

# **Simple deconvolution of time varying signal of gas phase variation in a 12 MW CFB biomass boiler**

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## **Introduction**

**A gas concentration measured by gas analyzer may not be the same as the concentration at the probe tip in the boiler. There is a time delay for gas transport and gas species can be dispersed in some extent during the transport.**

**The measured concentration can be restored (deconvoluted) to its original concentration by help of a flow model that would give the mean residence time and RTD of fluid elements which describes the extent of gas dispersion.**

**A simple deconvolution scheme for time series signals by tanks-in-series model has been developed in this study.**

## The convolution of signals

A blurring or smoothing by a weighted function, RTD of fluid  
Tracer response represents convolution of the unit impulse of the system

$$C_{out}(t) = \int_0^t C_{in}(t-t')\mathbf{E}(t')dt'$$

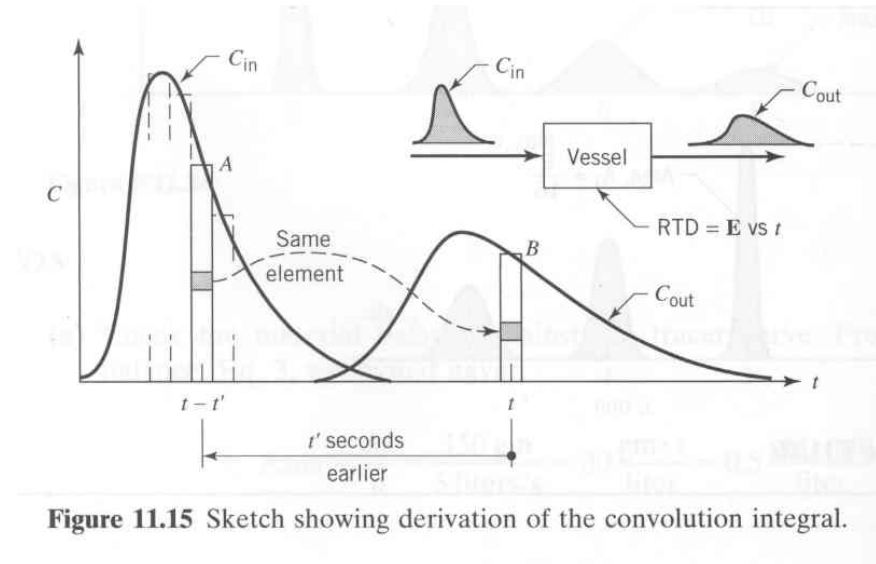


Figure 11.15 Sketch showing derivation of the convolution integral.

To deconvolute output, to find  $\mathbf{E}(t)$  or input under integral is difficult [Levenspiel, 1999].

## Deconvolution methods

### 1. Direct method

$$y_k = z_k + \varepsilon_k = \sum_{j=0}^{N-1} x_{k-j} e^j \quad k = 0, 1, \dots, N-1$$

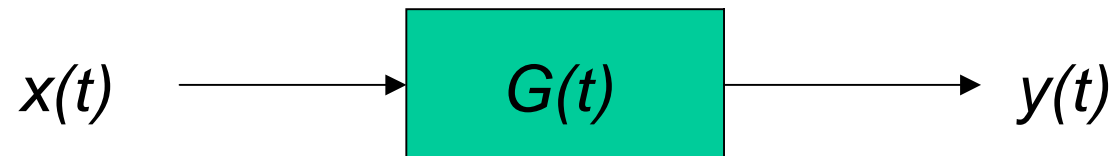
$$\mathbf{y} = \mathbf{x}\mathbf{e}$$

$$\mathbf{x} = \mathbf{e}^{-1}\mathbf{y}$$

Matlab has its Fourier transform version  
But very limited success

*Mills and Dudukovic, Computers Chem. Engng., 13(8), 881-898 (1989)*

### 2. Linear flow model and parameter estimation (AR or ARX model)



*Nameche and Vassel, Water Sci. Tech., 33(8), 105-124 (1996)*

### **3. Flow models**

*Levenspiel, O., 'Chemical Reaction Engineering', 3rd, John Wiley & Sons, New York (1999)*

**Models are useful for representing flow in vessels and other purposes. There are two models for small deviations from plug flow, they are roughly equivalent.**

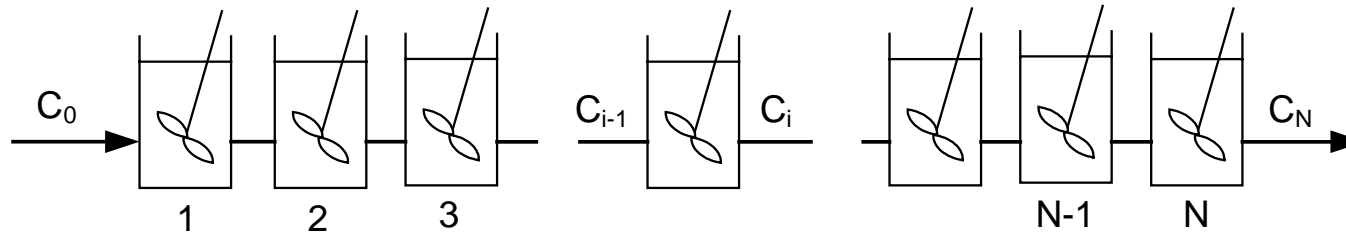
#### **3-1. Dispersion model**

**All correlations for flow in real reactors use this model.**

#### **3-2. Tanks-in-series model**

**Simple, it can be easily extended to other arrangements.**

## Tanks-in-series model



A mass balance at  $i$ th tank, in a series of  $N$  tanks, gives:

$$\frac{V_{tot}}{N} \frac{dC_i}{dt} = vC_{i-1} - vC_i$$

$$C_{i \neq 0} = 0 \quad C_{i=0} = \frac{M}{v} \delta(t), \quad \text{at } t = 0$$

