Multiloop PI Controller Design Based on Disturbance Rejection of Multivariable Processes Using Ms Criterion

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Introduction

The multiloop PID/PI controller has already been used for the last two decades and it is still popular in process control nowadays. Therefore, there are many theories and practices about the control of multivariable systems. The proposed design method proceeds from the generalized IMC-PID approach [1], design of multiloop PI controller for disturbance rejection [2], and Ms tuning. In multiloop IMC control systems, the controller is based directly on the closed-loop time constant (λ), the PID tuning parameters are then a function of this factor, the selection of closed-loop time constant is directly related to the robustness of closed-loop system. The optimal values of closed-loop time constant can be obtained by using Ms Criterion. The proposed method can cancel the influence of disturbance because the feedback signal is equal to this influence and modifies the controller setpoint accordingly and it is more convenient for measurements of system sensitivity to noises and parameter variations.

1. Generalized IMC-PID method based on disturbance rejection for multiloop PI controller design

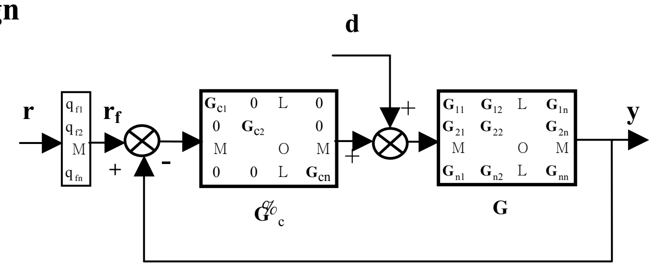

Figure 1. Block diagram for multi-loop control system

In multiloop control system, the *n* x *n* open-loop stable G(s) is expressed as

$$\mathbf{G}(s) = \mathbf{G}_{ij}(s) \qquad i=1,...,n \quad ; \quad j=1,...,n \tag{1}$$

It is controlled by a multi-loop diagonal controller $\tilde{\mathbf{G}}_c(s)$

$$\tilde{\mathbf{G}}_c(s) = diag[G_{c11}, G_{c22}, ..., G_{cnn}] \tag{2}$$

The multivariable process and the multi-loop controller with an integral term can be written in a Maclaurin series as

$$\mathbf{G(s)} = \mathbf{G_0} + \mathbf{G_1}s + \mathbf{G_2}s^2 + O(s^3) \tag{3}$$

$$\tilde{\mathbf{G}}_{ci}(s) = \frac{1}{s}\left[\tilde{\mathbf{G}}_{c0} + \tilde{\mathbf{G}}_{c1}s + \tilde{\mathbf{G}}_{c2}s^2 + O(s^3)\right] \tag{4}$$

where, $\mathbf{G}_0 = \mathbf{G}(0)$, $\mathbf{G}_1 = \mathbf{G}'(0)$, and $\mathbf{G}_2 = \mathbf{G}''(0)/2$. $\tilde{\mathbf{G}}_{c0}$, $\tilde{\mathbf{G}}_{c1}$ and $\tilde{\mathbf{G}}_{c2}$ keep up a correspondence with the integral, proportional, and derivative terms of the multi-loop PID controllers, respectively. The closed-loop transfer function of the multiloop control system which is shown in Fig. 1 is

$$\frac{\mathbf{y}(s)}{\mathbf{r}_f(s)} = \mathbf{H}(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{G}_c(s))^{-1}\mathbf{G}(s)\mathbf{G}_c(s) \tag{5}$$

Consider to the generalized IMC-PID approach, adding the IMC filter which can reject dominant pole in process control model, the desired closed-loop response $\mathbf{R}_i$ of the *i*th loop typically is represented as

$$\frac{\mathbf{y}_i}{\mathbf{r}_i'} = \tilde{\mathbf{R}}_i = \mathrm{diag}\{R_1, R_2, ..., R_i\} = \frac{G_{ii+}(s)\sum_{j=1}^{m}\left(\beta_{ij}s^j+1\right)}{\left(\lambda_i s+1\right)^{n_i}} \tag{6}$$

The multiloop controllers are then represented by

$$G_{ci}(s) = \frac{Q_i(s)}{1 - G_{ii}(s)Q_i(s)} = \frac{[G_{ii-}(s)]^{-1}\left(\sum_{j=1}^{m}\beta_{ij}s^j + 1\right)}{(\lambda_i s+1)^{n_i} - G_{ii+}(s)\left(\sum_{j=1}^{m}\beta_{ij}s^j + 1\right)} \tag{7}$$

It can be expressed in Maclaurin series with an integral term as

$$\mathbf{G}_{ci}(s) = \frac{1}{s}(f_i(0) + f_i'(0)s + \frac{f_i''(0)}{2}s^2 + O(s^3)) \tag{8}$$

Where, $f_i(s) = G_c(s)s$, $G_{ii+}(0) = 1$,.$\mathbf{Q}_i(s)$ is IMC controller, G_{ii+} is the nonminimum of G_{ii} , λ_i is an adjustable constant for system performance and stability, and n_i is selected large enough that makes the IMC controller would be proper, , m is number of poles to be cancelled .

