## **Design of IMC Filter for Improved Disturbance Rejection of PID Controller**

M. Shamsuzzoha, Kihong Lee, Moonyong Lee<sup>\*</sup>, Jietae Lee<sup>1</sup>

School of Chemical Engineering and Technology, Yeungnam University, <sup>1</sup>Department of Chemical Engineering, Kyungpook National University  $(mv nlee@vu.ac kr^*)$ 

## **1. Introduction**

The PID (proportional, integral, and derivative) control algorithm is widely used in the process industries because of its simplicity, robustness and successful practical application. The well-known IMC (internal model control) PID tuning rules have the advantage that a clear tradeoff between closed-loop performance and robustness to model inaccuracies is achieved with a single tuning parameter (Rivera et al., 1986). The IMC-PID tuning method (Rivera et al., 1986; Morari and Zafiriou, 1989, Lee et al., 1998) and the direct synthesis method (Smith et al., 1975) are typical of the tuning methods based on achieving a desired loop response for set point tracking. However, several workers (Åström et al., 1993; Bergh and MacGregor, 1987; Chien and Fruehauf, 1990; Ho et al., 1994; and Horn et al., 1996) reported that the suppressing load disturbance is poor, when the process dynamics are significantly slower than the desired closed-loop dynamics. Horn et al. (1996) developed an IMC-PID tuning rule of the form of PID with the second order lead lag filter. It has clear advantage over the conventional IMC filter structure, but still lagging with Ziegler and Nichols (1943).

Therefore, we have proposed a filter structure that has the better performance for the lag time dominant process than Horn et al. as well as Ziegler and Nichols. This tuning rule has the PID controller in series with a filter and is easily implemented on a modern control hardware. The basic approach is similar to Brosilow and Markate (1992) and Scali et al., (1992).

# **2. Development of Controller Design Algorithm**

The stable (no right half plane pole) process model of the form:

$$
\widetilde{\mathbf{g}}_{p}(s) = \widetilde{\mathbf{g}}_{p^{+}}(s)\widetilde{\mathbf{g}}_{p^{-}}(s)
$$
\n(1)

where  $g_p(s)$  is the portion of the model inverted by the controller,  $g_{p+}(s)$  is the portion of the model not inverted by the controller and  $g_{p+}(0)=1$ . The idealized internal model controller is then designed as the inverse of the invertible portion of the process model.

$$
\widetilde{q}(s) = \widetilde{q}_{p^{-1}}(s) \tag{2}
$$

To make the controller proper, it is needed to add the filter. A transfer function is proper if the order of the denominator is at least as high as the numerator polynomial.

$$
q(s) = \tilde{q}(s) f(s) = \tilde{q}_P^{-1}(s) f(s)
$$
\n<sup>(3)</sup>

In the IMC, the complementary sensitivity is equal to (Garica and Morari, 1982; Rivera et al., 1986 and Horn et al., 1996)

$$
T=\widetilde{g}_{p^{+}}(s) f(s)
$$
\n<sup>(4)</sup>

where the *f(s)* is the filter.

It is important to select the proper filter structure that gives a good disturbance rejection. The conventional IMC filter *f(s)* is

Type I: 
$$
f(s) = \frac{1}{(\lambda s + 1)^n}
$$
 (5)

The type I filter ensures a stable and efficient setpoint tracking. The filter order *n* is selected large enough to make  $q(s)$  proper. Horn et al. (1996) proposed the alternative filter form:

Type II: 
$$
f(s) = \frac{\beta s + 1}{(\lambda s + 1)^n}
$$
 (6)

where  $\beta$  is chosen so that the slow pole of process model is canceled by a zero.

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The alternative filter for the proposed study for better performance is as:

Type III: 
$$
f(s) = \frac{\beta s + 1}{(\lambda^2 s^2 + 2\lambda \zeta s + 1)}
$$
 (7)

where ζ is a damping factor. It provides a measure of the amount of damping in the system. The IMC structure can be rearranged to the feedback control structure and the corresponding feedback controller  $\frac{1}{10}$  is  $\frac{1}{10}$   $g_c (s) = \frac{q(s)}{1 - \tilde{g}_p(s)q(s)}$ 

(8)

Since the model for chemical processes are usually of low order, the IMC controllers based on these models are of low order and can be written in the form of a PID controller in series with a secondorder lead-lag filter.

$$
G_c(s) = k_c \left( 1 + \tau_{D} s + \frac{1}{\tau_{I} s} \right) \frac{1 + cs + ds^2}{1 + as + bs^2}
$$
 (9)

where  $k_c$  is the proportional gain,  $\tau_L$ ,  $\tau_D$  are integral and derivative time constants respectively. a, b, c, and d are filter parameters. The second-order filter ensures that the nominal PID controller is proper and is easily implemented using modern control hardware.

## **3. Example: Tuning Rules for First Order Plus Dead Time (FOPDT) Model.**

The most commonly used approximate model for chemical processes is the first-order plus dead time model given below

$$
G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \tag{10}
$$

where K is the steady-state gain,  $\tau$  is the time constant,  $\theta$  is the time delay.