Let  $\tau$  = total residence time for all  $N$  tanks,

$$\frac{\tau}{N} \frac{dC_i}{dt} = C_{i-1} - C_i \quad (\text{B-3})$$

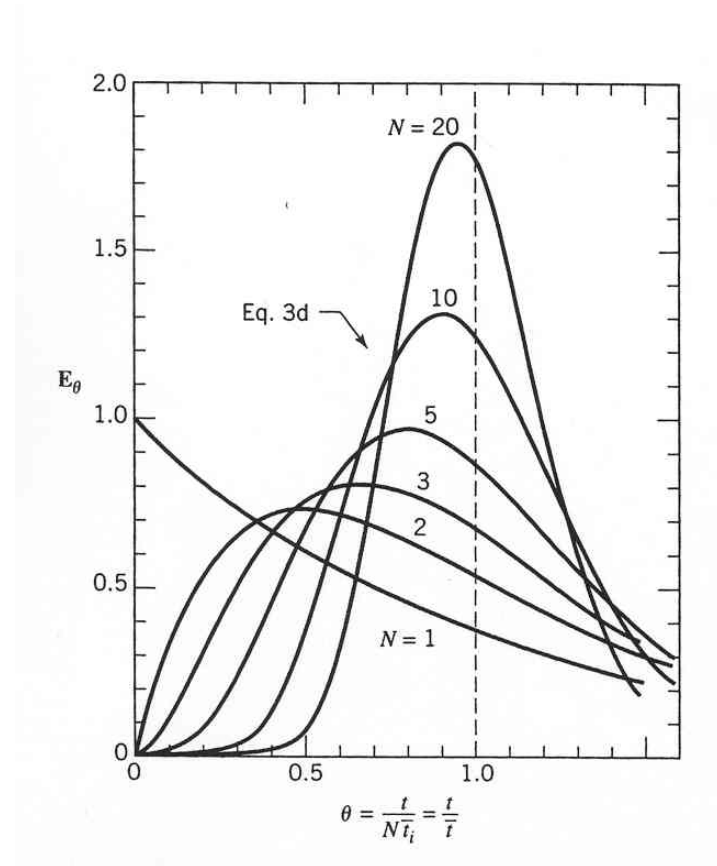
with Laplace transform, the final output is

$$\overline{C_N} = \frac{\overline{C_0}}{\left(1 + \frac{\tau}{N} s\right)^N}$$

In dimensionless forms

$$\theta = \frac{t}{\tau}$$
$$\mathbf{E}(\theta) = \frac{N(N\theta)^{N-1}}{(N-1)!} e^{-N\theta} \quad (\text{B-7})$$

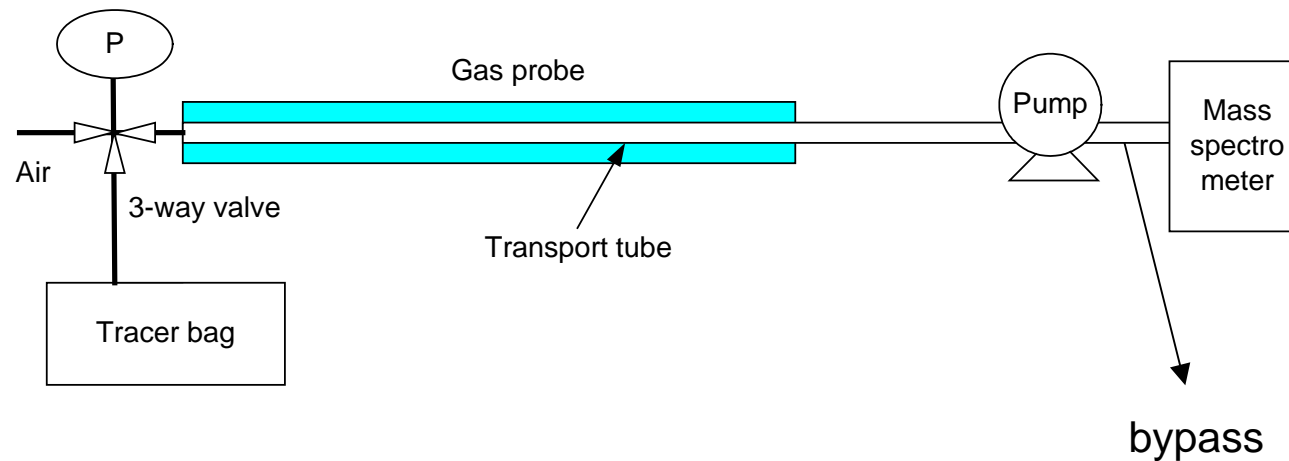
$$\sigma_{\theta}^2 = \frac{\sigma^2}{\tau^2} = \frac{1}{N} \quad (\text{B-8})$$



**Fig. B10. RTD curves for the tanks-in-series model.**

## Step response test

The gas transport system for measurement of gas concentration in a 12 MW CFB boiler with a tracer input part.



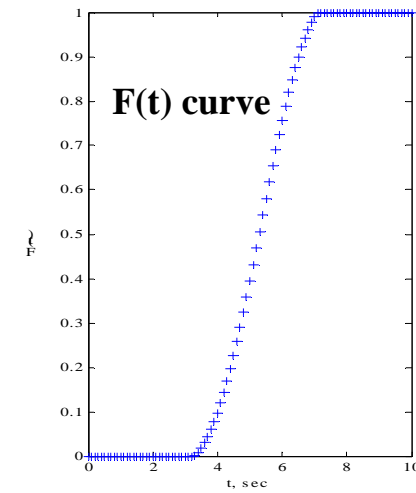
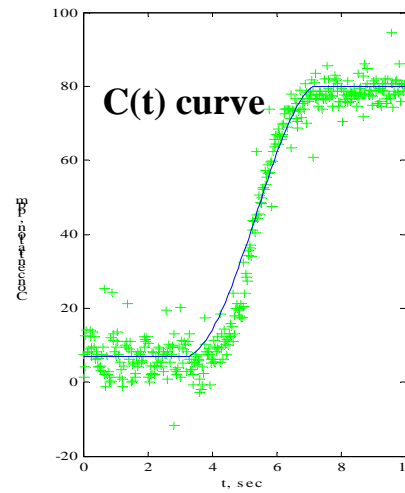
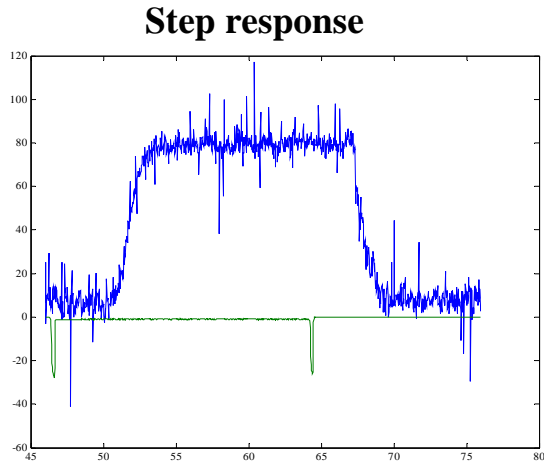
**Tracer = methane of 100 ppm**

