IMC-PID Controller Tuning for Improved Disturbance Rejection of Unstable Time Delay Processes

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Introduction

The PID control algorithm is widely used in the process industries for unstable process because of its simplicity, robustness and successful practical application. The well-known IMC-PID tuning rules have the advantage that a clear trade-off between closed-loop performance and robustness to model inaccuracies is achieved with a single tuning parameter. It is well known that the IMC-PID controller provides good setpoint tracking but sluggish disturbance response especially for the process with a small time-delay/time-constant ratio. However, for many process control applications, disturbance rejection for the unstable processes is much more important than setpoint tracking. Therefore, controller design that emphasizes disturbance rejection rather than setpoint tracking is an important design problem that has received renewed interest recently.

The IMC-PID tuning methods (Rivera *et al.*, 1986; Morari and Zafiriou, 1989; Chien and Fruehauf, 1990; Horn *et al.*, 1996; Lee *et al.*, 1998; Lee *et al.*, 2000) and the direct synthesis method (Smith *et al.*, 1975; Chen and Seborg, 2002) are the typical tuning methods based on achieving a desired closed-loop response. They obtained the PID controller parameters by computing the controller which gives the desired closed-loop response. Lee *et al.* (1998) proposed the IMC-PID controller based on a two-degree-of-freedom (2DOF) control structure to improve disturbance performance. Lee *et al.* (2000) extended their tuning method for the unstable process such as FODUP and SODUP models.

In this study, we propose an optimum IMC filter to design an IMC-PID controller for disturbance rejection of unstable processes. The proposed controller gives better performance than earlier reported Lee *et al.* (2000). The concept of 2DOF controller is used to cope with setpoint performance.

Controller Design Algorithm

The details of IMC structure is presented in Morari and Zafiriou (1989), where $G_P(s)$ process, $\tilde{G}_P(s)$ process model and $q(s)$ is the IMC controller. The controlled variable are related as

$$C = \frac{G_P(s)q(s)}{1+q(s)\left(G_P(s)-\tilde{G}_P(s)\right)}R + \left[\frac{1-\tilde{G}_P(s)q(s)}{1+q(s)\left(G_P(s)-\tilde{G}_P(s)\right)}\right]G_D(s)d \quad (1)$$

For the nominal case (*i.e.*, $G_P(s)=\tilde{G}_P(s)$), the setpoint and disturbance responses are simplified as

$$\frac{C}{R} = G_P(s)q(s) \quad (2)$$

$$\frac{C}{d} = \left[1-G_P(s)q(s)\right]G_D \quad (3)$$

The equivalent of IMC for classical feedback control structure the setpoint and disturbance responses are represented by

$$\frac{C}{R} = \frac{G_C(s)G_P(s)}{1+G_C(s)G_P(s)}\text{; and } \frac{C}{d} = \frac{G_D(s)}{1+G_C(s)G_P(s)} \quad (4)$$

where $G_c(s)$ denotes the feedback controller.

The IMC controller design involves two steps:

Step 1: A process model $\tilde{G}_P(s)$ is factored into

$$\tilde{G}_P(s) = P_M(s)P_A(s) \quad (5)$$

where $P_M(s)$ is the portion of the model inverted by the controller; $P_A(s)$ is the portion of the model not inverted by the controller (it is usually a non-minimum phase and contains dead times and/or right half plane zeros); $P_A(0)$= 1.

Step 2: The idealized IMC controller is the inverse of the invertible portion of the process model.

$$\tilde{q}(s) = P_M^{-1}(s) \quad (6)$$

To make the controller proper, it needs to add the filter. Thus, the IMC controller is designed by

$$q(s) = \tilde{q}(s) f(s) = P_M^{-1}(s) f(s) \quad (7)$$

The ideal feedback controller equivalent to the IMC controller can be expressed in terms of the internal model, $\tilde{G}_P(s)$, and the IMC controller, $q(s)$:

$$G_C(s) = \frac{q(s)}{1 - \tilde{G}_P(s) q(s)} \quad (8)$$

Since the resulting controller has not a standard PID controller form, the remaining issue is to design the PID controller that approximates the equivalent feedback controller most closely. Lee *et al.* (1998) proposed an efficient method for converting the ideal feedback controller $G_c(s)$ to a standard PID controller. Since $G_c(s)$ has an integral term, it can be expressed

$$G_c(s) = \frac{f(s)}{s} \quad (9)$$

Expanding G_c in Maclaurin series in s gives

$$G_c(s) = \frac{1}{s}\left(f(0) + f'(0)s + \frac{f''(0)}{2}s^2 + \ldots \right) \quad (10)$$

The first three terms of the above expansion can be interpreted as the standard PID controller given by

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s + \ldots \right) \quad (11)$$

