레이놀즈수가 작은 영역에서의 단일구 종말 속도 측정

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Terminal Velocity Measurement of a Settling Sphere with a Low Reynolds Number

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Introduction

Objects falling through a fluid will attain a "terminal velocity" determined by gravity force, buoyant force and kinetic force. Our experiment was designed to investigate the effect of those various factors on the terminal velocity. In this experiment, fluids with high viscosity such as silicone oil and glycerine are used to slow down the terminal velocities. For the sake of the simple calculations, spherical balls were chosen. If the densities of balls and fluids, and the viscosity of fluids are known as well as measured are the radii and terminal velocities of the balls, we can calculate the drag coefficient (C_D) and Reynolds Number (Re)

Apparatus

Two sorts of fluids used in the experiment are silicone oil and glycerine as we need high viscous fluids. Silicone oil has 9.5 $g/sec \cdot cm$ of viscosity and 0.950 of specific gravity while glycerine has 14.0 $g/sec \cdot cm$ and 1.24, respectively. One kind of sphere is aluminum oxide ball of diameter 0.48 cm , mass 0.3 g and density 5.12 g/ml, and the another kind is made of zirconium oxide with 0.49 cm of diameter, 0.4 g of mass and 6.38 g/ml of density. Transparent messcylinder is used to observe the falling motion through the fluid in it.

Procedure

We obtain the radii of balls with a caliper and its mass from a scale. Care should be taken in all mass measurements since mass differences will enter as an important factor in later calculations. We use two fluids such as silicone oil and glycerine through which a ball goes down. Dropping the ball we record the falling sphere with a camcorder to get accurate data. We repeat the above procedure with two kinds of balls of aluminum oxide and zirconium oxide.

Theory

The magnitude of the kinetic force is proportional to a characteristic area A and a characteristic kinetic energy K per unit volume; thus

$$F_k = AKf \tag{1}$$

in which the proportionality constant f is called *friction factor*.

We can use C_D as a friction factor when an object falls through a fluid. Projected area (A_p) of a sphere to the fluid direction is defined by $(\pi/4)D_p^2$ where D_p is a diameter of projected area. The average drag force per unit projected area is F_k/A_p where F_k is kinetic force. Thus we can get C_D from

$$C_D = \frac{F_k/A_p}{\rho \cdot u_0^2/2}\,. \tag{2}$$

Here the kinetic force F_k is given by [1]

$$F_k = F_{form} + F_{friction} \tag{3}$$

where F_{form} and $F_{friction}$ mean form and friction drags respectively.

Thus, terminal falling velocity if Re < 1.0 is given [1] by

$$u_o = \frac{gD_p^2(\rho_p - \rho)}{18\mu}\,. \tag{4}$$

If Re < 1.0, C_D of a sphere follows the Stokes law [1]

$$C_D = \frac{24}{Re}\,. \tag{5}$$

Data

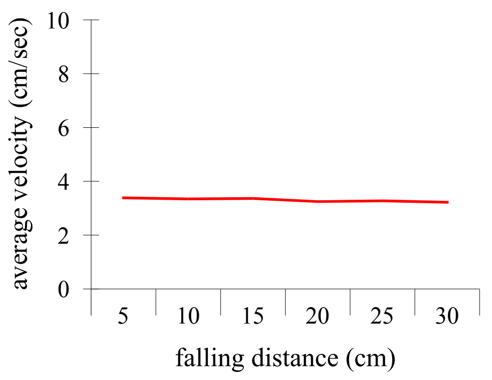

Figure 1. Average velocity vs. falling distance of an aluminum oxide with 0.48 cm diameter in glycerine.

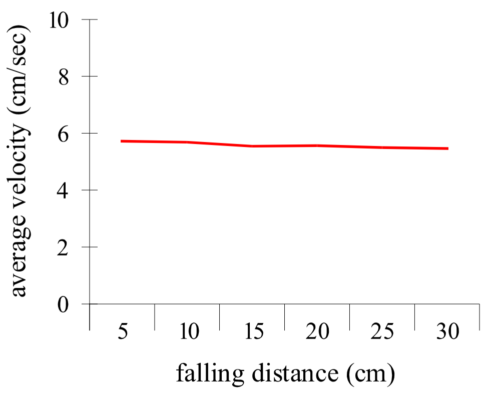

Figure 2. Average velocity vs. falling distance of a zirconium oxide with 0.49 cm diameter in glycerine.