**Dimension of tube = 0.004 m dia x 6 m length (probe=1.5m)**

**Flow rate of 1.5 L/min gives residence time of 3.02 sec**



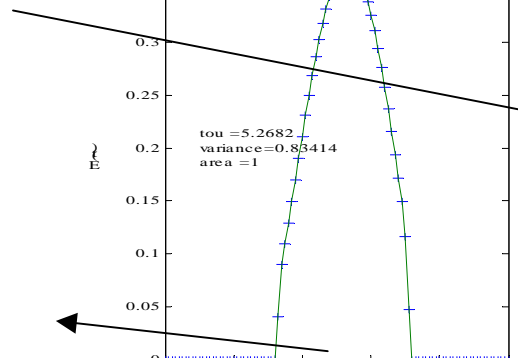
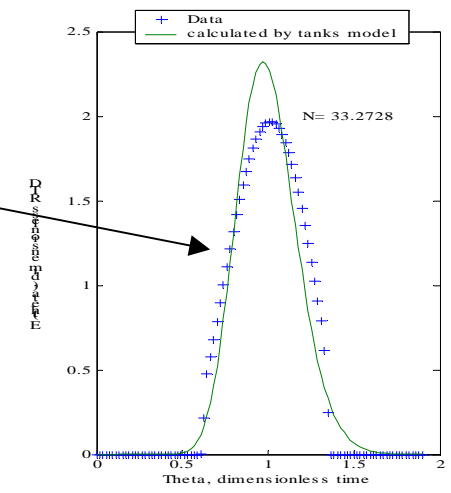
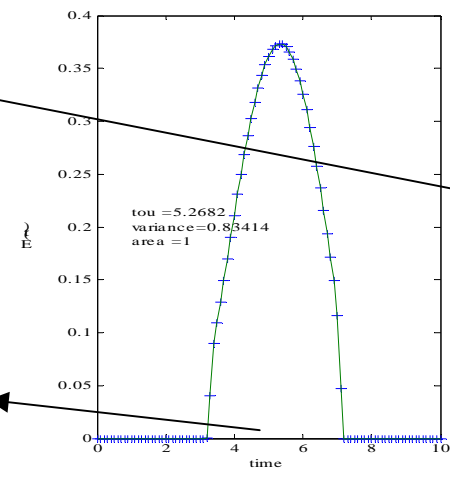
# E(t) curve from experiment



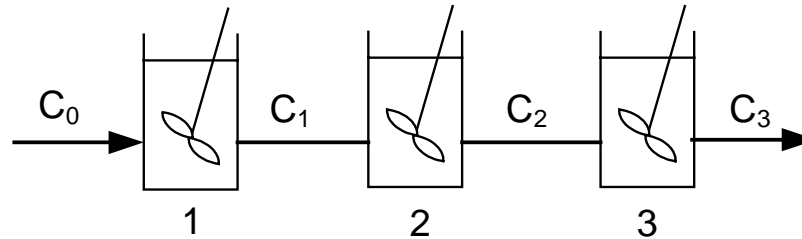
The experimental E curve gives  $N=33$  by tank model,

$$\sigma_{\theta}^2 = \frac{\sigma^2}{\tau^2} = \frac{1}{N}$$

E curve gives residence time of 5 sec.



## Deconvolution by tanks-in-series model



A mass balance at  $i$  th tank, in a series of  $N$  tanks, gives:

$$C_{i-1} = \frac{\tau}{N} \frac{dC_i}{dt} + C_i \quad (\text{B-3})$$

If  $N = 3$  the following equations can be used to obtain input concentration  $C_0$  from  $C_3$ .

$$C_2 = \frac{\tau}{N} \frac{dC_3}{dt} + C_3$$

$$C_1 = \frac{\tau}{N} \frac{dC_2}{dt} + C_2$$

$$C_0 = \frac{\tau}{N} \frac{dC_1}{dt} + C_1$$

In somecase the signal get noise at a certain point and the noise grow quickly to make the signal vibrated too much.

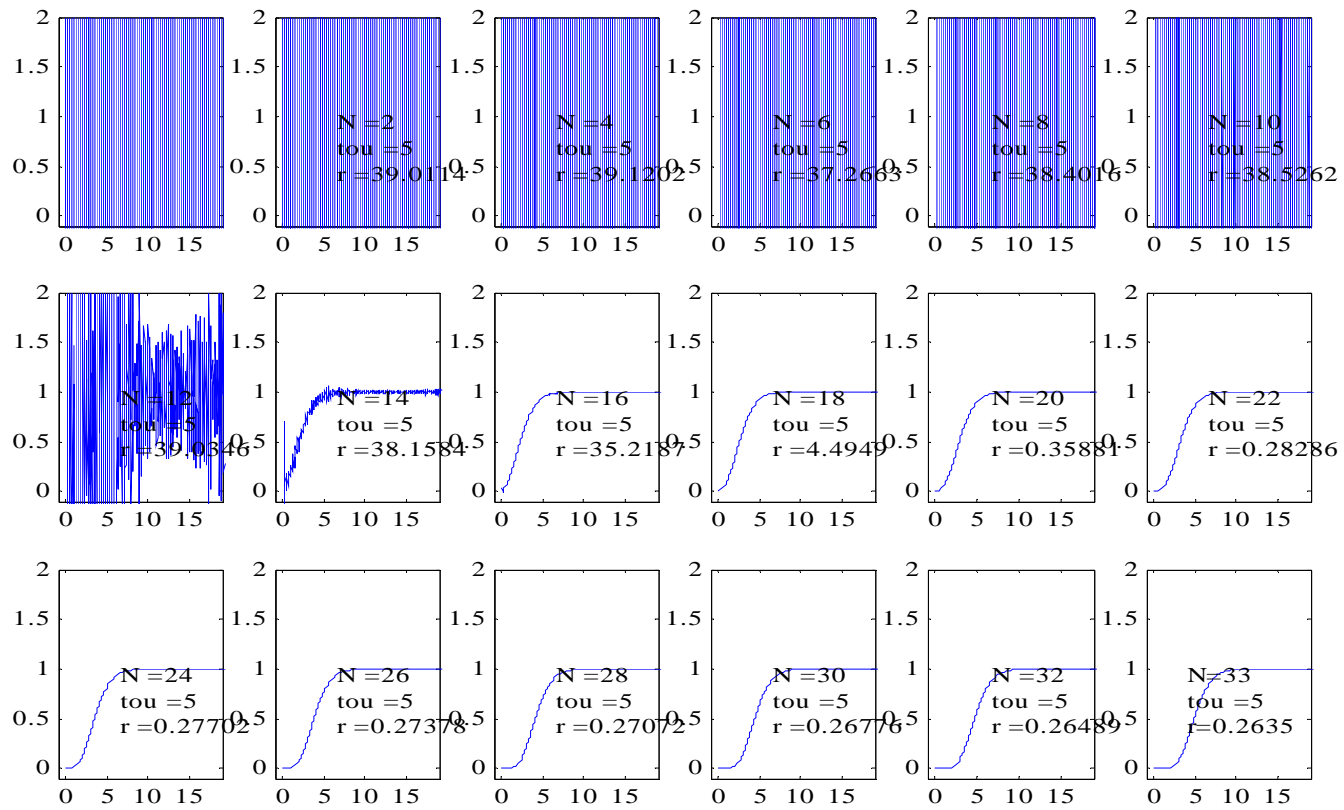
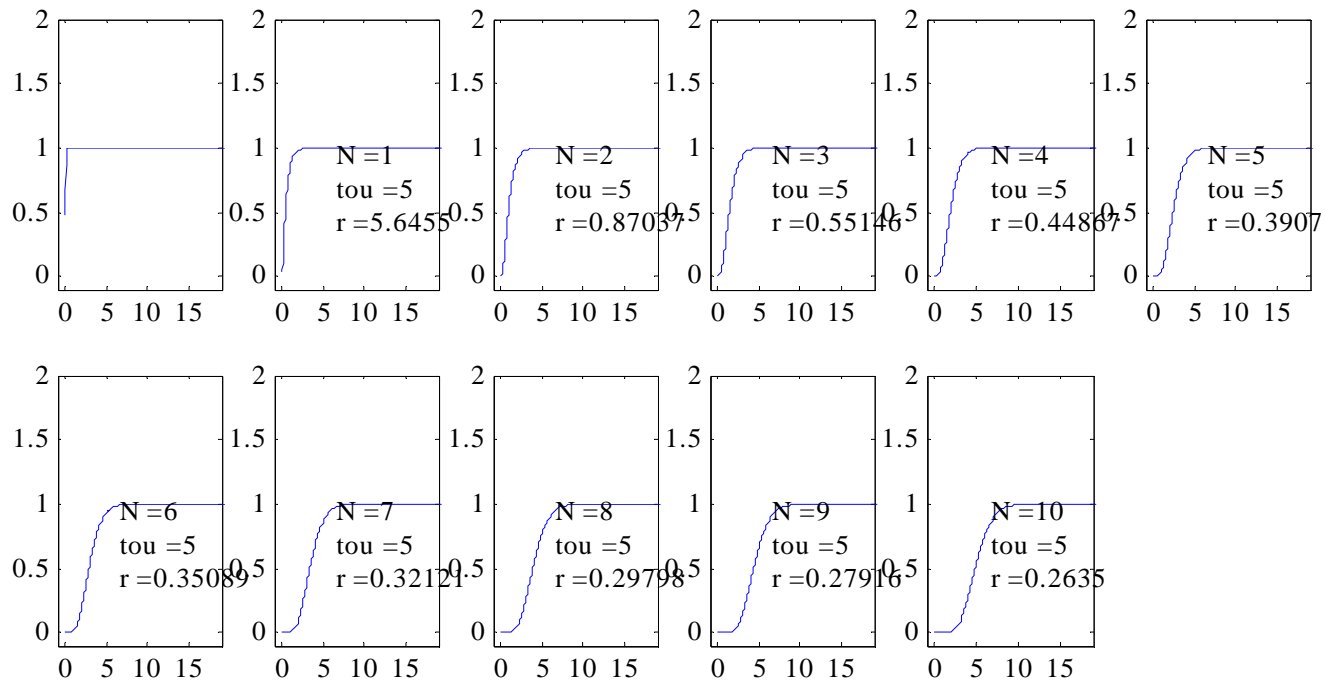