Where, $K_c = f'(0)$; $\tau_I = f'(0)/f(0)$; $\tau_D = f''(0)/2f'(0)$; $\tau_D \geq 0$; $\tau_I \geq 0$ (12)

Selection of Filter for Design of IMC-PID Controller Tuning Rules

One common problem in the conventional IMC-PID controller design approaches is that they select the IMC filter merely based on the resulting IMC performance while the ultimate goal of the filter design in the IMC-PID approach is to find the best PID controller. In the conventional approach for the filter design, it is assumed that the best IMC controller results in the best PID controller. However, since all the IMC-PID approaches utilize some kind of model reduction techniques to convert the IMC controller to the PID controller, approximation error occurs essentially. This error becomes severe for the process with time delay. Therefore, if some IMC filter gives best IMC performance but structurally causes a significant error in conversion to the PID controller, then the resulting PID controller could have poor control performance. Performance of the resulting PID controller depends on this combined effect which is also directly related to filter structure and process model. Therefore, there exists the optimum filter structure for each specific process model to give the best PID performance. For a given filter structure, as λ decreases, the discrepancy between the ideal and the PID controller increases while the nominal IMC performance improves. It indicates that an optimum λ value also exists which compromises these two effects to give the best performance. Thus what we mean by the best filter structure is the structure to give the best PID performance for the optimum λ value.

Our investigation shows that the high order filter structures generally give better PID performance over the low order filter structures. For example, for a *FODUP* model, it is found that the high order filter, $f(s) = (\beta s + 1)^2/(\lambda s + 1)^3$, provides the best disturbance rejection in terms of minimum IAE.

First Order Delayed Unstable Process (FODUP)

The most commonly used approximate unstable process model for chemical processes is the *FODUP* model

$$G_p(s) = G_D(s) = \frac{Ke^{-\theta s}}{\tau s - 1} \quad (13)$$

The optimum IMC filter structure is found as $f(s) = (\beta s + 1)^2/(\lambda s + 1)^3$. Then, the IMC controller becomes $q(s) = (\tau s - 1)(\beta s + 1)^2 / K(\lambda s + 1)^3$. Thus, the ideal feedback controller equivalent to the IMC controller is

$$G_C = \frac{(\tau s - 1)(\beta s + 1)^2}{K\left[(\lambda s + 1)^3 - e^{-\theta s}(\beta s + 1)^2\right]} \quad (14)$$

Expanding $G_c(s)$ in a Maclaurin series in s gives

$$k_C = -\frac{\tau_I}{K(3\lambda - 2\beta + \theta)}; \quad \tau_I = (-\tau + 2\beta) - \frac{\left(3\lambda^2 - \theta^2/2 + 2\beta\theta - \beta^2\right)}{(3\lambda - 2\beta + \theta)}; \quad \tau_D = \frac{(-2\tau\beta + \beta^2) - \frac{\left(\lambda^3 + \theta^3/6 - \beta\theta^2 + \beta^2\theta\right)}{(3\lambda - 2\beta + \theta)}}{\tau_I} - \frac{\left(3\lambda^2 - \theta^2/2 + 2\beta\theta - \beta^2\right)}{(3\lambda - 2\beta + \theta)} \tag{15}$$

The value of β is calculated by solving $\left[1 - \frac{(\beta s+1)^2}{(\lambda s+1)^3} e^{-\theta s}\right]_{s=-\frac{1}{\tau}} = 0$ and the value is $\beta = \tau\left[\left(\left(1 + \frac{\lambda}{\tau}\right)^3 e^{\theta/\tau}\right)^{1/2} - 1\right]$

Second Order Delayed Unstable Process (SODUP)

The process model is

$$G_p(s) = G_D(s) = \frac{Ke^{-\theta s}}{(\tau s - 1)(as + 1)} \tag{16}$$

The optimum filter is found as $f(s) = (\beta s+1)^2/(\lambda s+1)^4$ and resulting tuning rules is listed in Table 1.

Table 1. PID controller tuning rules for *SODUP*

$G(s) = G_D(s) = \frac{Ke^{-\theta s}}{(\tau s - 1)(as + 1)}$	$k_C = -\frac{\tau_I}{K(4\lambda - 2\beta + \theta)}$	$\tau_I = (a - \tau + 2\beta) - \frac{\left(6\lambda^2 - \theta^2/2 + 2\beta\theta - \beta^2\right)}{(4\lambda - 2\beta + \theta)}$
$\tau_D = \frac{\left(\beta^2 - 2\beta(-a + \tau) - a\tau\right) - \frac{\left(4\lambda^3 + \theta^3/6 - \beta\theta^2 + \beta^2\theta\right)}{(4\lambda - 2\beta + \theta)}}{\tau_I} - \frac{\left(6\lambda^2 - \theta^2/2 + 2\beta\theta - \beta^2\right)}{(4\lambda - 2\beta + \theta)}$		$\beta = \tau\left[\left\{\left(\frac{\lambda}{\tau} + 1\right)^4 e^{\theta/\tau}\right\}^{1/2} - 1\right]$