After rearranging and solving for *q(s)* we should have:

$$
q(s) = \frac{1}{K} \frac{(\tau s + 1)(\beta s + 1)}{(\lambda^2 s^2 + 2\lambda \zeta s + 1)}
$$
(11)

Substituting the value in Eq. (8) and rearranging, we get the equivalent standard feed back controller.  $(12)$ 

$$
g_c(s) = \frac{(\tau s + 1)(\beta s + 1)}{K(\lambda^2 s^2 + 2\lambda\zeta s + 1 - e^{-\theta s}(\beta s + 1))}
$$
(12)  
The time delay has been modeled with the first order Pade approximation and can be rearranged to be

The time delay has been modeled with the first order Pade approximation and can be rearranged to be in the form of Eq. (9) with different parameters as:

$$
k_c = \frac{2\tau + \theta}{2(2\varsigma\lambda + \theta - \beta)K}, \tau_1 = \tau + \frac{\theta}{2}, \tau_D = \frac{\tau\theta}{2\tau + \theta} \ a = \frac{2\lambda\varsigma\theta + 2\lambda^2 + \beta\theta}{2(2\varsigma\lambda + \theta - \beta)}, b = \frac{\lambda^2\theta}{2(2\varsigma\lambda + \theta + \beta)}, c = \beta, d = 0 \quad (13)
$$

The extra degree of freedom  $\beta$  is selected to cancel the open-loop pole, which causes the sluggish response to load disturbance:

$$
\beta = \frac{(8\varsigma\lambda + 4\theta)\tau + 2\theta^2 - 4\lambda^2}{4(\theta + \tau)}
$$
\n(14)

The  $\beta$  in Eq. (14) is important to cancel the slow pole of the process, which cause the sluggish response in load disturbance.

#### **4. Simulation Study**

Simulation study is carried out to the wide range of  $\theta/\tau$  (i.e., delay/lag time) ratios. The adjusted value of closed loop time constant *λ,* and integral of the squared error (ISE) is presented in Table 1. The comparison of ISE for equal maximum peak of tuning methods is one of the fair performance indexes. Fig. 1 shows the variation of ISE with  $\theta/\tau$  for equal peak, as getting by other tuning methods by adjusting the *λ*. As from the figure, the proposed tuning rule gives the smallest ISE among all tuning rules over the entire θ/τ range. The difference in ISE values for various filter structures decreases as the process is going to dead time dominant range. It also gives very less difference with the ISE by the Ziegler and Nichols as the process have extreme time dominant range, but still shows better

performance. Fig. 2 shows the output disturbance *vs.* time for θ/τ=0.1, the proposed method has rapid disturbance rejection than Ziegler and Nichols and ISE value is also less. The tuning method by Horn et al. gives highly oscillatory response. On the other hand, for the conventional filter, it is impossible to get the equal maximum peak and is also very sluggish. The performance of PID controller by Ziegler and Nichols rapidly deteriorates as the process goes into dead time dominant region while it seems to be satisfactory for the lag time dominant process. The proposed filter structure shows better performance over Ziegler and Nichols even for the lag time dominant process such as  $\theta/\tau$ =0.05. Since the process have slow pole, which cause the sluggish response,  $\beta$  in Eq. (14) is used to cancel this slow pole. The proposed filter structure provides control engineer more flexible options from under-damped but fast response to over-damped but stable response. In this study, the over-damped option is chosen for stability and small *λ* for getting a fast output response.

#### **5. Robustness** *vs* **Performance Tradeoff**

The magnitude of *T* in Eq. (4) quantifies the robustness of single-loop systems. The Bode plots of *T* for different controller design are shown in Fig. 3 for  $\theta/\tau=0.1$ . The figure shows that the conventional filter and proposed tuning have almost similar robustness, but the Horn et al., has poor robustness. The conventional filter results in very poor load disturbance suppression, even the very low value of *λ.* The proposed tuning method provides a tradeoff between robustness and best performance. The conventional filter provides a tradeoff between robustness and poor performance and in Horn et al., neither good performance nor robustness for controller.

#### **6. Conclusions**

Key conclusions from the present study are as follows:

a). The tuning rule based on the conventional filter has good disturbance tracking in the dead time dominant process. As the ratio of dead time to time constant is less then one, the conventional filter gives very sluggish response.



*Fig. 1 Variation of ISE in lag time lag time process.* 

#### **Table: 1 Tuning Rule and corresponding ISE value**





*(4).* 

b). The proposed filter has clear advantage over the Horn et al., as well as Ziegler and Nichols tuning rule, since the ISE of the proposed tuning rule is minimum. For the equal maximum peak, the proposed tuning rule has rapid disturbance rejection as well as less ISE over all other tuning rule. c). Due to approximation (1/1 Pade approximation) error in dead time, the derived IMC-PID controller

shows a little bit worse performance than the ideal IMC controller. But it closely approximates the IMC controller and thus gives better performance than others.

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