Figure D3. Deconvolution of a simulated curve (FT4) by 33 tanks.



**Figure D4. Deconvolution of a simulated curve (FT4) by 10 tanks (tou = 5)**

## Measure of error

$$e_j = \left( \frac{\text{standard deviation of } \frac{dy}{dt}}{\text{standard deviation of } y} \right) \quad \text{in } j \text{ th tank} \quad (\text{C-4})$$

$$f_j = \left( \frac{\text{standard deviation of } \left( \frac{\tau}{N} \right) \frac{dy}{dt}}{\text{standard deviation of } y} \right) \quad \text{in } j \text{ th tank} \quad (\text{C-5})$$

$$E = \sum_{j=1}^N e_j \quad F = \sum_{j=1}^N f_j \quad (\text{C-6})$$

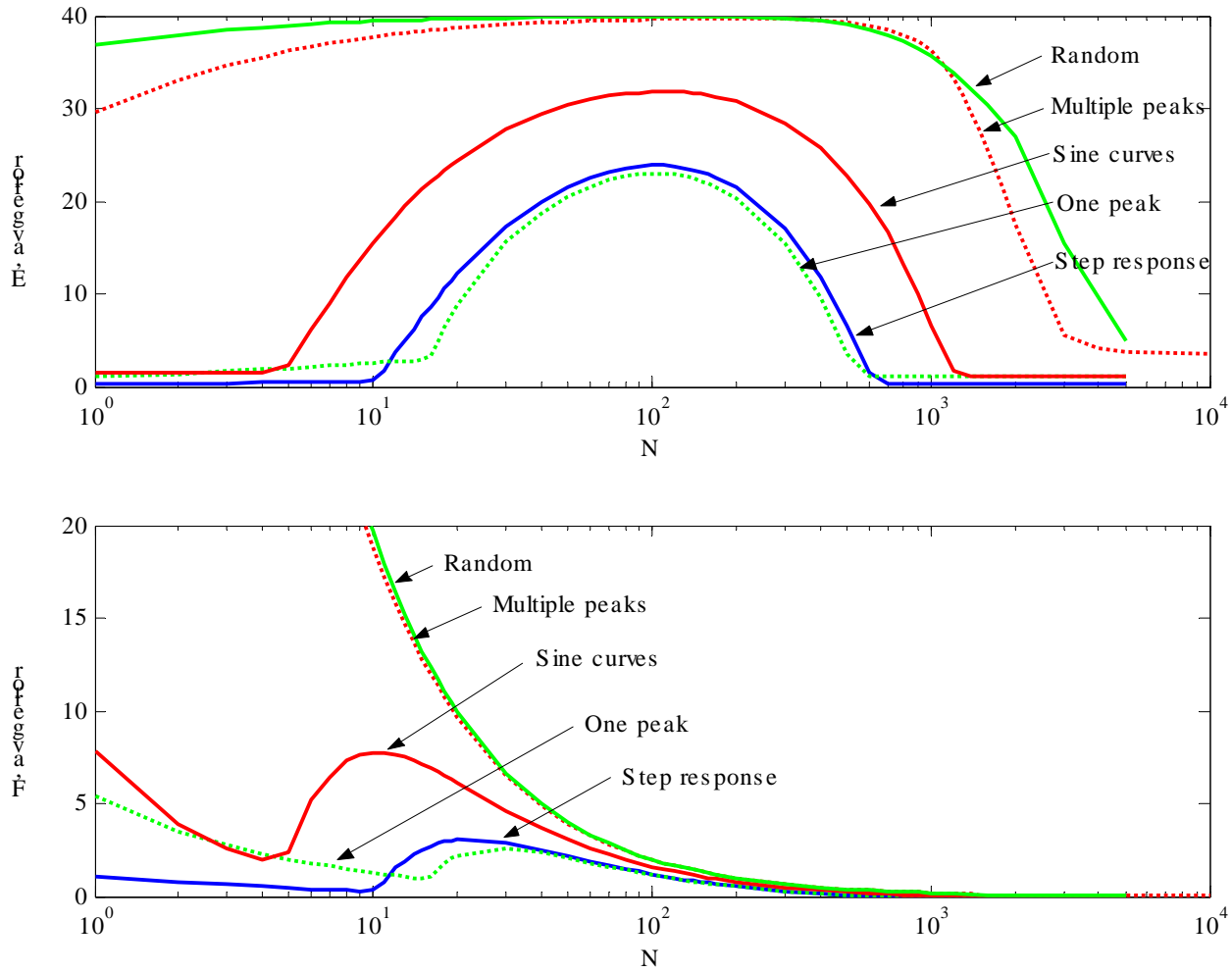


Figure D9. The changes of average ratio E and F with variation of N from the deconvolution of the various output signals with total residence time of 5 s.

