IMC Based Method for Control System Design of PID Cascaded Filter

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Introduction

The ability of PID controllers to compensate most practical industrial process has led to their wide acceptance in industrial applications. In spite of its wide spread use there exists no generally accepted design method for the controller tuning. The PID tuning rule based on the IMC (internal model control) principle has advantage to keep the desired closed-loop response by adjusting the closed-loop time constant. The tuning methods based on the IMC-PID design algorithm like Rivera *et al.* (1986), Chien and Fruehauf (1990) and Lee *et al*. (1998) and the direct synthesis method by Smith *et al*. (1975) are typical of the tuning methods based on achieving a desired closed-loop response. They obtain the PID controller parameters by first computing the controller which gives the desired closed-loop response. Since these controllers are more complicated but, by clever approximations of the process model's dead time, the controller form can be reduced to that of a PID controller, or a PID controller cascaded with a first-or second-order lead lag filter.
In all the above mentioned tuning methods, the design procedure is based on IMC principle and the resulting PID controller is somewhat different by approximating the dead time with different series expansion (1/1 Pade, first order Taylor series etc). The performance based on these IMC-PID tuning methods for the lag time dominant process shows sluggish as well as overshoot, which cause undesirable time to reach its set-point value.
Therefore, in the proposed study, we developed IMC-PID cascaded filter for getting the desired closed-loop response. In the proposed control algorithm the resulting control system becomes a control of the fast dynamic process (or gain process on neglecting this process) to integral control, which causes the performance improvement drastically. The λ guideline has been suggested for FOPDT and SOPDT for different M_S value. The example is provided for the comparing the result with alternative PID tuning methods based on IMC principle.

Controller Design Algorithm

The detail of IMC structure is presented in Morari and Zafiriou (1989), where $G_P(s)$ process, $\tilde{G}_P(s)$ process model and $q(s)$ is the IMC controller. The controlled variable are related as

$$C=\frac{G_P(s)q(s)}{1+q(s)\left(G_P(s)-\tilde{G}_P(s)\right)}R+\left[\frac{1-\tilde{G}_P(s)q(s)}{1+q(s)\left(G_P(s)-\tilde{G}_P(s)\right)}\right]G_D(s)d \quad (1)$$

For the nominal case (*i.e.*, $G_P(s)=\tilde{G}_P(s)$), the setpoint and disturbance responses are simplified as

$$\frac{C}{R}=G_P(s)q(s) \quad (2)$$

$$\frac{C}{d}=[1-G_P(s)q(s)]G_D \quad (3)$$

The IMC controller design involves two steps:
Step 1: A process model $\tilde{G}_P(s)$ is factored into

$$\tilde{G}_P(s)=P_M(s)P_A(s) \quad (4)$$

where $P_M(s)$ is the portion of the model inverted by the controller; $P_A(s)$ is the portion of the model not inverted by the controller (it is usually a non-minimum phase and contains dead times and/or right half plane zeros); $P_A(0)$= 1.
Step 2: The idealized IMC controller is the inverse of the invertible portion of the process model.

$$\tilde{q}(s)=P_M^{-1}(s) \quad (5)$$

To make the controller proper, it needs to add the filter. Thus, the IMC controller is designed by

$$q(s)=\tilde{q}(s)f(s)=P_M^{-1}(s)f(s) \tag{6}$$

The ideal feedback controller equivalent to the IMC controller can be expressed in terms of the internal model, $\tilde{G}_P(s)$, and the IMC controller, $q(s)$:

$$G_C(s)=\frac{q(s)}{1-\tilde{G}_P(s)q(s)} \tag{7}$$

Since the resulting controller has not a standard PID controller form, the remaining issue is to design the PID controller that approximates the equivalent feedback controller most closely. The dead time contain in the denominator of Eq. (7) can be approximated by the Pade expansion and the resulting form of the controller can easily be converted into the PID cascaded filter. The form of the proposed PID cascaded filter is

$$G_c=K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1+cs+ds^2}{1+as+bs^2} \tag{8}$$

where K_c is the gain, τ_I is the integral time, τ_D is the derivative time constant, a, b, c and d are filter parameters. The second order filter ensures that the nominal PID controller is proper and is easily implemented using the modern DCS control hardware system. This controller form is an extension to the modified PID controller proposed by Rivera *et al.* (1986) and Lee *et al.* (1998), and also suggested by Horn *et al.* (1996).