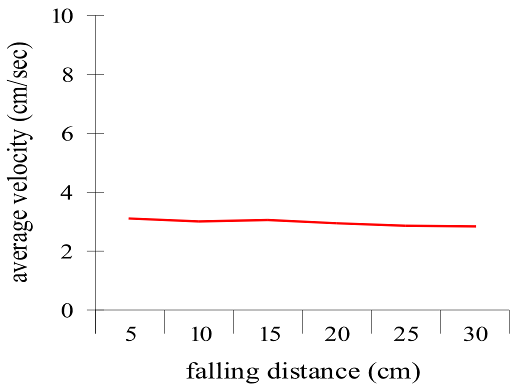

Figure 3. Average velocity vs. falling distance of an aluminum oxide with 0.48 cm diameter in silicone oil

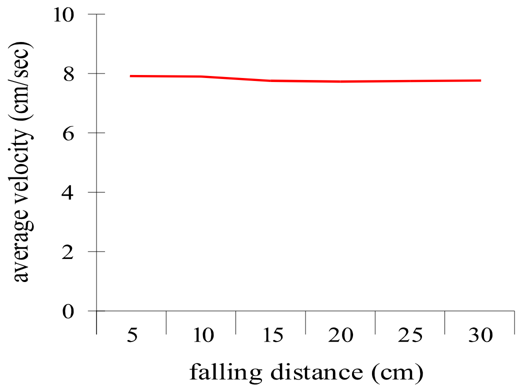

Figure 4. Average velocity vs. falling distance of a zirconium oxide with 0.49 cm diameter in silicone oil

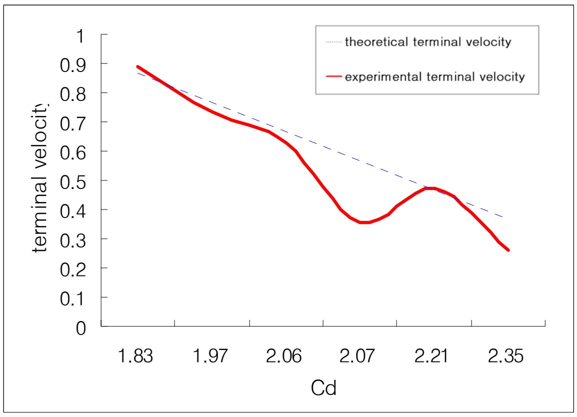

Figure 5. The relationship between C_D and terminal velocity.

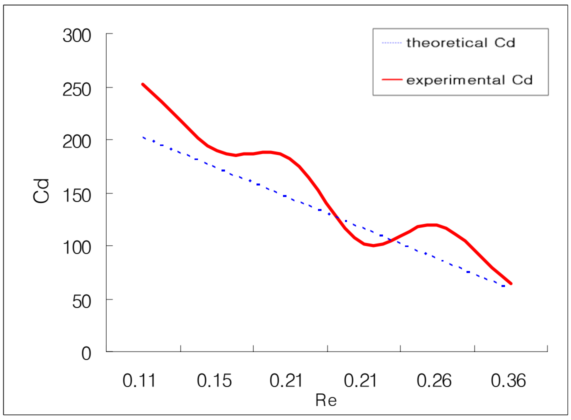

Figure 6. The diagram proves the equation (5) especially in the range of Re less than 1.

Conclusions

The experiment of a sphere sedimentation was designed to evaluate terminal velocity in order to get the relationship between C_D and Re. The experiments conducted in several different conditions let us know the fact that a settling sphere has a fast terminal velocity as it is more weighed or goes down through lower viscous fluid. The fluids used in the experiment are highly viscous enough to result in very low Re such as 0.126 and 0.199 in case of aluminum balls. In the conclusion, if the densities of a ball and fluids, and the viscosity of fluids are known, as well as measured are the radii and terminal velocities of the balls, we are allowed to find out how C_D and Re are related each other. Especially in the low Re like the experiment, it follows the Stoke's law leading the equation $C_D = 24/Re$

as proven in the Figure 6.

The results we have obtained from the experiment is not perfectly same as the theoretic values as shown in the Figure 5 and 6. We have concerned a lot about every possible errors that could drive us a wrong experiment result, but a few incorrect manipulations were made.

The experiment has been conducted from March to June as changing the temperature of the laboratory. The density and the viscosity of fluids are very sensitive variables to lead the experiment result as functions of temperature. The fact that we could not help keeping the temperature of the laboratory constant is the main reason for the error.

Recording the time of the falling balls was fully relied on a camcorder, and we played it back in a computer to measure the falling velocity. The presumption is that the small difference between the real one and the recorded one caused the error as every computer has its own output speed. We had considered the factor before doing it, but could not find any better choice.

Discussion

In this experiment, we have not allowed the interaction between the falling spheres which will interfere with each other and certainly provide much different results from present ones. This another assumption would be very useful as it is more likely to happen in the real world although that must be a harder trial with far more variables. Moving cars in the road and falling raindrops are the good examples in our future experiment.

References

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