The signal can be smoothed before the signal is differentiated.

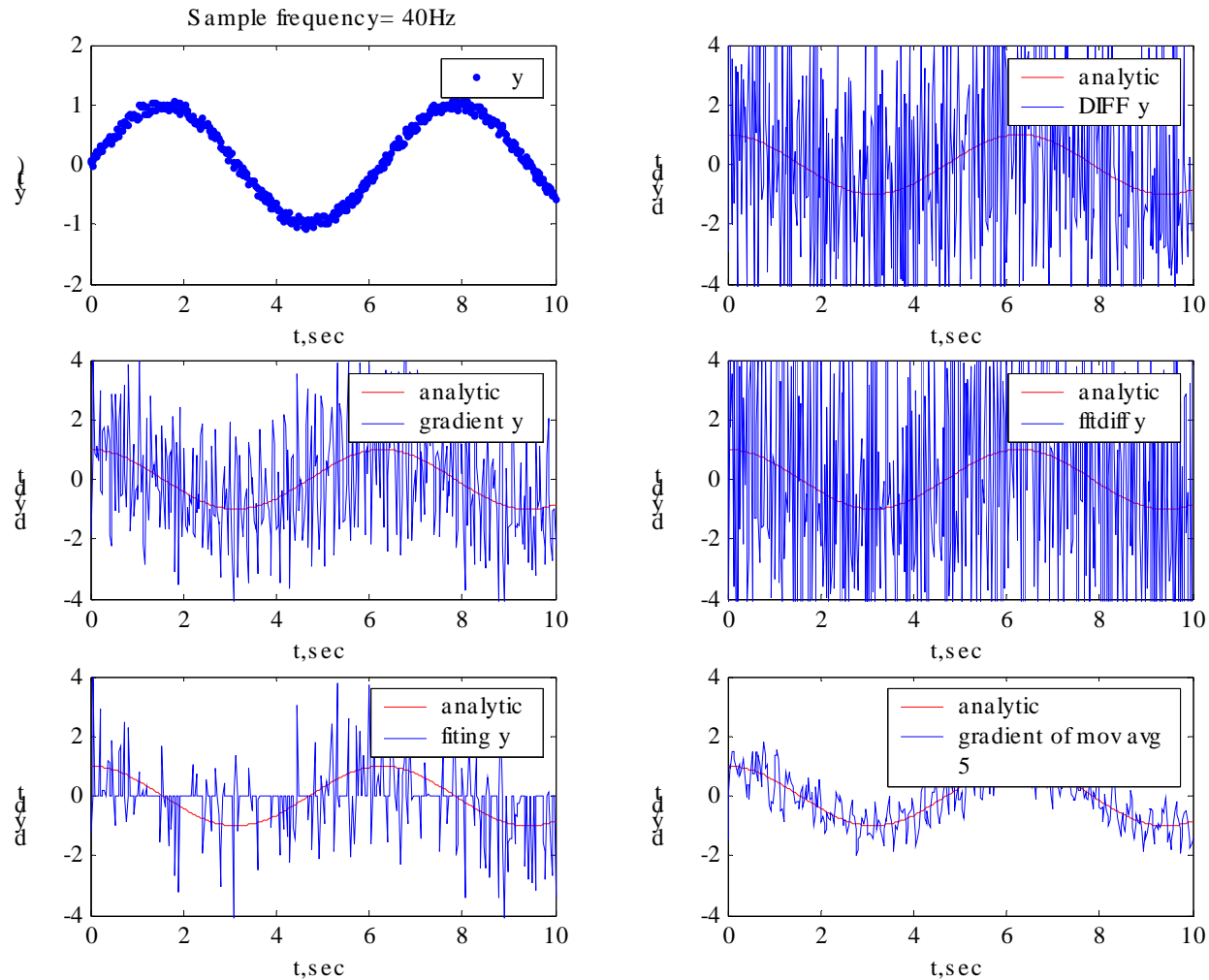


Figure E4. The effect of signal smoothing on the differentiation of a sine curve with a noise (sample frequency = 40 Hz).

