## 증류탑의 비선형 제어

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# CONSTRAINED NONLINEAR CONTROL OF A DISTILLATION COLUMN

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## Introduction

Various linear control techniques have been applied to the control of a distillation column. However, high nonlinearity of the column limits the performance of the linear control and attempt to implement nonlinear control is recently introduced to improve the control performance. In the meantime, the effectiveness of the constrained control is already proved in the linear control technique, such as the dynamic matrix control[1] and it is especially suitable to chemical processes.

In this study, a nonlinear model of a distillation column is formulated and applied to the control of the column. Also, input constraints are included and control computation is conducted by the successive quadratic programming (SQP)[2]. Numerical investigation of the control performance is carried out through rigorous simulation and the performance is compared with the result of the quadratic dynamic matrix control[3] having the same tuning parameters.

## Nonlinear Model

The size of dynamic model for control is limited by the computation time, since the calculated result has to be obtained within sampling time. In case of binary distillation column, mostly two controlled variables are adjusted by two manipulated variables. However, many stages are connected in the column and measurement and manipulation of those variables are conducted at the both ends of the column. Though neither measurement nor manipulation has done in the stages between two end stages, solving material and energy balances of those stages is necessary to find the solution of two end stages. In other words, a tremendous computation load is imposed in hidden stages without measurements like liquid and vapor flow rates and compositions. Moreover, the prediction of

product composition in the future steps is excised for various combinations of the expected manipulated variables. Therefore, the control model can not be such complex.

In this study, material balance is only used and the assumption of constant molal over flow is applied. Equilibrium calculation utilizes constant relative volatility and the Francis weir equation is employed for the stage liquid holdup.

Component material balance at each tray is formulated as

$$\frac{dx_n}{dt} = \frac{1}{M_n} \left[ L_{n+1} x_{n+1} + V_{n-1} y_{n-1} - L_n x_n - V_n y_n \right]$$
 (1)

where  $M_n$  is liquid holdup at the  $n^{th}$  stage. There are two way to integrate Eq. (1) to find future compositions of top and bottom products. One is to integrate with possible future input variation and to find the sensitivity of output to the input. The sensitivity is used to optimize control objective. A major problem of this procedure is that constraints are not included in the process of optimization. The other is to employ the orthogonal collocation method and the SQP. Solution is found only at collocation points and simultaneous optimization is conducted. This one is also not suitable to the case of this study which doesn't have an explicit form of differential equation. In Eq. (1), the compositions are connected to the adjacent trays and equilibrium relation also required.

Therefore, a different approach is implemented in this study. Integration of Eq. (1) is conducted at every sampling moment of future prediction horizon and the integration employs the simple trapezoidal rule which reduces integration time. Initial conditions are found from measurements, but they are not sufficient for the integration. There constant molal overflow assumption is used to find liquid and vapor flow rates and liquid holdup is found using the Francis weir equation. While the holdup is assumed to be constant during the prediction horizon, vapor and liquid flow rates vary during optimization. Integration of Eq. (1) is conducted at every prediction step by updating composition recursively for the control computation. Initial steady state is assumed at the beginning of integration. The initial liquid and vapor flow rates are computed from reflux flow and vapor boilup rates with the constant molal overflow assumption.

#### Control algorithm

The control objective is the sum of weighted absolute error of predicted output and is given as

$$J = \Gamma^{T} (y_{s} - \hat{y})^{T} (y_{s} - \hat{y}) \Gamma$$
 (2)

where  $\Gamma$  is output error weight and adjusts significance between outputs and prediction steps. Minimization of Eq. (2) along with input movement constraint is conducted by the successive quadratic programming. Since the prediction of future output involves many differential equations and nonlinear equilibrium formula, output and its derivative are numerically computed and reformulated to be applied the standard quadratic programming.

The control objective is reformulated as

In order to apply QP procedure, a subproblem of Eq. (3) is composed as

$$\text{min.} \quad \frac{1}{2} \ d^T B d \ + \ \nabla J^T d$$

s. t. 
$$\nabla g^T d + g \ge 0$$
,  
 $u_1 - u \le d \le u_n - u$  (4)

where  $B = \nabla^2 L$ 

and L = J - u g

The solution of Eq. (3) is found from the solution of Eq. (4) like

$$\mathbf{u}_{\mathbf{k}+\mathbf{l}} = \mathbf{u}_{\mathbf{k}} + \mathbf{r} \, \mathbf{d} \tag{5}$$

where r is weighting factor for the recursive update of the solution.

#### Results and discussion

The performance of the proposed control scheme is examined through rigorous simulation of a binary distillation column for step changes of top and bottom product compositions and feed flow rate change. The simulation model includes energy balance, the van Laar equilibrium equation and the nonlinear Francis weir equation and these are not shown in the control process model. The performance of this study is compared with the result of the QDMC.

Major tuning parameters of the both controls are taken as same. The step number of control horizon is 4 and error weights of top and bottom are 1 and 3. The input limit are 120% and 80% of steady state values for upper and lower limits, respectively. The limit of input movement is 0.01 in normalized value. of input movement limit.

In the set-point change of top product composition, the NLC of this study shows less deviation than the QDMC in both top and bottom compositions. Also, fast approach to the set point is observed in the NLC. For the change of bottom product, slow response and high deviation are obtained in top product composition, but no oscillation is shown. In the bottom composition, fast approach with no deviation is found with the NLC. In the change of feed flow rate, the NLC gives higher deviation, but no oscillation is observed.

While the control performances of both controls result similar deviation, the QDMC gives oscillatory response in all cases. When the variation of input is examined, the response makes sense. The is not shown in the NLC. A large number of prediction horizon is utilized in the QDMC because of long trailing step response of the distillation column, but control horizon is relatively small owing to the short computation time. By increasing the control horizon the width of oscillation is lowered. However, it is limited by the sampling time.

#### Conclusion

The control performance of this study and the QDMC is compared and it indicates that the NLC gives same performance to the QDMC in general. However, the oscillatory response of the QDMC is removed and long preparation time of dynamic matrix required in the QDMC is not necessary.

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## References

- 1. Cutler, C. R. and B. L. Ramaker, Proc. ACC, Green Valley, AZ, (1980)
- 2. Edgar, T. F. and D. M. Himmelblau, "Optimization of Chemical Processes," p.348, Mc-Graw Hill Book Co., N. Y. (1988).
- 3. Garcia, C. E. and A. M. Morshedi, Proc. ACC, San Diego, CA (1984)