Design of Multiloop PI Controller for Disturbance Rejection in Multivariable Processes

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Introduction

In practice, the multi-loop PID controllers have designed that depended on disturbance rejection more desirable and more often encountered than set-point tracking in multivariable processes but most design methods are considered to the direct synthesis method that makes identical PID controller for set-point tracking. Therefore, in case of disturbance, these PID controllers always give sluggish disturbance rejection for the process systems.

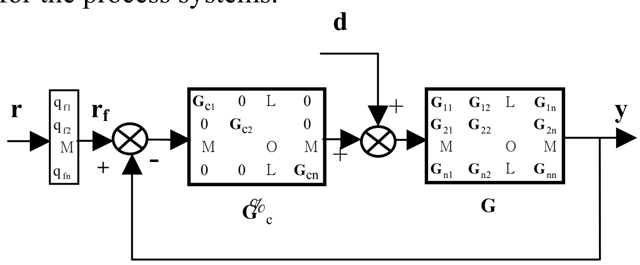

Figure 1. Block diagram for multi-loop control system

The proposed method is interesting idea that reflects on disturbance rejection in MIMO processes, the multi-loop PI controllers that is obtained by extending the generalized IMC-PID method, a successful design method for SISO systems by Lee et al [1] for SISO systems, to MIMO systems provide several advantages such as canceling the influence of disturbances because the feedback signal is equal to this influence and modifies the controller set-point accordingly and the IMC structure offers the opportunity of implementing complex nonlinear control algorithms without generating complex stability issues.

1. The general tuning algorithm for multiloop PID controllers

Consider to multi-loop feedback control system in Fig. 1, the closed-loop transfer function can be expressed as

$$\mathbf{y}(s) = \mathbf{H}(s)\mathbf{r}_f(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{G}_c(s))^{-1}\mathbf{G}(s)\mathbf{G}_c(s)\mathbf{r}_f(s) + (\mathbf{I} + \mathbf{G}(s)\mathbf{G}_c(s))^{-1}\mathbf{G}(s)\mathbf{d} \quad (1)$$

where $\mathbf{G}(s)$ is the n x n open-loop stable processes, $\tilde{G}_c(s)$ is multi-loop controller, and y (s), $r_f(s)$ and d (s) are the controlled variable vector, the setpoint vector, and disturbance vector, respectively.
The closed-loop setpoint transfer function for the multivariable feedback control system

$$\mathbf{y}(s) = \mathbf{H}(s)\mathbf{r}_f(s) = (\mathbf{I} + \mathbf{G}(s)\mathbf{G}_c(s))^{-1}\mathbf{G}(s)\mathbf{G}_c(s)\mathbf{r}_f(s) \quad (2)$$

Deriving from the design strategy of multi-loop IMC controller [3] and putting the IMC filter that reject dominant pole in each element of the process transfer function, the desired closed-loop response $\mathbf{R}_i$ of the ith loop typically chosen by

$$\frac{y_i}{r_i'} = \mathbf{R}_i = \frac{G_{ii+}(s)\sum_{j=1}^{m}\left(\beta_{ij}s^j+1\right)}{(\lambda_i s+1)^{n_i}} \tag{3}$$

where G_{ii+} is the nonminimum of G_{ii} and it is chosen to be the all-pass form, λ_i is an adjustable constant for system performance and stability, and n_i is selected large enough that makes the IMC controller would be proper, β_{ij} is defined to cancel the dominant poles in each element of processes, m is number of poles to be cancelled and usually on the order of one or two. The requirement of $G_{ii+}(0) = 1$ is essential for the controlled variable to track its set-point.

The desired closed-loop response matrix $\tilde{\mathbf{R}}(s)$ can be articulated as

$$\tilde{\mathbf{R}}(s)=\mathrm{diag}\left[R_1,R_2...,R_n\right] \tag{4}$$

The multi-loop controller with an integral term, can be expanded in a Maclaurin series as

$$\tilde{\mathbf{G}}_{ci}(s) = \frac{1}{s}\left[\tilde{\mathbf{G}}_{c0} + \tilde{\mathbf{G}}_{c1}s + \tilde{\mathbf{G}}_{c2}s^2 + O(s^3)\right] \tag{5}$$

where $\tilde{\mathbf{G}}_{c0}$, $\tilde{\mathbf{G}}_{c1}$ and $\tilde{\mathbf{G}}_{c2}$ keep up a correspondence with the integral, proportional, and derivative terms of the multi-loop PID controllers, respectively.

The multivariable process, $\mathbf{G}(s)$, can be written in a Maclaurin series as

$$\mathbf{G(s)} = \mathbf{G_0} + \mathbf{G_1}s + \mathbf{G_2}s^2 + O(s^3) \tag{6}$$

where $\mathbf{G}_0 = \mathbf{G}(0)$, $\mathbf{G}_1 = \mathbf{G}'(0)$, and $\mathbf{G}_2 = \mathbf{G}''(0)/2$.