First order plus dead time process (FOPDT)

The most commonly used approximate model for chemical process is the first-order plus dead time model as given below

$$G_P=\frac{Ke^{-\theta s}}{\tau s+1} \tag{9}$$

The process model $\tilde{G}_P$ is factored into minimum phase and non-minimum phase as: $P_M=K/(\tau s+1)$ and $P_A=e^{-\theta s}$. The IMC controller is given as $q=\tilde{q}f=P_M^{-1}f=(\tau s+1)/K(\lambda s+1)$ for the desired closed-loop response $G_d=e^{-\theta s}/(\lambda s+1)$. The ideal feedback controller should be $G_c=(\tau s+1)/K\left[\lambda s+1-e^{-\theta s}\right]$ and dead time $e^{-\theta s}$ is approximated by 2/2 Pade approximation. The resulting tuning rule after some simplification is given in Table 1. The dead time has been approximated in this paper either 2/1 or 2/2 Pade approximation to give the control structure PID cascaded filter as suggested and maximum accuracy of the dead time, which we can get for proposed controller form.

Table 1. IMC-PID cascaded filter tuning rules for various processes

Process model	K_C	τ_I	τ_D	a	b	c	d
$G_P=\frac{Ke^{-\theta s}}{\tau s+1}$	$\frac{\theta}{2K(\lambda+\theta)}$	$\theta/2$	$\theta/6$	$\frac{\theta\lambda}{2(\lambda+\theta)}$	$\frac{\theta^2\lambda}{12(\lambda+\theta)}$	τ	-
$G_p=\frac{Ke^{-\theta s}}{(\tau^2s^2+2\xi s+1)}$	$\frac{\theta}{2K(\lambda+\theta)}$	$\theta/2$	$\theta/6$	$\frac{\theta\lambda}{2(\lambda+\theta)}$	$\frac{\theta^2\lambda}{12(\lambda+\theta)}$	τ^2	$2\xi\tau$
$G_p=\frac{Ke^{-\theta s}}{s(\tau s+1)}$	$\frac{1}{K(\lambda+\theta)}$	-	$\theta/3$	$\frac{(\lambda\theta/3-\theta^2/6)}{(\theta+\lambda)}$	-	τ	-

Simulation Study

Example 1.

Consider the following FOPDT process for the step set-point change studied by the Lee *et al.* (1998).

$$G_P=\frac{1e^{-3s}}{10s+1} \tag{10}$$

The proposed tuning, Lee *et al.* (1998) and Rivera *et al.* (1986) method were used to design the controller as shown in Table 2. For the proposed method, a value of closed-loop time constant $\lambda=2.031$

was chosen so that $Ms = 1.60$. To obtain the fair comparison, λ=1.774 was selected for the Lee *et al.* (1998) and $\lambda = 2.592$ for Rivera *et al.* (1986) method.
The simulation results in Fig. 1 and the IAE, ISE, IATE, overshoot and TV values in Table 2, indicate that the set-point response for the proposed controller is much better and faster than the other tuning methods. The proposed tuning rule has no overshoot as well as the settling time is negligible compare to other tuning methods as shown in Fig. 1. The overshoot is high for the Lee *et al.* (1998) whereas the settling time for the Rivera *et al.* (1986) method looks to be more as shown in Fig. 1. Based on the performance matrix as listed in Table 2 and Fig. 1, it is clear that the proposed method has superior performance compare to other tuning methods.
Fig. 2 shows the comparison of the IAE for set-point change using the proposed tuning rule with those of the Lee *et al.* (1998) and Rivera *et al.* (1986) for varying process dead time to time constant ratios. The process model which is taken into consideration is $G_p = 1e^{-\theta s}/(1s+1)$, where θ is adjusted to get the desired θ/τ ratio. To obtain the fair comparison the λ value is adjusted for every case $Ms = 1.6$, which means keeping all controller at equal robustness level. As seen from the Fig. 2, the proposed tuning rule gives the smallest IAE value among other methods up to approximately $\theta/\tau = 2.0$. When the θ/τ ratio increases and goes beyond the $\theta/\tau = 2.0$ Lee *et al.* (1998) method having clear advantage among other tuning methods. The tuning method proposed by Rivera *et al.* (1986) cascaded filter showing always inferior result among other tuning rule.