The β_{ij} is defined to cancel the dominant poles in each element of processes, it is can be obtained as

$$\beta = \tau(1-(1-\frac{\lambda}{\tau})^2 e^{-\frac{\theta}{\tau}} \tag{9}$$

τ , θ is the time constant and delay time of each elements in process model.

Accordingly the design of multiloop PI controller for disturbance rejection which is proposed by T. Vu, J.T. Lee, and M.Y. Lee [2], the proportional gain and the integral time constant of the multi-loop PI controller can be obtained as

$$K_{ci} = f_i'(0) \tag{10}$$

$$\tau_{Ii} = -\frac{\left[G_{ii+}'(0) - n_i\lambda_i + \beta_{i1}\right]\mathbf{G}_{ci}}{\left[\mathbf{G}^{-1}(0)\right]_{ii}} \tag{11}$$

2. Ms Criterion for the multiloop control system

Ms tuning is one of the frequency-domain methods which related to the resonant peak Ms. The Ms values is related to the resonant peak of the sensitivity function, the relative stability of a stable closed-loop system can be suggested by the magnitude of Ms. In practical, the desirable value of Ms is usually between 1.2 and 2 [Astrom et al., 1998]. Ms tuning provides the limited of closed-loop time constant for a fixed-model and filter time constant, the optimal model and controller parameter can be found.

The sensitivity function in multiloop control system can be represented by

$$\mathbf{S}(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{G}_c(s))^{-1} \tag{12}$$

The sensitivity function can be expressed by the matrix form

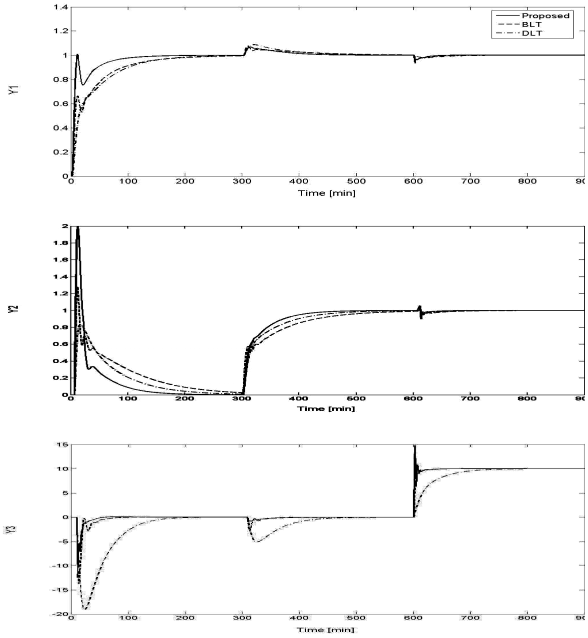

Figure 2- The response of PI control system in step setpoint change for OR column.

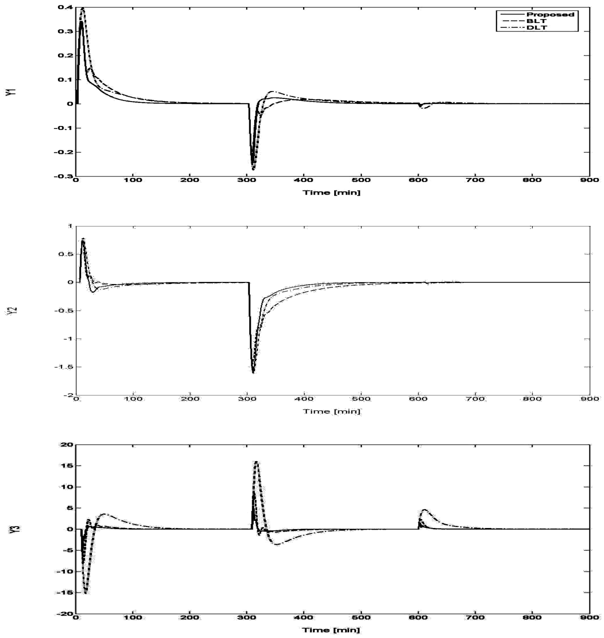

Figure 3 - The response of PI control system in step disturbance for OR column.

