PID 제어기의 자동 조절을 위한 전달 함수 근사를 이용한 실시간 모델 식별법

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On-Line Identification using the Approximation of Transfer Function for Autotuning of PID Controller

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Introduction

Most of the industrial process control system are composed of the conventional PID(Proportional-Integral-Derivative) controller despite of a number of the advanced control strategies have been proposed. As a representative of many proposed methods for automatic tuning of the PID controller, P-controller method(Yuwana and Seborg, 1982) which identifies the first-order plus time delay model can be stated. This method involves the on-line closed-loop identification technique based on a single experimental test. Various modifications and extensions of the P-controller method have been proposed(Jutan and Rodriguez(1984), Hwang and Chang(1987), Lee(1989), Chen(1989), Hwang(1993), Sung et al.(1994), Hwang and Shiu(1994)).

However, since all the previous methods use the first order plus time delay model, there is a structural limitation to approximate the broad range of the closed-loop response frequency and the performance of control is not satisfactory. Lee(1990) proposed the closed-loop identification method using the second-order plus time delay model employing a Taylor series to treat time delay term using ultimate data matching technique. Hwang(1995) proposed the identification method using the second-order plus time delay model under the arbitrary test input signal.

All the above identification techniques using the P-controller method have a strong constraint that is, the gain(K_c) of P-controller should be chosen very carefully not to produce the overdamped response. And since the identification of the method parameters relies on a few points of the controlled response which is usually contaminated with noise, the accuracy of the identified model and the control performance can be degraded significantly.

Zervos et al.(1988) and Dumont et al.(1989) proposed the PID controller tuning method which identifies the process using a least squares method with a Laguerre series and determines the optimal controller parameters by the iterative optimization. However, the use of this method is confined with the application in off-line manner.

In this work, we propose a new on-line identification method which preserves the advantages of the previous works and avoids the above-mentioned shortcomings. The approximation of the closed-loop response was made using Laguerre polynomials and least squares method. And the second-order plus time delay model was derived by a simple model reduction method based on the frequency data.

On-Line Identification

Let's consider the typical feedback control system as Fig 1. It is assumed that the process transfer function, $G_p(s)$, is unknown. And the closed-loop transfer function, $G_{CL}(s)$, is as follows.

$$G_{CL}(s) = \frac{y(s)}{y_{sp}(s)} = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \tag{1}$$

It is assumed that the test signal is the step change in set point whose magnitude is A. From Eq(1), the open-loop transfer function, $G_p(s)$, can be calculated. It is assumed that the controller is a proportional controller($G_c(s) = K_c$).

$$G_p(s) = \frac{s \cdot y(s)}{K_c(A - s \cdot y(s))} \tag{2}$$

The steady-state gain can be estimated directly from the measurements of the steady-state values of controlled variable and manipulated variable(i.e., $K_p = \frac{y_\infty}{u_\infty}$).

Since the Laguerre series converge to zero in $L_2[0,\infty)$ space, let's introduce new variable, $\tilde{y}(s)$, when the P-controller with step set point change is used.

$$\tilde{y}(s) = y(s) - \frac{K_c K_p}{1+K_c K_p} \tag{3}$$

The new variable, $\tilde{y}(s)$ means the deviation from the new steady-state value after the step change in the set point. Then we can approximate the closed-loop response which has a zero steady-state value using the linear combination of Laguerre series via a simple least squares technique.

$$\underset{a_1,\cdots,a_n}{Minimize} \sum_{i=1}^{M} (\tilde{y}(t_i) - \hat{y}(t_i))^2 \tag{4}$$

where $\tilde{y}(t_i)$ is the i-th measurement of the deviated controlled variable from steady-state value. and $\hat{y}(t_i)$ denotes the approximation using the N Laguerre polynomials at i-th sampling time as follows.

$$\hat{y}(t_i) = \sum_{n=1}^{N} a_n l_n(t) \quad \text{where, } l_n(t) = \sqrt{2p}\frac{e^{-pt}}{(n-1)!}\frac{d^{n-1}}{dt^{n-1}}(t^{n-1}e^{-2pt}) \tag{5}$$

where a_n, $l_n(t)$ is the n th coefficient of Laguerre polynomial and Laguerre polynomial in time domain respectively and p is the time scale parameter.

