An introduction to sedimentation theory in wastewater treatment

Bengt Carlsson
Systems and Control Group
Uppsala University

Nov 96, rev Okt 98

Abstract

This material is made for the course "Wastewater treatment" in the Aquatic and Environmental Engineering program. The sections which are marked with a $*$ are not central in the course.

Contents

1 Introduction

Sedimentation (settling) is the separation of suspended particles that are heavier than water. The sedimentation of particles are based on the gravity force from the differences in density between particles and the fluid. Sedimentation is widely used in wastewater treatment systems. A successful sedimentation is crucial for the overall efficiency of the plant. Common examples include the removal of;

- Grit and particulate matter in the primary settling basin (settling tanks that receive raw wastewater prior to biological treatment are called primary tanks, försedimentering).
- Sludge from the bioreactor (activated sludge process).
- Chemical flocs in the chemical step.

Often, the settler connected to the activated sludge process is the main bottle neck in the plant. The seemingly simple process has proven to be the weak link in many wastewater treatment plants.

The implementation of nitrogen removal in many Swedish plants emphasis the importance of the settler. The slow growth of nitrifying bacteria means that a high sludge age is necessary in the activated sludge process. For a give volume of the aeration basin, the sludge age may be increased by using a higher sludge concentration in the basin. However, by increasing the sludge concentration in the aeration basin, the capacity of the settler may be reached, the sludge blanket level will then increase which finally results in an uncontrolled sludge escape in the effluent water. Hence, there is a possible conflict between operation for good nutrient removal (high sludge age) and operation for good sludge sedimentation. Further, nitrogen removal in the activated sludge process gives also a risk for sludge rise in the secondary settler due to denitrification in the bottom of the settler. The sludge may rise due to flotation of solids when nitrogen gas is released.

Note also that the settler has two functions; clarification and sludge separation. That is to remove essentially all of the solids from suspension and to concentrate theses solids (eg for recycling to the aeration basin).
Depending on the particles concentration and the interaction between particles, four types of settling can occur, see also Figure 1:

- *Discrete particle settling.* The particles settle without interaction and occurs under low solids concentration. A typical occurrence of this type of settling is the removal of sand particles.
- *Flocculent settling.* This is defined as a condition where particles initially settle independently, but flocculate in the depth of the clarification unit. The velocity of settling particles are usually increasing as the particles aggregates. The mechanisms of flocculent settling are not well understood.
- *Hindered settling.* Inter-particle forces are sufficient to hinder the settling of neighboring particles. The particles tend to remain in a fixed positions with respect to each others. This type of settling is typical in the settler for the activated sludge process (secondary clarifier).
- *Compression settling.* This occurs when the particle concentration is so high that so that particles at one level are mechanically influenced by particles on lower levels. The settling velocity then drastically reduces.

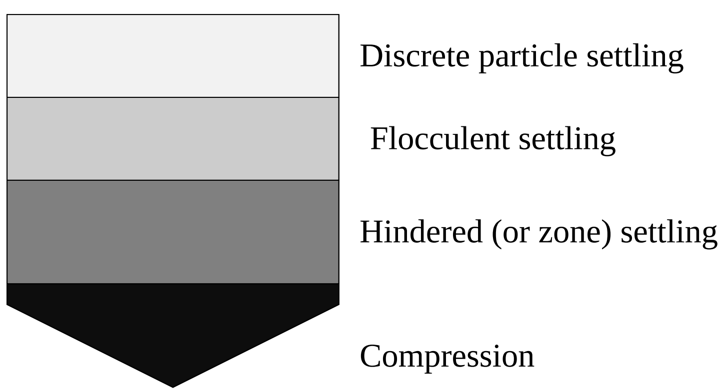

Figure 1: Settling phenomena in a clarifier

In the following, we will restrict the discussion to discrete particle settling and hindered settling.

2 Discrete particle settling

Consider the settling of a discrete particle, see Figure 2. The sedimentation is obtained by the Newton and Stokes law.

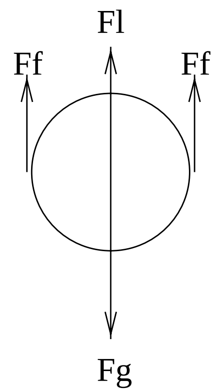

Figure 2: Forces on a discrete particle, F_g is the gravitational force, F_l is the "lifting" force from the liquid, and F_f is the frictional force between the particle and the liquid.

Newton second law gives

$$m\frac{dv}{dt} = F_g - F_l - F_f \tag{1}$$

where v is the velocity of the particle, and m is the mass.

The gravity force F_g is given by

$$F_g = mg = \rho_p V_p g \tag{2}$$

where ρ_p is the density of the particle and V_p is the volume.

The lifting force F_l is given by

$$F_l = \rho_f V_p g \tag{3}$$

where ρ_f is the density of the fluid.