The closed-loop transfer function, H(s), can be expressed in a Maclaurin series by substituting Eqs. 5 and 6 into Eq. 2.

$$\begin{aligned}\mathbf{H}(s) = {} & \mathbf{I} - (\mathbf{G}_0\tilde{\mathbf{G}}_{c0})^{-1}s + (\mathbf{G}_0\tilde{\mathbf{G}}_{c0})^{-1}(\mathbf{I} + \mathbf{G}_0\tilde{\mathbf{G}}_{c1} + \mathbf{G}_1\tilde{\mathbf{G}}_{c0})(\mathbf{G}_0\tilde{\mathbf{G}}_{c0})^{-1}s^2 + \\ & (\mathbf{G}_0\tilde{\mathbf{G}}_{c0})^{-1}[\mathbf{G}_0\tilde{\mathbf{G}}_{c2} + \mathbf{G}_1\tilde{\mathbf{G}}_{c1} + \mathbf{G}_2\tilde{\mathbf{G}}_{c0} - (\mathbf{I} + \mathbf{G}_0\tilde{\mathbf{G}}_{c1} + \mathbf{G}_1\tilde{\mathbf{G}}_{c0})(\mathbf{G}_0\tilde{\mathbf{G}}_{c0})^{-1} \\ & (\mathbf{I} + \mathbf{G}_0\tilde{\mathbf{G}}_{c1} + \mathbf{G}_1\tilde{\mathbf{G}}_{c0})(\mathbf{G}_0\tilde{\mathbf{G}}_{c0})^{-1}]s^3 + O(s^4)\end{aligned} \tag{7}$$

$\tilde{\mathbf{R}}(s)$ can also be expressed in a Maclaurin series as

$$\tilde{\mathbf{R}}(s)=\tilde{\mathbf{R}}(0)+\tilde{\mathbf{R}}'(0)s+\frac{\tilde{\mathbf{R}}''(0)}{2}s^2+\frac{\tilde{\mathbf{R}}'''(0)}{6}s^3+O(s^4) \tag{8}$$

where $\tilde{\mathbf{R}}(0) = \mathbf{I}$, given that $G_{ii+}(0) = 1$.

1.2. Design proportional gain K_c

The influence of $\tilde{\mathbf{G}}_{c0}$, $\tilde{\mathbf{G}}_{c1}$ on the PID formula is very considerable at high frequencies, but it is negligible at low frequencies. Consequently, the design of multi-loop PI controller should be followed the process characteristics at high frequencies.

Given that $\left|\mathbf{G}(jw)\tilde{\mathbf{G}}_c(jw)\right| = 1$ at high frequencies, $\mathbf{H}(s)$ can be approximated to

$$\mathbf{H}(s)=(\mathbf{I}+\mathbf{G}(s)\tilde{\mathbf{G}}_c(s))^{-1}\mathbf{G}(s)\tilde{\mathbf{G}}_c(s) \approx \mathbf{G}(s)\tilde{\mathbf{G}}_c(s) \tag{9}$$

Dropping the off-diagonal terms in $\mathbf{G}(s)$, the ideal multi-loop controller $\tilde{\mathbf{G}}_c(s)$ can be obtained as

$$\tilde{\mathbf{G}}_c(s)=\tilde{\mathbf{G}}^{-1}(s)\tilde{\mathbf{R}}(s)(\mathbf{I}-\tilde{\mathbf{R}}(s))^{-1} \tag{10}$$

where $\tilde{\mathbf{G}}(s)=\mathrm{diag}\left[G_{11},G_{22},...G_{nn}\right]$.

The ideal controller $\tilde{\mathbf{G}}_c(s)$ of the *i*th loop can be designed as

$$G_{ci}(s) = \frac{Q_i(s)}{1 - G_{ii}(s)Q_i(s)} = \frac{\left[G_{ii-}(s)\right]^{-1}\left(\sum_{j=1}^{m}\beta_{ij}s^j + 1\right)}{(\lambda_i s+1)^{n_i} - G_{ii+}(s)\left(\sum_{j=1}^{m}\beta_{ij}s^j + 1\right)} \tag{11}$$

where $\mathbf{Q}_i(s)$ is IMC controller given by $[\mathbf{G}_{ii-}(s)]^{-1}\left(\sum_{j=1}^{m}\beta_{ij}s^j+1\right)/(\lambda_i s+1)^{n_i}$

Because $G_{ii+}(0)$ is 1 , Eq. 11 can be rewritten in a Maclaurin series with an integral term as

$$\mathbf{G}_{ci}(s)=\frac{1}{s}(f_i(0)+f_i^{'}(0)s+\frac{f_i^{''}(0)}{2}s^2+O(s^3)) \quad (12)$$

where $f_i(s) = G_c(s)s$.