Filter for the dead time dominant process

In the case of dead time dominant process (*i.e.*, $\theta/\tau >> 1$), the filter time constant should be chosen as $\lambda \approx \theta >> \tau$ for stability. Therefore, the process pole at $-1/\tau$ is not a dominant pole in the closed-loop system. Instead, the pole at $-1/\lambda$ by the controller determines overall dynamics. Thus, to cancel the process pole by introducing the lead term $(\beta s+1)$ into the filter has little impact to speed up the disturbance rejection response. Furthermore, the lead term generally makes the IMC controller form more complicated, which in turn leads to performance degradation of the resulting controller by causing a large discrepancy between the ideal and the PID controller. As a result, in the case of dead time dominant process, the conventional filter without any lead term offers the best performance.

Simulation Study

Example 1.

The following *FODUP* model (Lee *et al.*2000) was studied.

$$G_p(s) = G_D(s) = \frac{1e^{-0.4s}}{s - 1} \tag{17}$$

For simulation, unit step changes are introduced into the setpoint and the disturbance, sequentially. The simulation result is shown in Fig. 1. Time response of proposed method is compared with those by Lee *et al.(*2000). For fair comparison, the optimum λ value to give the best achievable performance in terms of IAE was also used for other simulations. The resulting PID tuning values and the IAE are listed in Table 2. As seen in the IAE values, the PID controller by the proposed method offers the better performance both for disturbance rejection and setpoint tracking. However, under the 1DOF control structure, any controller for good disturbance rejection essentially accompanies an excessive overshoot in the setpoint response. To avoid this water-bed effect, a 2DOF control structure is used and the corresponding responses are shown in the figures.

Example 2.

The unstable process was considered as $G_p(s) = G_D(s) = \frac{1e^{-0.5s}}{(5s - 1)(2s + 1)(0.5s + 1)}$

The model can be approximated to the *SODUP* model (Lee *et al.*, 2000) as

$$G_p(s) = G_D(s) = \frac{1e^{-0.939s}}{(5s - 1)(2.07s + 1)} \tag{18}$$

The resulting PID tuning values and the IAE are listed in Table 3. Fig. 2 compares the closed-loop responses by the proposed method and by Lee *et al.* (2000).

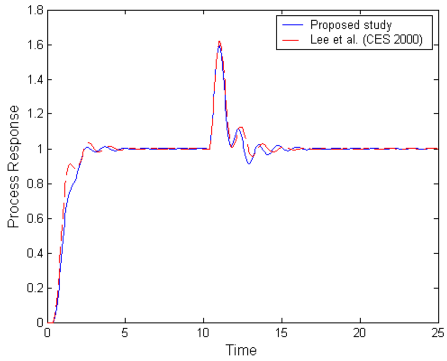

Fig. 1. Simulation results for Example 1

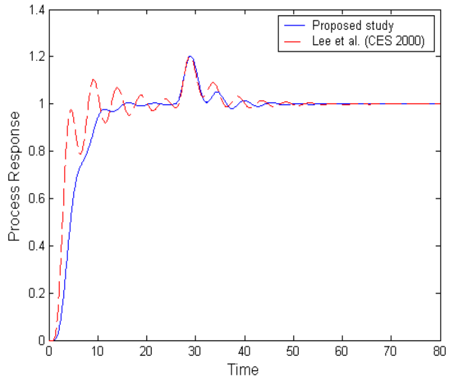

Fig. 2. Simulation results for Example 2

Table 2. PID controller setting for Example 1

Method	k_c	τ_I	τ_D	Dist.	Set.	2DOF
Le2 (λ=0.30)	3.28	1.71	0.171	0.580	1.808	1.046
PS (λ=0.39)	3.24	1.41	0.201	0.559	1.838	1.197

Table 3. PID controller setting for Example 2

Method	k_c	τ_I	τ_D	Dist.	Set.	2DOF
Le2 (λ=1.1)	7.63	6.41	1.63	0.965	6.034	3.946
PS (λ=0.88)	7.46	5.46	1.48	0.857	5.933	5.329

**PS=Proposed study; Le2=Lee *et al.*(2000)

Conclusions

Optimum IMC filter structures are proposed for unstable process such as FODUP and SODUP models to improve disturbance rejection performance of the PID controller. Based on the proposed filter structures, tuning rules for the PID controller was derived by using the generalized IMC-PID method by Lee *et al.* (1998). The simulation results demonstrated superiority of the proposed method.

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