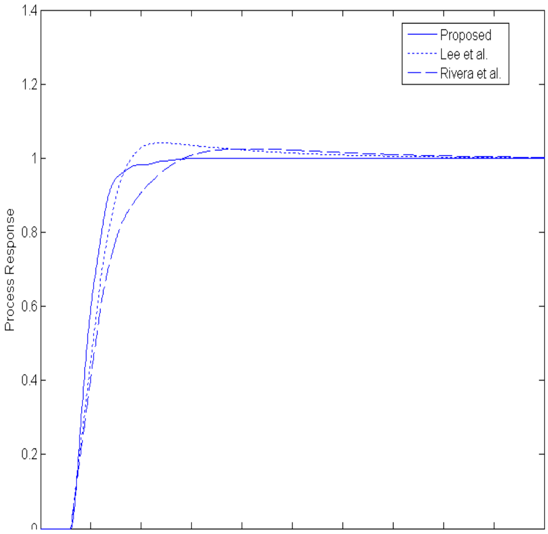

Fig. 1 Output response of Example 1

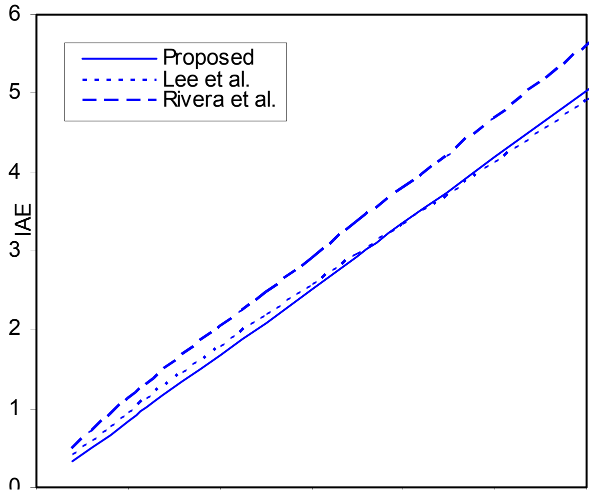

Fig. 2 Comparison of the IAE for various tuning rule

Table 2 PID controller setting for set-point (Example 1)

Tuning methods	K_c	τ_I	τ_D	a	b	c	set-point (time=50) nominal process			
							IAE	*ISE*	*IATE*	*overshoot*
Proposed $\lambda = 2.031$	0.29	1.5	0.50	0.60	0.3	10	5.03	4.22	13.9	0
Lee *et al.* $\lambda = 1.774$	2.29	10.9	0.85	-	-	-	5.93	4.51	27.3	0.04
Rivera *et al.* $\lambda = 2.592$	2.05	11.5	1.30	0.69	-	-	6.54	4.75	35.0	0.02

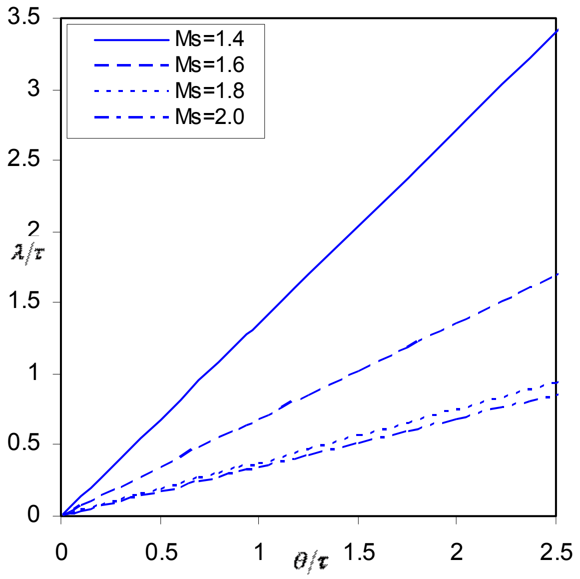

Fig. 3 λ guide lines plot at different Ms

Conclusions

The proposed method is based on IMC principle for the PID cascade filter. Resulting structure is suggested by Horn *et al.* (1996), and is extension of Rivera *et al.* (1986). Due to availability of PID cascaded filter form in modern DCS control hardware, it is easy to apply this form of the PID controller. The proposed method makes the original process model to fast process and control by resulting integral controller. The proposed method having best response in time lag process, where resulting new controller become small time constant process having fast dynamic compare to original process. Based on the proposed method the simulation results demonstrate the superiority of the proposed method with great impact.

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Literature Cited

Chien, I.-L., Fruehauf, P. S., (1990), Consider IMC Tuning to Improve Controller Performance, *Chem. Eng. Prog., **86***, 33.

Horn, I.G., Arulandu, J. R., Christopher, J. G., VanAntwerp, J. G., Braatz, R. D., (1996), Improved Filter Design in Internal Model Control, *Ind. Eng. Chem. Res*., **35,** 3437.

Lee, Y., Park, S., Lee, Moonyong, Brosilow, C., (1998), PID Controller Tuning for Desired Closed-Loop Responses for SI/SO Systems, *AIChE Journal*, **44**, No.1, pp. 106-115.

Morari, M., Zafiriou, E., (1989) *Robust Process Control,* Prentice-Hall: Englewood Cliffs, NJ,.

Rivera, D. E., Morari, M., Skogestad, S., (1986), Internal Model Control.4. PID Controller Design, *Ind. Eng. Chem. Process Des. Dev*., **25**, 252.

Smith, C. L., Corripio, A. B., and J. Martin, Jr., (1975), Controller Tuning from Simple Process Models, *Instrum. Technol.,* **22**, 12, 39.