Table 1. PID parameters and IAE values by the various methods for OR column

PID parameter	Proposed method	BLT	DLT
$\boldsymbol{K}_{ci}$	1.19,-0.14, 8.66	1.51,-0.29, 2.63	0.61, -0.14, 0.3
$\boldsymbol{\tau}_{Ii}$	8.65, 5.52, 12.47	16.40, 4.18, 6.61	8.00, 6.50, 6.85
λ	11.7, 12.1, 2.45	-	20, 20, 20
β	4.06, -0.53, 3.48	-	-
Step setpoint change			
IAE$_t$	213.419	350.96	1434.90
Step disturbance change			
IAE$_t$	146.62	277.15	1077.10

Note: $n_i = 2$, $m_i = 1$; IAEt: sum of IAE_i in each loop.

$$\mathbf{S}(s) = \{\mathbf{S}_{ij}\} = \begin{bmatrix} S_{11} & S_{12} & \cdots & {}_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{1n} & S_{2n} & \cdots & {}_{nn} \end{bmatrix} \quad i = 1,\ldots,n \quad j = 1,\ldots,n \tag{13}$$

where $\mathbf{S}_{ij}$ is the sensitivity response of the *i*th loop

The sensitivity frequency response can be found by setting s =j ω and Eq.(15) is given by the formula of ω and λ as following,

$$\mathbf{S}(j\omega,\lambda) = (\mathbf{I} + \mathbf{G}(j\omega,\lambda)\mathbf{G}_c(j\omega,\lambda))^{-1} \tag{14}$$

The peak magnitude of the sensitivity function is given by

$$\mathbf{Ms}_{ij} = \max_{\omega,\lambda\geq 0}\{\mathbf{S}(j\omega,\lambda)\} = \max_{\omega,\lambda\geq 0}\{\mathbf{I} + \mathbf{G}(j\omega,\lambda)\mathbf{G}_c(j\omega,\lambda))^{-1}\} \qquad (15)$$

The proposed Ms tuning method which aims to make the closed-loop frequency responses of multiloop control system good performance and robustness is found a set of optimal λ by the optimization problem in the frequency domain:

$$J = \min\{\mathbf{Ms}_{ij}\}$$
$$\text{s.t. } \mathbf{Ms}_{ii} \geq Ms_{low} \qquad (16)$$

where Ms_{low} is the given value, the lower bound of diagonal **Ms,** recommended values of lower bound are in the range of 1.2 to 2.0. In this paper, Ms_{low} =1.3 is selected.

According to Eq. 15, Eq.16, it is easy to find the optimal value of λ which makes multiloop control system not only stable and robust in setpoint tracking but also in disturbance.

3. Simulation study

In case study, we consider to the well-known OR column that was studied by Luyben in 1986.

$$\mathbf{G}(s)=\begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.6e^{-3.5s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1} \\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3s}}{5s+1} & \frac{-0.01e^{-12s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.4s}}{10.9s+1} & \frac{0.89(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \qquad (17)$$

By using optimization function Eq. 15 and Eq. 16, the optimal values of λ for the 1st, 2nd, and 3rd loops are 11.7, 12.1, and 2.45, respectively.

The close-loop frequency response of multiloop PI control system in step setpoint change and step disturbance is showed in Figs. 2 and 3. They indicate that the proposed tuning method make control system performance well and satisfy the robustness. In this example, sequential step change of magnitude 1, 1, and 10 in step setpoint in and step disturbance for the 1st, 2nd, and 3rd loops was used. All control parameters are listed in Table 1. By considering the integral absolute error (IAE) values, it shows that proposed method is better than the others.

Conclusions

The proposed method is suitable for the most multiloop control systems because it implies the generalized IMC-PID approach for disturbance rejection and Ms tuning. The generalized IMC-PID approach can reject the disturbance in most cases of control systems and Ms tuning can apply to select a set of the closed-loop time constants which have been included in the optimization formulas of closed-loop transfer function of multivariable control system, so the controller parameters will be determined for minimizing the interaction and overshoot in control loops. The favourabe comparison with BLT, DLT method in the case study shows that proposed method is effective in MIMO control system.

Acknowledgement

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