**Moving average was found to be very effective to reduce the noise in the calculated gradient.**

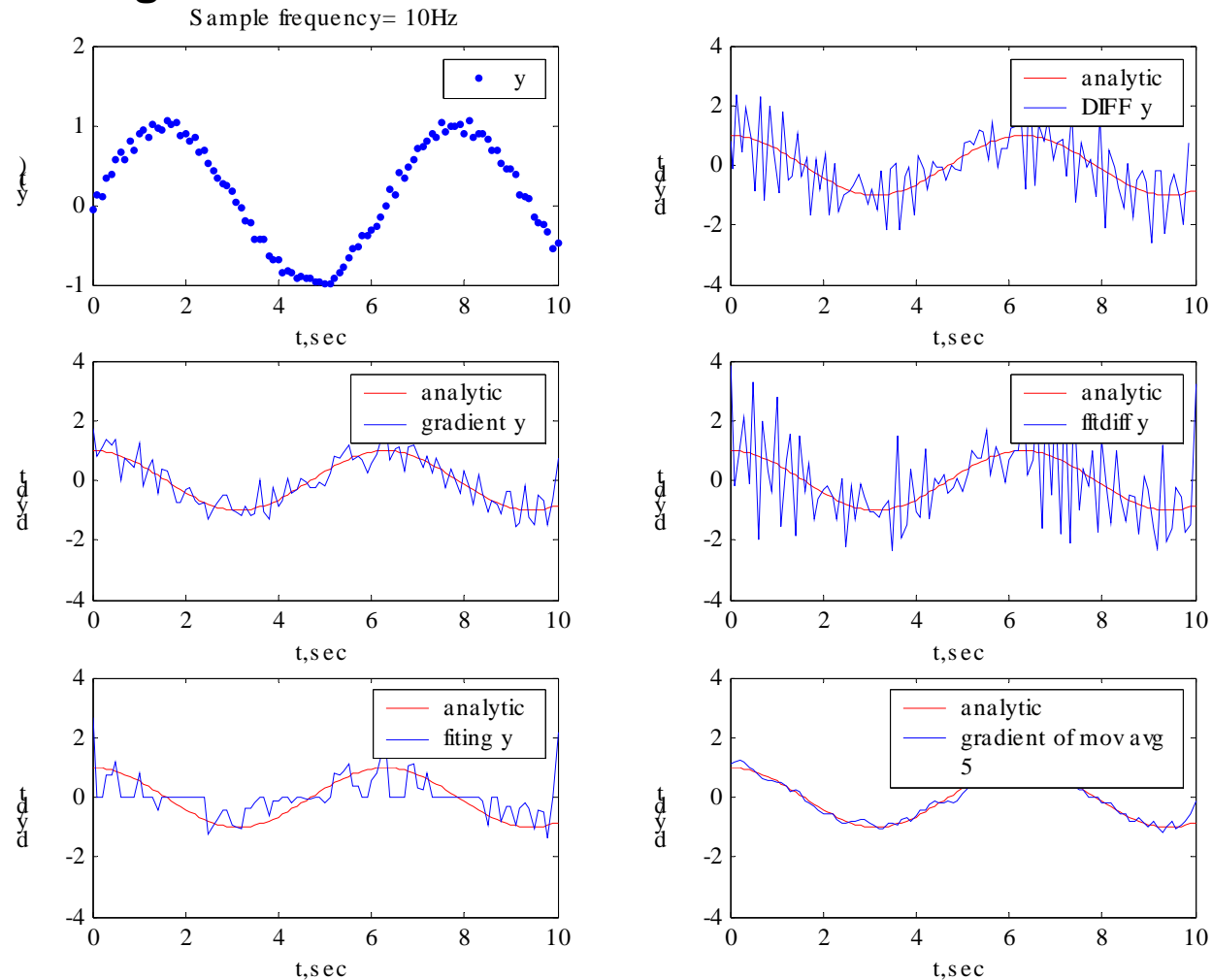


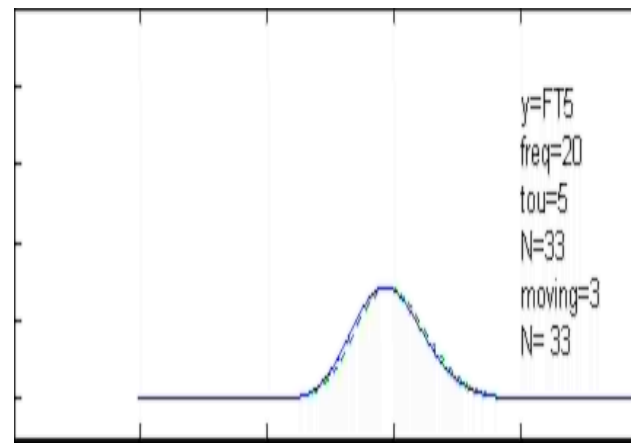
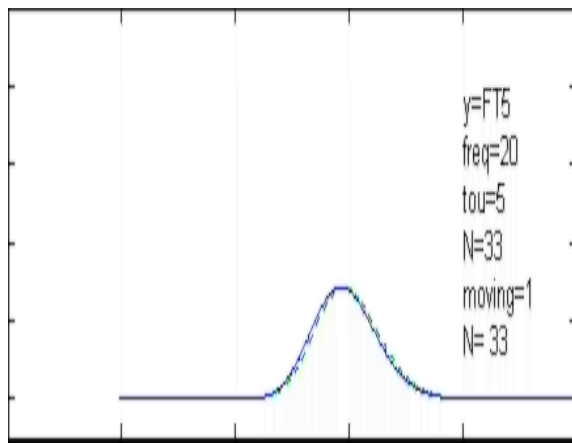
Figure E5. The effect of signal smoothing on the differentiation of a sine curve having a noise (sample frequency = 10 Hz).



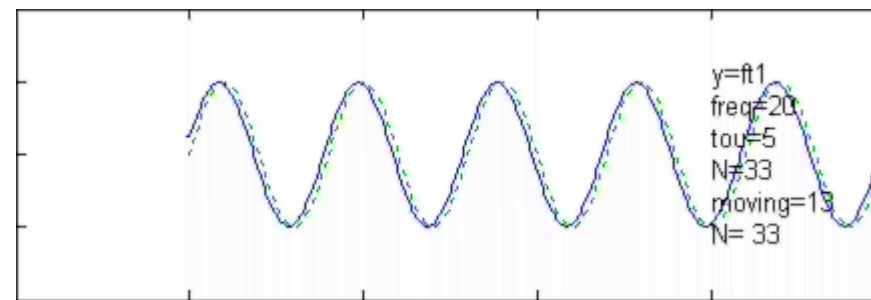
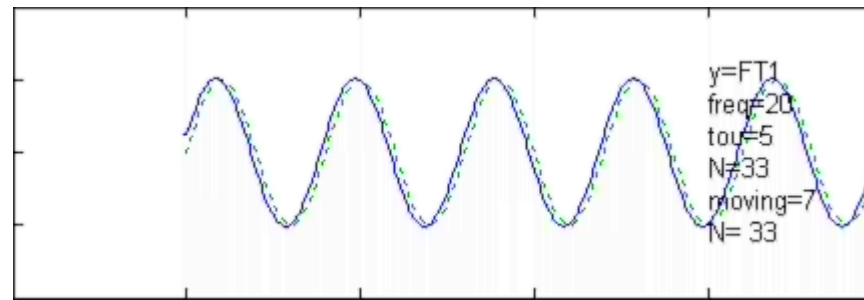
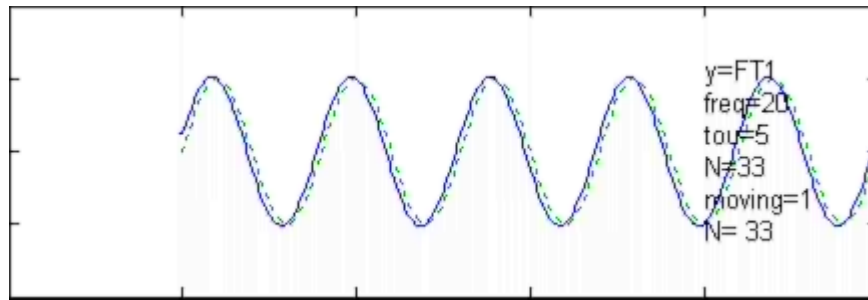
## *Application of moving average to the signal*

The best size of window for the moving average should be determined by trial and error.

One peak signal could be deconvoluted without any noise by adapting 3 points (window size) average to the signal before the differentiation.