The frictional drag force F_f depends on the particle velocity, fluid density, projected area, and a drag coefficient. The following *empirical* expression is used

$$F_f = \frac{C_D A_p \, \rho_f \, v^2}{2} \tag{4}$$

where C_D is the drag coefficient[1] and A_p is the projected area of the particle perpendicular to the velocity.

Inserting (2), (3) and (4) in (1) yields

$$m\frac{dv}{dt} = g(\rho_p - \rho_f)V_p - \frac{C_D A_p \, \rho_f \, v^2}{2} \tag{5}$$

In steady state ($\frac{dv}{dt} = 0$), we have

$$v = \sqrt{\frac{2g(\rho_p - \rho_f)V_p}{C_D A_p \, \rho_f}} \tag{6}$$

[1] Newtons motståndskoefficient.

2.1 Stokes law*

For a spherical particle with diameter d, we have the volume $V_p = \frac{\pi d^3}{6}$ and a *projected* area $A_p = \frac{\pi d^2}{4}$ Inserting this in (6) gives

$$v = \sqrt{\frac{4g(\rho_p - \rho_f)d}{3C_D\rho_f}} \tag{7}$$

For laminar flows it holds that

$$C_D = \frac{24}{R_N} \tag{8}$$

where R_N is the Reynolds number.

$$R_N = \frac{vd\rho_f}{\mu} \tag{9}$$

In (8), the viscosity μ is introduced. This gives a measure of a fluids resistance to tangential or shear stress.

Inserting (8) in (7) using (9) gives Stokes law

$$v = \frac{g(\rho_p - \rho_f)d^2}{18\mu} \tag{10}$$

2.2 The surface loading rate

The settling velocity of a particle can be used in the design of settling (sedimentation) basins. The key idea is to find a lower limit on the settling velocity for the particle to settle before it reach the outlet. Consider an ideal settling basins according to Figure 3.

Figure 3: Settling in an *ideal* settling basin. The basin depth is h, the surface area is A.

The time a unit element is residencing in the (ideal) settling basin is given by

$$T = \frac{V}{Q} = \frac{Ah}{Q} \tag{11}$$

where V is the basin volume, A is the surface of the basin, h is the basin depth, and Q is the flow rate. The minimum settling velocity for a particle (entering the basin at height h) to settle is thus given by

$$v_{min} = h/T = \frac{Q}{A} \tag{12}$$

The ratio $\frac{Q}{A}$ is the surface loading rate[2] and is one of the key parameters in the operation and design of settling basins. Note, however, that for normal settling basins the above

[2] Ytbelastningen.

relation only gives a crude approximation. Typically, only about 60% of the theoretical settling capacity (12) is achieved in practice. Different empirical relations exist to compensate for non ideal situations. Note that the basin depth h does not influence the (theoretical) minimum velocity. Common experience suggests that the basin depth should exceed 2-3m.

3 Hindered settling

3.1 The solid flux theory

Most models for hindered settling are based on the solid flux theory. Pioneering work, using solid flux concept for settler calculations, was done by Cloe and Clevenger (1916) and Kynch (1952).

In general, the total flux (mass/(area$\times$ time) of solids is obtained by

$$J = Xv \tag{13}$$

where X is the solid (sludge) concentration[3] and v is the settling velocity which in general depends on X.

Now, consider a settling basin according to Figure 4.

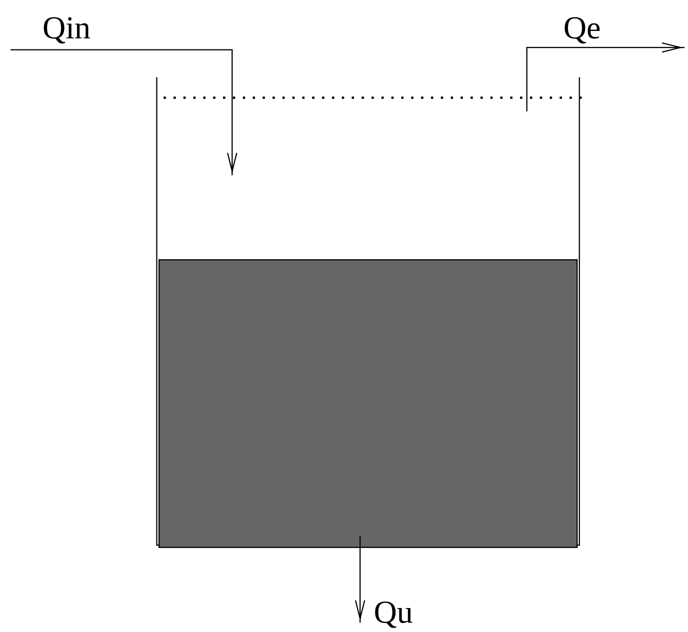

Figure 4: A settling basin. The basin has a cross sectional area A. At the bottom of the basin, sludge is withdrawal at the rate Q_u.

