PID Controller Design for Unstable Process with Negative/ Positive Zero

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Introduction

The proportional integral derivative (PID) controller is unquestionably the most commonly used control algorithm. It is mainly because they can assure satisfactory performances with simple algorithm and the cost/benefit ratio is difficult to achieve by other controllers similar to PID. It is well-known that control system design for the first-order delayed unstable process (FODUP) and second-order delayed unstable process (SODUP) system with negative/positive zero is difficult. If the zero is negative, it causes large overshoot, and, if the zero is positive, it shows an inverse response. Due to the effectiveness of internal model control (IMC), many efforts have been made to exploit the IMC principle to design the equivalent feedback controllers for various types of process models (Morari and Zafiriou [4]; Lee et al. [1-2]).

The literature review shows that designing for SODUP model with either one RHP pole or two RHP poles include reports from Lee et al. [2], Yang et al. [6], Tan et al. [5], and Liu et al. [3]. The controller design for SODUP systems with two unstable poles and a negative zero has been addressed by Lee et al. [1]. However, controller design for SODUP processes with one/two RHP poles and a RHP zero has not yet been addressed. Some of the recently reported [3, 5] methods use a two-degrees-of-freedom control structure with a greater number of controllers; also, the design of controllers is complicated.

Therefore, the present work is directed to get the analytical tuning formula for the IMC-PID controllers design for the above said process models, which gives the best disturbance rejection and setpoint tracking as well. However the setpoint response is usually accompanied with excessive overshoot, that effect can be eliminated with set-point filter. The controller setting gives a robust performance for uncertainty in the process model parameters. The simulation studies show that the proposed design method provides better disturbance rejection when the controllers are tuned to have the same degree of robustness by a measure of maximum sensitivity (Ms).

The IMC has been shown to be a powerful method for control system synthesis [4]. However, for unstable processes the IMC structure cannot be implemented exactly similar to stable process, since any input will make output grow without bound if process is unstable. Nevertheless, as discussed in [4], we could still use IMC approach to design a controller for an unstable process, if only the following conditions are satisfied for the internal stability of the closed-loop system (i) q stable, (ii) $G_p q$ stable, (iii) $(1-G_p q)G_p$ stable.

IMC controller design step

The IMC controller design involves two steps:

Step 1: A process model $\tilde{G}_p$ is factored into invertible and non invertible parts

$\tilde{G}_p = P_M P_A$, where P_M is the portion of the model inverted by the controller; P_A is the portion of the model not inverted by the controller (it is usually a non-minimum phase and contains dead times and/or right half plane zeros); $P_A(0)=1$.

Step 2: The IMC controller is set as $q = P_M^{-1} f$. Here, q has zeros at $up_1, \ldots, up_k$ because P_M^{-1} is the inverse of the model portion with unstable poles. The filter for the IMC controller can be designed to satisfy two criteria, one is that to make the IMC controller proper and another to cancel the unstable poles or stable poles near zero of G_D.

$f=\left(\sum_{i=1}^{m}\beta_i s^i+1\right)\big/\left(\lambda^2 s^2+2\lambda\xi s+1\right)^m$ where β_i are determine to cancel the unstable poles of G_D and m is the number which can be adjusted to make the IMC controller proper. It has function a of adjustable time constant λ and damping coefficient ξ. $1-G_p q\big|_{s=dup_i,\ \ldots,\ dup_m}=0$ where $dup_i \neq 0$. Thus, the IMC controller is

$$q=P_M^{-1}\frac{\left(\sum_{i=1}^{m}\beta_i s^i+1\right)}{\left(\lambda^2 s^2+2\lambda\xi s+1\right)^m} \tag{1}$$

The lead term in IMC filter $\left(\sum_{i=1}^{m}\beta_i s^i+1\right)$ causes an overshoot in the closed-loop response to a setpoint change. This problem can be resolved if we add a setpoint filter. $f_R=1\big/\left(\sum_{i=1}^{m}\beta_i s^i+1\right)$ The resulting IMC controller in Eq. (1) has stable response and the classical feedback controller exactly equivalent to IMC can be obtained from the following relationship

$$G_c=\frac{q}{1-\tilde{G}_p q} \tag{2}$$

The resulting closed-loop output response in Eq. (2) is physically realizable, but it does not have the standard PID controller form. For the PID controller from the ideal controller G_c, is discussed in detailed Lee *et al.* [2].