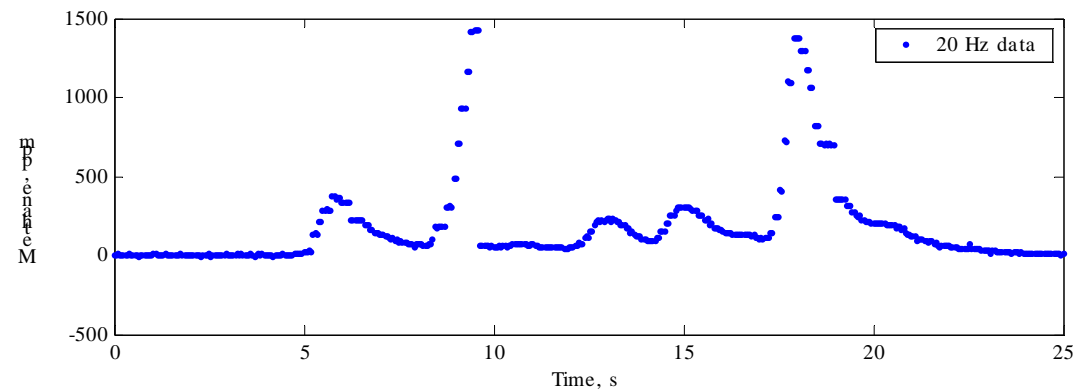
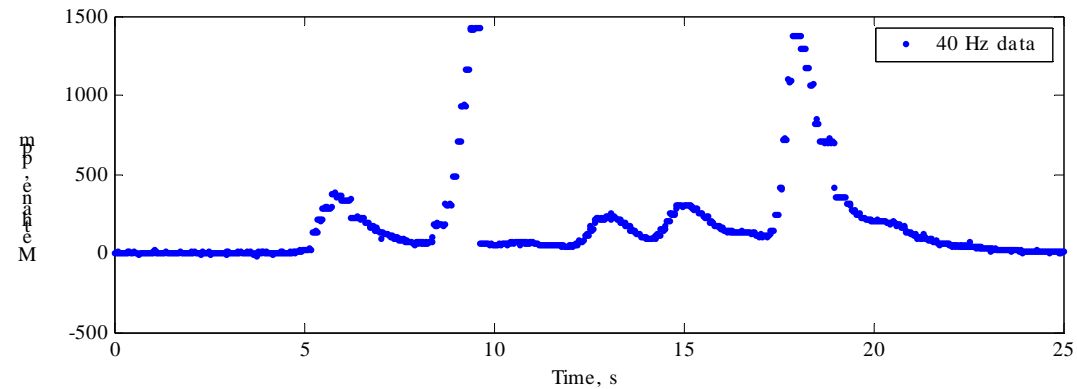
Larger window size gives more stable signal, whereas the peak height decrease so much that the obtained peaks may not be able to count gas dispersion properly.



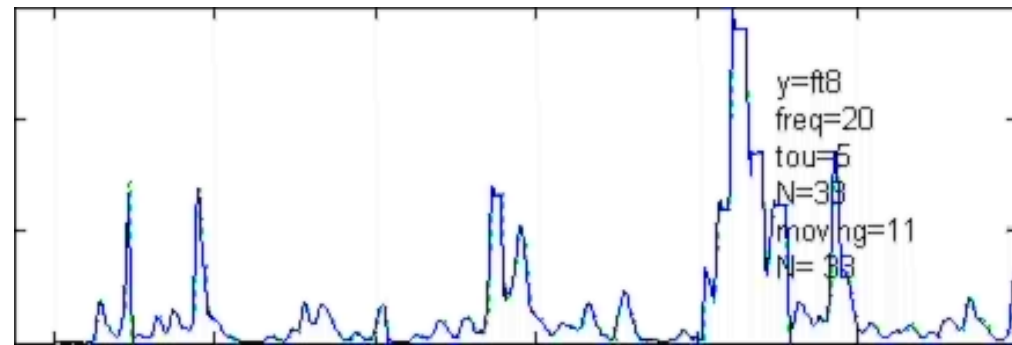
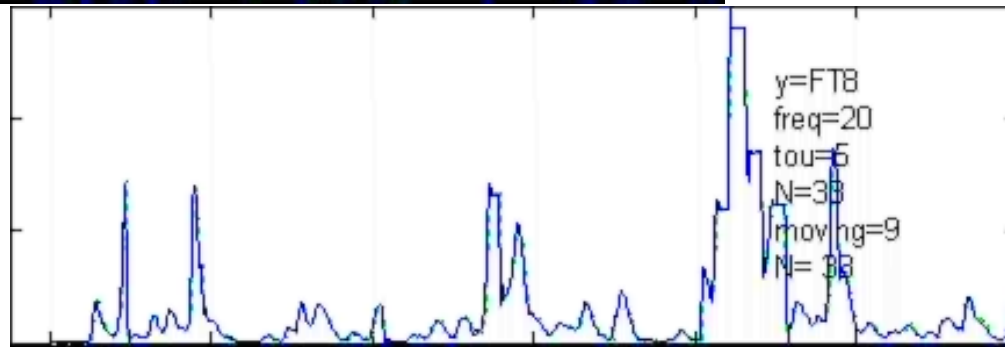
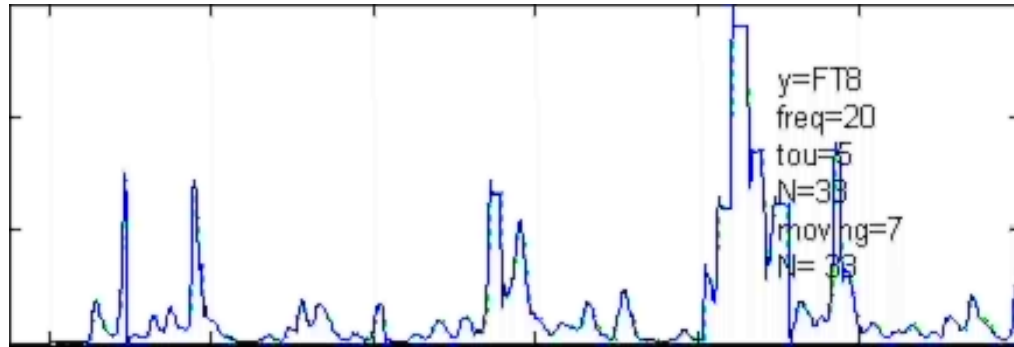
## The time series gas concentration (methane)

The length is usually 899 sec and sampling frequency 40 Hz.

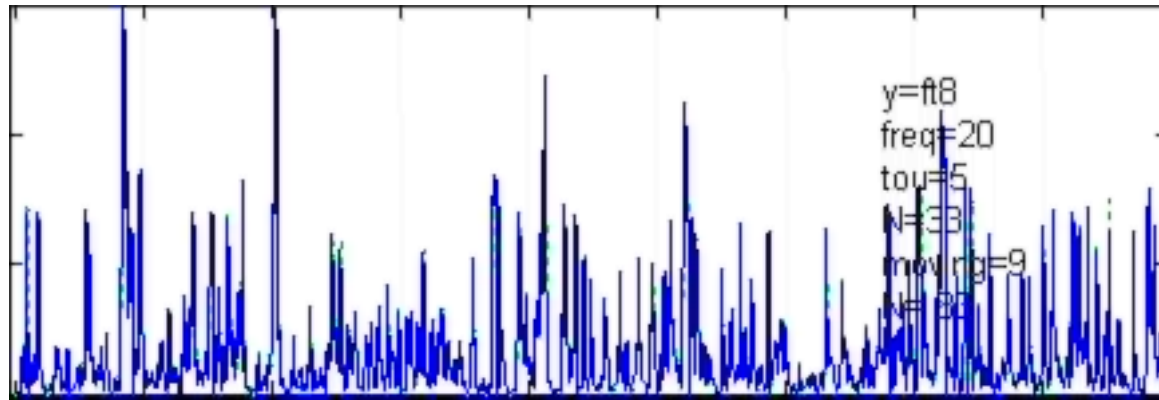
It was clear that the effect of sampling frequency is very strong. Smaller frequency is better for the stable deconvolution. First the frequency of the real signal has been reduced half and fortunately *the change of 40 to 20 Hz almost does not affect the signal shape*