The total flux of solids through a segment is

$$J_t = J_g + J_u \tag{14}$$

where J_t is the total flux, J_g is flux due to gravitational settling, and J_u is the flux resulting from the sludge withdrawal at the bottom of the basin (often called bulk flux). From Figure 4 we have that

$$J_u = v_u X = \frac{Q_u}{A} X \tag{15}$$

[3]The concentration of suspended solids is normally given as MLSS, mixed liquor suspended solids

where v_u is the velocity resulting from the removal of sludge at the bottom of the tank..

The solid flux theory states that

$$J_g = v_g(X)X \tag{16}$$

That is, the gravitational settling velocity only depends on the local concentration of solids. Hence, we can write the total flux as

$$J_t = (v_g(X) + \frac{Q_u}{A})X \tag{17}$$

3.2 Veslinds formula

Several empirical relations have been suggested to describe the relation between the v_g and X. A commonly used relation is the Veslind formula:

$$v_g(X) = v_o \exp^{-nX} \tag{18}$$

where v_o is the maximum settling velocity and n gives a measure on how fast the settling velocity decreases with increasing concentration of particles. In practice, these parameters can be found by multiple batch settling experiments where $\log v_g$ is measured for different sludge concentrations. Then, v_o and n can be found by a simple least squares fit to the data (linear regression). Also, several empirical relations exist where one tries to relate parameters like SVI (sludge volume index) to v_o and n.

Inserting the Veslind formula (18) in (17) yields

$$J_t = (v_o \exp^{-nX} + \frac{Q_u}{A})X \tag{19}$$

An illustration of the relation (19) is given in Figure 5.

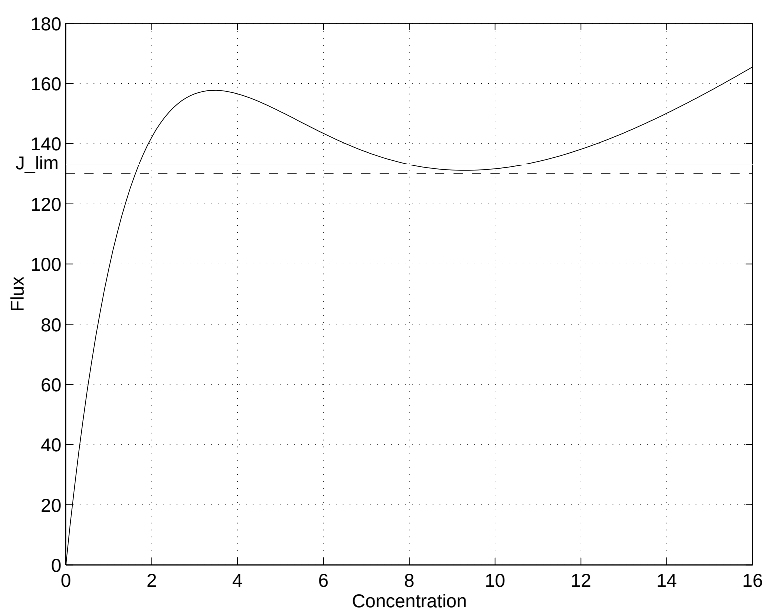

Figure 5: Total flux as a function of the concentration.

Notice that the flux curve has a local minima denoted J_{lim}. This flux is the maximum allowable flux loading if the settling is to be successful. If the influent flux to the settler is larger than J_{lim}, the sludge blanket will increase, resulting in solids (sludge) in the effluent.

For a general total flux model $J_t(X)$, the limiting flux can be obtained by solving $\frac{J_t(X)}{dX} = 0$. To find the minimum we have to check which of the extreme points that fulfill $\frac{J_t^2(X)}{dX^2} > 0$. These calculations may have to be solved numerically. Obviously, graphical solutions are also possible to use.

The limiting flux can be used for the design of the area of the settler. The flux at the level where the influent is located is

$$J_{in} = \frac{Q_{in}}{A} X_{in} \tag{20}$$

where X_{in} is the solid (sludge) concentration of the influent water. We must require that $J_{in} < J_{lim}$ and hence

$$A > \frac{Q_{in}}{J_{lim}} X_{in} \tag{21}$$

Note that J_{lim} in general depends on A.

3.3 Numerical solution of the settler*

In order to model the settler during non-steady state conditions, the involved mass balance have to be solved. This is done by applying the conservation law (continuity equation) which states that the increase of mass per time unit equals the incoming flux minus the outgoing flux. This can be stated as a partial differential equation as

$$\frac{\partial X}{\partial t} = -\frac{\partial J}{\partial z} \tag{22}$$

where z is the height coordinate.

The most popular approach to solve (22) numerically is to divide the settling basin in a number of horizontal slices (typically 10 to 100 slices). Each slice is regarded as well mixed. A further description of numerical methods is, however, outside the scope of this note.