By following Eq. 12, it is clear that the first three s-terms are more important part of controller; otherwise they are insignificant. The proportional gain of the multi-loop PI controller can be obtained by

$$\boldsymbol{K}_{ci}=f_i^{'}(0) \quad (13)$$

1.3. Design of integral time constant τ_I

The integral term $\tilde{G}_{c0}$ is predominant at low frequencies, so that the interaction characteristics at low frequencies have been considered particularly. By comparing the off-diagonal element of the first-order s-terms in Eqs. 7 and 8, the integral time constant can be obtained as

$$\boldsymbol{\tau}_{Ii}=-\frac{\left[G_{ii+}^{'}(0)-n_i\lambda_i+\beta_{i1}\right]\mathbf{G}_{ci}}{\left[\mathbf{G}^{-1}(0)\right]_{ii}} \quad (14)$$

2. Simulation study

In case study, the simulation results show that the proposed method compares favorably with the biggest log-modulus tuning (BLT) method of Luyben (1986) and the decentralized λ tuning (DLT) method of Jung et al., (1998).

Example

Consider the Ogunnaike and Ray (OR) column was studied by Luyben in 1986.

$$\mathbf{G}(s)=\begin{bmatrix}\frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.6e^{-3.5s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1}\\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3s}}{5s+1} & \frac{-0.01e^{-12s}}{7.09s+1}\\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.4s}}{10.9s+1} & \frac{0.89(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)}\end{bmatrix} \quad (15)$$

Figs. 2 and 3 show the response of PI control system in step disturbance and step setpoint change for OR column. Sequential step change of magnitude 1, 1, and 10 in step disturbance and in step setpoint were made to the 1st, 2nd, and 3rd loops, respectively. The simulation results show that the proposed method gives the MIMO control systems with well-balanced and robust responses.

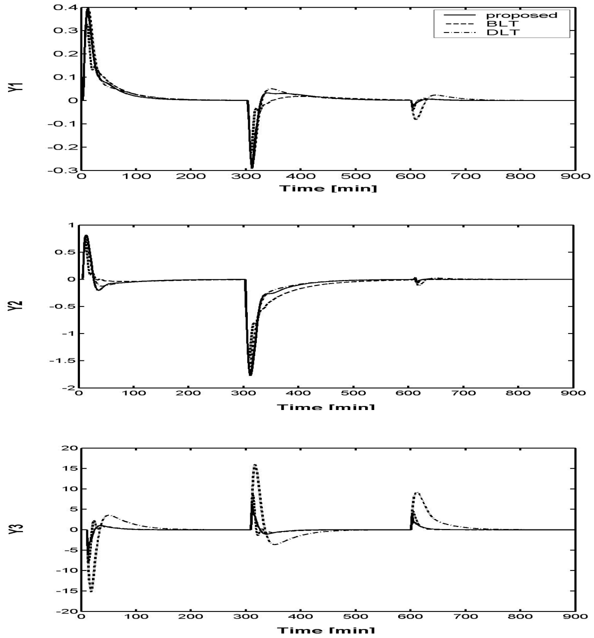

Figure 2. Output responses to a step disturbance for OR column

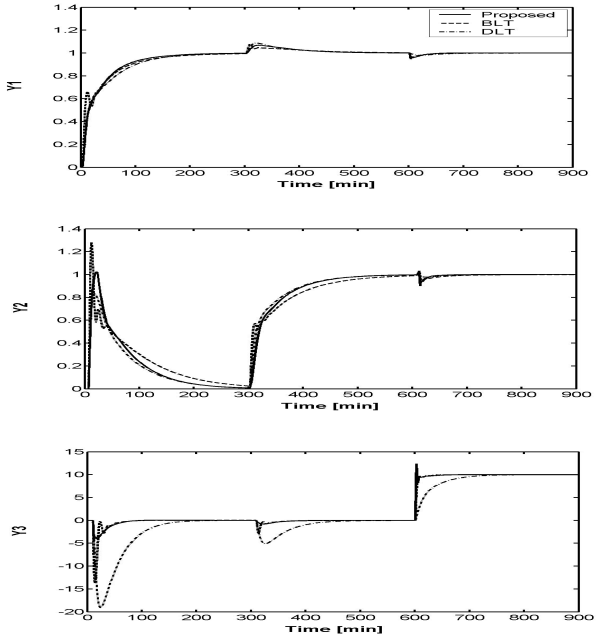

Figure 3. Responses to a step setpoint change for OR column

Conclusions

The tuning of multi-loop PID controllers for multivariable process is usually a complex problem. Therefore, the simplification of tuning rule is very necessary in MIMO system. In this paper, Maclaurin series expansion has used to create the simple formulas of multi-loop PI controllers for that goal. Some advantages of proposed method are found by illustrative example. Firstly, it is clear, simple, and relative easy to use in forms of controller's parameter. Secondly, it can achieve better control performance than existing methods (BLT, DLT) with both disturbance rejection and setpoint tracking because of the proportional terms and integral term which can be derived from neglecting the off-diagonal elements and taking the off-diagonal elements fully into account, respectively. Thirdly, it makes the closed-loop responses of PI control system stability and robustness.

Acknowledgement

This research was supported by the Korea Research Foundation Grant (KRF-2003-002-D00068)

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