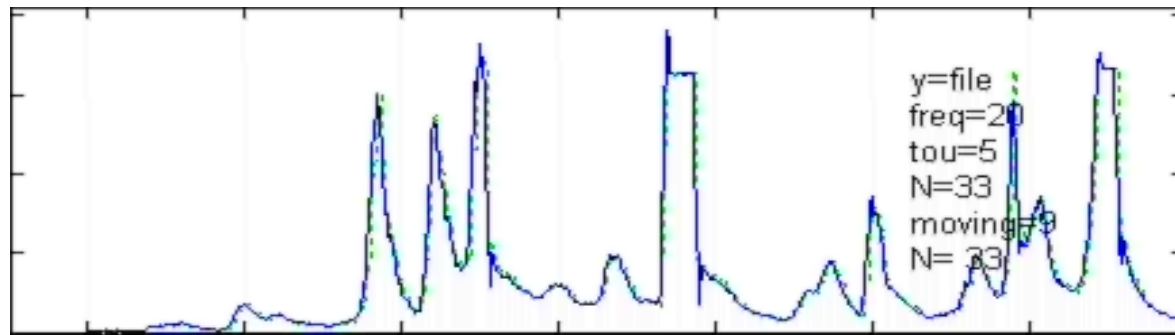
The gas concentration signal of 120 s (methane) was deconvoluted by tanks of  $\tau = 5$  and  $N=33$  with various window size of moving average.



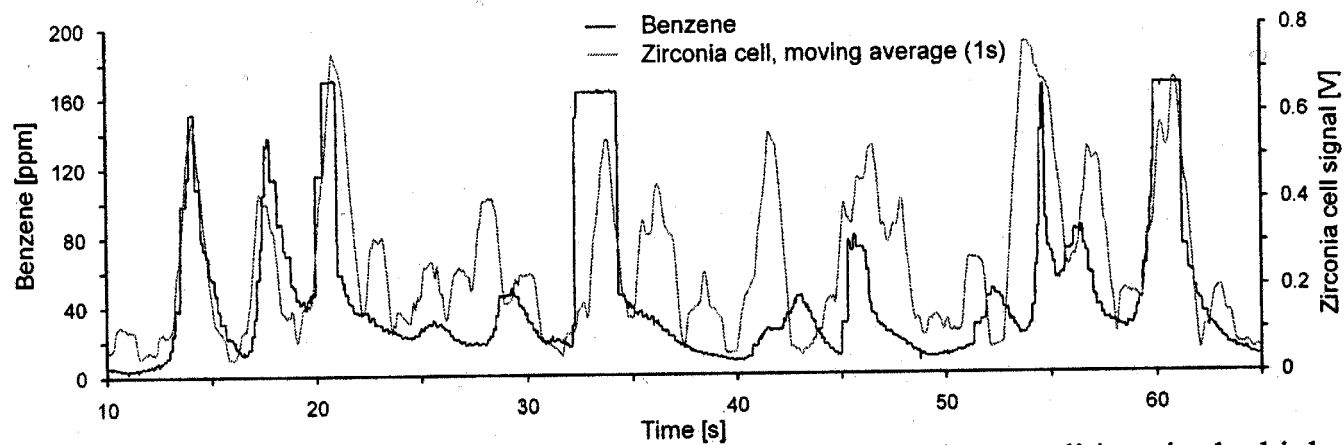
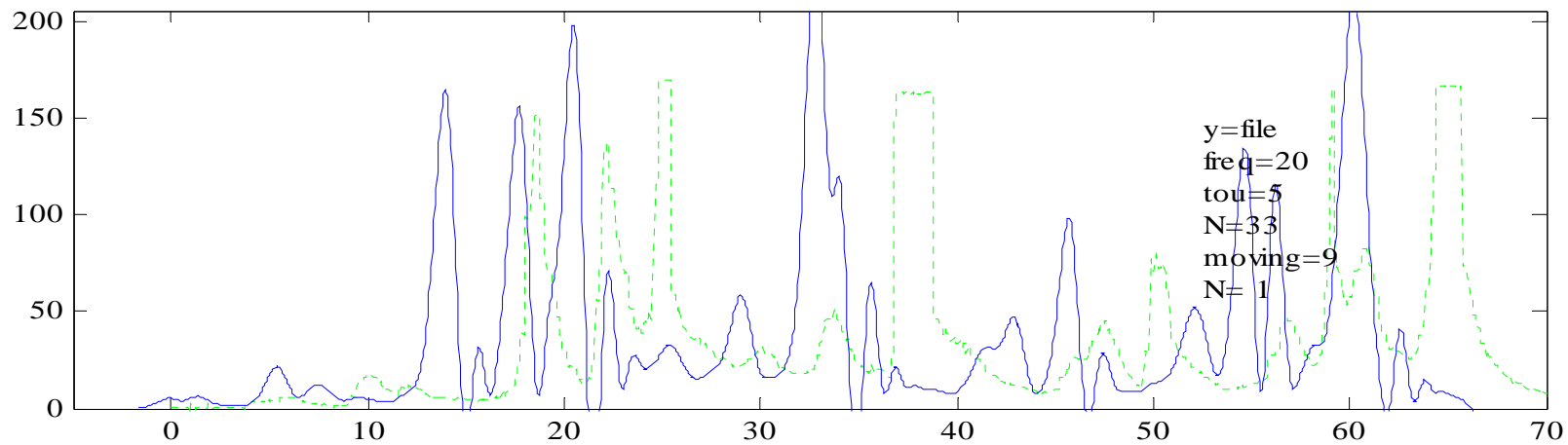
Full length of signal, 899 s can be treated, but just not so good to look at.



The gas concentration signal of 70 s (benzene) was deconvoluted by tanks of  $\tau = 5$  and  $N=33$  with window size of 9.



### Deconvolution by tanks



**Figure 8.** Time series of concentration of benzene and reducing conditions in the high load case ( $h = 3.7$  m).

## **Conclusions**

**Fast varying signal is easy to have inherent signal perturbation error that is coming from the repeated numerical differentiation of signal through each tank. The signal growth error greatly depends on the shape of signal to be recovered.**

**The parameter study tells that the numerical error can be reduced if tanks system has very small variance (very short residence time and larger number of tanks), however it would be a plug flow tube.**

**A compartment model consisted of a plug tube and tanks-in-series has been devised to reduce the number of differentiation in the system. This model was effective in reducing the chance of signal perturbation, but the model could not remove signal error completely.**

**Fast time varying signal of gas concentration from the fluidized bed boiler could be successfully recovered to its original shape by the deconvolution procedure with a moving average filter proposed in this study.**