Proposed Tuning Rule

Second Order Unstable Process (SODUP) with two unstable poles

Let's consider the following second order plus unstable process

$$G_P=G_D=\frac{Ke^{-\theta s}}{(\tau_1 s-1)(\tau_2 s-1)} \tag{3}$$

The proposed IMC filter is found as $f=\left(\beta_2 s^2+\beta_1 s+1\right)\big/\left(\lambda^2 s^2+2\lambda\xi s+1\right)^2$. The resulting IMC controller becomes $q=(\tau_1 s-1)(\tau_2 s-1)\left(\beta_2 s^2+\beta_1 s+1\right)\big/K\left(\lambda^2 s^2+2\lambda\xi s+1\right)^2$. Therefore, the ideal feedback controller equivalent to the IMC controller is

$$G_c=\frac{(\tau_1 s-1)(\tau_2 s-1)\left(\beta_2 s^2+\beta_1 s+1\right)}{K\left[\left(\lambda^2 s^2+2\lambda\xi s+1\right)^2-e^{-\theta s}\left(\beta_2 s^2+\beta_1 s+1\right)\right]} \tag{4}$$

Expanding dead time in a Maclaurin series in s and simplification the tuning formula is given as:

$$k_C=\frac{\tau_I}{K(\theta-\beta_1+4\lambda\xi)}\qquad \tau_I=(\beta_1-\tau_1-\tau_2)-\frac{\left(-\theta^2/2+\theta\beta_1-\beta_2+2\lambda^2+4\lambda^2\xi^2\right)}{(\theta-\beta_1+4\lambda\xi)}\qquad \tau_D=\frac{(\beta_2+(-\tau_1-\tau_2)\beta_1+\tau_1\tau_2)-\dfrac{\left(\theta^3/6-\dfrac{\beta_1\theta^2}{2}+\theta\beta_2+4\lambda^3\xi\right)}{(\theta-\beta_1+4\lambda\xi)}}{\tau_I}-\frac{\left(-\theta^2/2+\theta\beta_1-\beta_2+2\lambda^2+4\lambda^2\xi^2\right)}{(\theta-\beta_1+4\lambda\xi)}$$

The extra degree of freedom β_2 and β_1 are calculated by solving $[1-Gq]\big|_{s=+1/\tau_1,s=+1/\tau_2}\ 0$. That means we want to choose β_i so that the term $[1-Gq]$ has a zero at the pole of G_D. Therefore, we have $\left[1-\left(\beta_2 s^2+\beta_1 s+1\right)e^{-\theta s}\big/\left(\lambda^2 s^2+2\lambda\xi s+1\right)\right]_{s=+1/\tau_1,s=+1/\tau_2}=0$. The value of β_i after some simplification is given as

$$\beta_1=\frac{\tau_1^2\left(\frac{\lambda^2}{\tau_1^2}+\frac{2\lambda\xi}{\tau_1}+1\right)^2 e^{\theta/\tau_1}-\tau_2^2\left(\frac{\lambda^2}{\tau_2^2}+\frac{2\lambda\xi}{\tau_2}+1\right)^2 e^{\theta/\tau_1}-\left(\tau_1^2-\tau_2^2\right)}{(\tau_1-\tau_2)}\qquad \beta_2=\tau_1^2\left(\frac{\lambda^2}{\tau_1^2}+\frac{2\lambda\xi}{\tau_1}+1\right)^2 e^{\theta/\tau_1}-\left(1+\frac{\beta_1}{\tau_1}\right)\qquad Lag\ filter(\tau_F)=\frac{1}{(\tau_a s+1)}$$