Let's introduce the following vectors such as Y($M \times 1$), θ ($N \times 1$) and matrix Φ ($N \times M$).

$$\mathrm{Y} = \begin{bmatrix} \tilde{y}(t_1) \\ \tilde{y}(t_2) \\ \vdots \\ \tilde{y}(t_M) \end{bmatrix}, \quad \Phi = \begin{bmatrix} \varphi^T(t_1) \\ \varphi^T(t_2) \\ \vdots \\ \varphi^T(t_M) \end{bmatrix}, \quad \varphi(t) = \begin{bmatrix} l_1(t) \\ l_2(t) \\ \vdots \\ l_N(t) \end{bmatrix}, \quad \theta = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} \tag{6}$$

where M denotes the number of measurements and N is the number of Laguerre series Then Eq.(4) can be rewritten in a simple form as follows

$$\underset{\theta}{Minimize}\|Y-\Phi\theta\|^2 \tag{7}$$

The unique minimum of Eq(7) can be found when $\theta=(\Phi^T\Phi)^{-1}\Phi^T Y$. From Eq(2), (3), the process transfer function can be rewritten in the form of rational polynomials with the known coefficients.

$$G_p(s)=\frac{s\cdot(\sum_{n=1}^{N}a_n l_n(s)+\frac{K_cK_p}{1+K_cK_p})}{K_c(A-s\cdot(\sum_{n=1}^{N}a_n l_n(s)+\frac{K_cK_p}{1+K_cK_p}))} \tag{8}$$

However, the obtained high order process transfer function should be reduced for the autotuning of PID controllers. We present a simple model reduction technique by Sung and Lee(1995) based on the frequency data. Let's consider the second-order plus time delay model which has the following form.

$$G_m(s)=\frac{K_m e^{-\theta_m s}}{\tau_m{}^2 s^2+2\xi_m\tau_m s+1} \tag{9}$$

The amplitude ratio of the model is given by Eq.(10).

$$|G_m(j\omega)|=\left|\frac{K_m e^{-\theta_m s}}{\tau_m{}^2 s^2+2\xi_m\tau_m s+1}\right|_{s=j\omega}=\frac{K_m}{\sqrt{(1-\tau_m{}^2\omega^2)^2+(2\xi_m\tau_m\omega)^2}} \tag{10}$$

The amplitude ratio of the high order process transfer function can be approximated to that of the desired model transfer function using a simple least squares technique with frequency response. The difference between the square of the amplitude ratio of the process transfer function and that of model transfer function can be given

$$|G_p(j\omega)|^2-|G_m(j\omega)|^2=\frac{|G_p(j\omega)|^2((1-\tau_m{}^2\omega^2)^2+(2\xi_m\tau_m\omega)^2)^2-K_m{}^2}{((1-\tau_m{}^2\omega^2)^2+(2\xi_m\tau_m\omega)^2)^2} \tag{11}$$

where $|G_p(j\omega_i)|$ is the amplitude ratio of the process transfer function, Eq.(9), which is approximated with the Laguerre polynomials. Let's define the numerator of Eq(11) as error at the arbitrary frequency, $e(\omega_i)$, as following,

$$e(\omega_i)=\{K_m{}^2-|G_p(j\omega_i)|^2\}-\{2(2\xi_m{}^2-1)\tau_m{}^2|G_p(j\omega_i)|^2\omega_i{}^2+\tau_m{}^4|G_p(j\omega_i)|^2\omega_i{}^4\} \tag{12}$$

While a rational approximation of the process with the time delay term requires the iterative nonlinear optimization technique(Levy(1959), Sanathanan and Koerner(1963), Seborg et al(1989)), we have the linear equation for unknown variables which can be solved by a simple least squares technique. Using the result of least squares technique, we can easily calculate time constant(τ_m) and damping factor(ξ_m) as follows.

$$\tau_m=\sqrt[4]{\theta_2}\quad\text{and}\quad\xi_m=\sqrt{\frac{1}{2}(1+\frac{\theta_1}{\sqrt{\theta_2}})} \tag{13}$$

Using the obtained time constant and damping factor, the time delay(θ_m) can be calculated by the following simple phase angle relation.

$$\theta_m=\frac{\tan 2^{-1}(-2\xi_m\tau_m\omega,1-\tau_m{}^2\omega^2)-\angle G_p(j\omega)}{\omega} \tag{14}$$

Conclusion

In this work, our main objective is to model the process dynamics in the second-order plus time delay model which can be applied to the autotuning of PID controller directly in SISO(Single Input-Single Output) system. In the approximation of the closed-loop response, the Laguerre series are employed as was done in Zervos et al.(1988) and Dumont et al.(1989). And a similar approach of Sung and Lee(1995)'s was made in that application of least square method and frequency response data. In the present method, the test signal is not restricted to the step change in the set point, so the other test signal such as pulse test can be used according to the appropriate purpose. This present identification also can be extended to the other control condition such as PI or PID controller. And this method does not use the approximation for the time delay term such as Padé approximation. Several simulation studies show that still preserving the flexibility and simplicity, the present method has the better performance and robustness in the identification and control in comparison with the previous methods.

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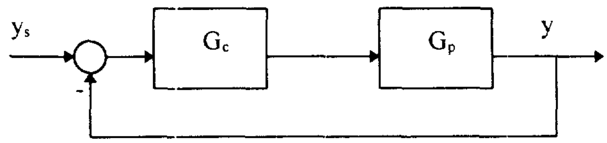

Fig. 1. Typical feedback control system