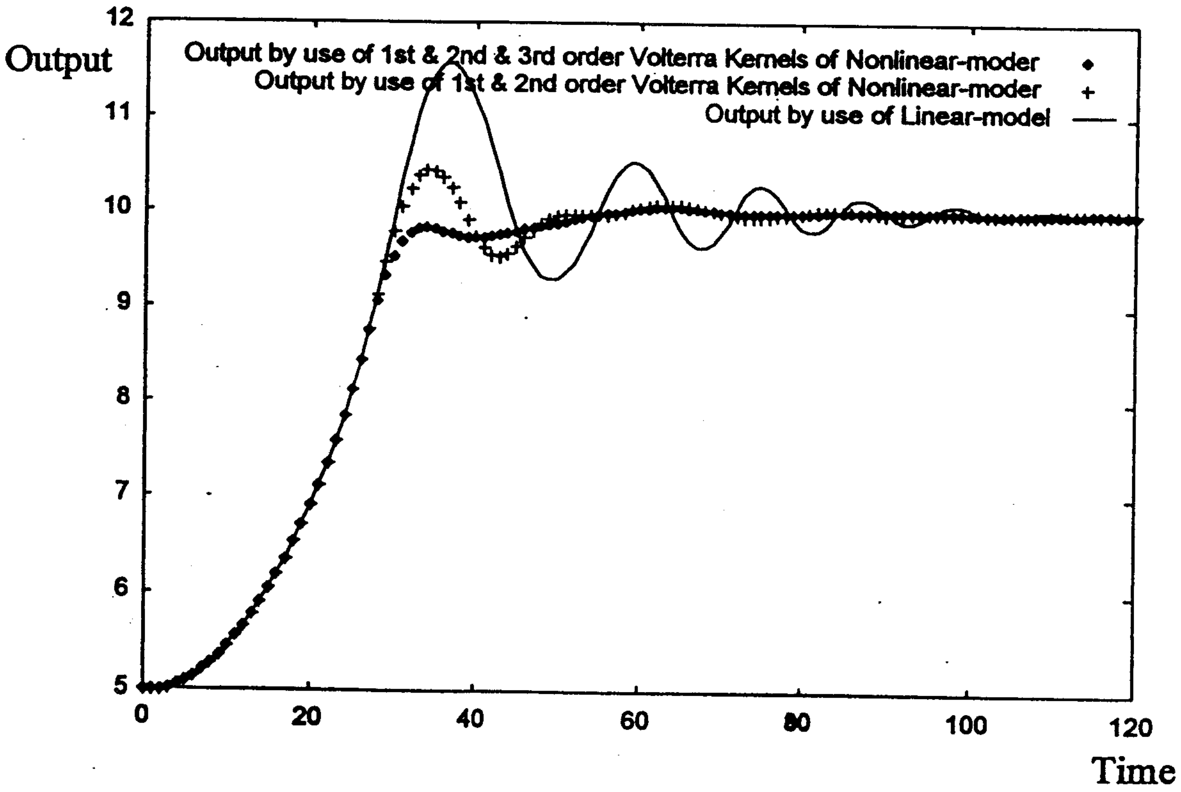
(39)



Comparison of output between actual and 3rd order Volterra model

(40)

Result of simulation



Result of simulation