<u>Robust Stability</u>

Norm-Bound Uncertainty Regions

Theorem (Robust Stability): Morari and Zafirou [4]. Assume that all plants p in the family Π

$$\Pi=\left\{p:\left|\frac{p(i\omega)-\tilde{p}(i\omega)}{\tilde{p}(i\omega)}\right|<\bar{l}_m(\omega)\right\} \tag{5}$$

have the same number of RHP poles and that a particular controller G_c stabilizes the nominal plant $\tilde{p}$. Then the system is robustly stable with the controller G_c if and only if the complementary sensitivity function $\tilde{\eta}$ for the nominal plant $\tilde{p}$ satisfies the following bound

$$\left\|\tilde{\eta}(s)\Delta_m(s)\right\|_\infty<1 \tag{6}$$

where $\Delta_m(s)$ defines the process multiplicative uncertainty bound. i.e., $\Delta_m(s)=\left(G_p-\tilde{G}_p\right)/\tilde{G}_p$. The complementary sensitivity function $\tilde{\eta}(s)$ can be obtained $\tilde{\eta}(s)=\left(\beta_2 s^2+\beta_1 s+1\right)e^{-\theta s}/\left(\lambda^2 s^2+2\lambda\xi s+1\right)^2$ for the proposed filter. If the process contains gain and delay uncertainty, then $\Delta_m(s)=\left(1+\frac{\Delta K}{K}\right)e^{-\Delta\theta s}-1$. Substituting $\tilde{\eta}(s)$ and $\Delta_m(s)$ into Eq. (6) yields the robust stability constraint for tuning adjustable parameters λ .

$$\left\|\frac{\left[\left(\left[\tau_1^2\left(\frac{\lambda^2}{\tau_1^2}+\frac{2\lambda\xi}{\tau_1}+1\right)^2 e^{\theta/\tau_1}-\left(1+\frac{\beta_1}{\tau_1}\right)\right]s^2+\left[\frac{\tau_1^2\left(\frac{\lambda^2}{\tau_1^2}+\frac{2\lambda\xi}{\tau_1}+1\right)^2 e^{\theta/\tau_1}-\tau_2^2\left(\frac{\lambda^2}{\tau_2^2}+\frac{2\lambda\xi}{\tau_2}+1\right)^2 e^{\theta/\tau_1}-\left(\tau_1^2-\tau_2^2\right)}{\left(\tau_1-\tau_2\right)}\right]s+1\right)\right]}{\left(\lambda^2 s^2+2\lambda\xi s+1\right)^2}\right\|_\infty<\frac{1}{\left\|\Delta_m(s)\right\|_\infty} \tag{7}$$

λ can be adjusted in Eq. (7) for the uncertainty in process parameters to obtain the stable response.

Simulation Study

Example 1: SODUP with a Negative Zero

Consider an unstable process with a negative zero with two unstable poles as follows:

$$G_P=\frac{2(5s+1)e^{-0.3s}}{(3s-1)(1s-1)} \tag{8}$$

Lee et al. [1] and proposed methods were used to design the PID controller. For the proposed method, a value of closed loop time constant, $\lambda=0.657$ was chosen so that $Ms=4.49$. The $\lambda=0.692$ has been adjusted to get the similar value of the Ms for Lee et al. [4] to obtain the fair comparison.

Figure 1 shows the closed-loop output response for a unit-step setpoint change occurring at *t=0*, and an inverse unit-step step change of load is added to the process input at *t=15*. Figure 1 shows that the disturbance rejection and set-point response for the proposed controller is better than the Lee et al. [1] tuning methods. The setpoint response of the proposed method is fast and settling time is less compare to above said method, which has very slow response and long settling time. In the disturbance rejection, both overshoot and undershoot are small in the proposed tuning method. Lee et al. [1] tuning method show bigger peak and large undershoot, which is highly undesirable in the control system. The modification of the IMC filter shows great advantage in the setpoint as well disturbance rejection for the unstable process with negative zero.

Example 2: SODUP with one Stable Pole and a Positive Zero

The following typical SODUP with one stable pole, and later modified for the positive zero as

$$G_P=\frac{e^{-0.5s}}{(5s-1)(2s+1)(0.5s+1)} \tag{9}$$

The approximated model of the above process is $G_P=e^{-0.939s}/(5s-1)(2.07s+1)$, which is used by [1,5]. Modifying the above process by introducing the strong positive zero is given as $G_P=(-1s+1)e^{-0.939s}/(5s-1)(2.07s+1)$. It is difficult to design the PID controller for the unstable process having a positive zero. For the controller design the above process can be approximated as

$$G_P=\frac{e^{-1.939s}}{(5s-1)(2.07s+1)} \tag{10}$$

"Inverse response time constant" (negative numerator time constant) has been approximated as a time delay $\left(-T_0^{inv}s+1\approx e^{-T_0^{inv}s}\right)$. This is reasonable since an inverse response has a deteriorating effect on control similar to that of a time delay.

The proposed IMC-PID tuning rule can be designs by modifying the IMC filter. For the fair comparison Ms=3.5 has been chosen for both the tuning method and accordingly λ=5.3 and λ=6.25 has been adjusted for the proposed and Lee et al. [1]. Figure 2 shows the closed-loop output responses for the above process with a unit-step setpoint change occurring at t=0, and a inverse unit-step disturbance at t=60. It is seen that the proposed method results in the improved load disturbance response and great advantage in the setpoint response as shown in the Fig. 2. The settling time is very less with smooth response in the proposed design method.

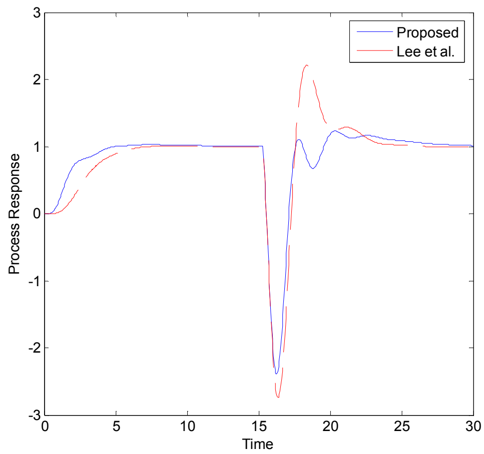

Fig. 1. Simulation results for Example 1

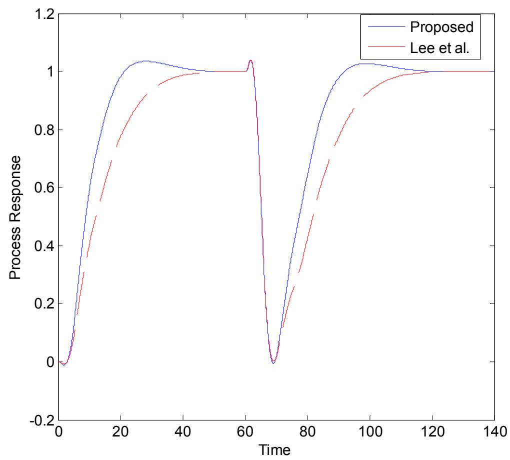

Fig. 2. Simulation results for Example 2

Conclusions

The IMC filter structure has been modified for several representative process models to improve disturbance rejection performance of the IMC-PID controller. Based on the proposed filter structures, tuning rules for the PID controller was derived by using the generalized IMC-PID method by Lee et al. [2]. The undershoot in the output response can be eliminated by the overdamped IMC filter for the system with a negative zero. The processes which have positive zero can be treaded by reducing them into FODUP/SODUP model. The model reduction techniques can be utilized to design the PID controller for the inverse response process maintaining the performance and robustness level. The simulation results demonstrated superiority of the proposed method.

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