A Simple Analytical Controller Design for Integrating and First Order Unstable Time Delay Process

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Introduction

In the process control, more than 95% of the control loops are of the proportional integral derivative (PID) type. The main reason is its relatively simple structure, which can be easily understood and implemented in practice. The numerous important chemical processing units are open-loop unstable process in industrial and chemical practice are well known to be difficult to control especially when there exists time delay, such as continuous stirred tank reactors, polymerization reactors and bioreactors are inherently open loop unstable by design.

The effectiveness of internal model control (IMC) design principle has attracted in process industry, which causes many efforts made to exploit the IMC principle to design the equivalent feedback controllers for stable and unstable processes Morari and Zafiriou [1]. The IMC based PID tuning rules have the advantage of only one tuning parameter to achieve a clear trade-off between closed-loop performance and robustness. It is well know that the IMC structure is very powerful for controlling stable processes with time delay and cannot be directly used for unstable processes by reason of the internal instability Morari and Zafiriou [1]. Some modified IMC methods of two-degree-of-freedom (2DOF) control such as Lee *et al*. [2], Yang *et al*. [3], Wang and Cai [4], Tan *et al*. [5], Liu *et al*. [6] had been developed for controlling unstable processes with time delay.

It is well known that in recent time the controller hardware support the microprocessor implementation for the PID cascaded filter. Therefore, PID controller cascaded with first order lead lag filter can be easily implemented using the modern control hardware. The PID controller cascaded first order filter type structure in Eq. (1) are suggested by Lee *et al.* [2], Horn *et al.* [7].

$$G_c = K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1+as}{1+bs} \tag{1}$$

where K_c, τ_I and τ_D are the proportional gain, integral time constant, and derivative time constant of the PID controller, respectively, and a & b are the filter parameters. Therefore, in the present study a simple method has been proposed for the design of the PID controller cascaded with first order filter to accomplish the improved performance for the first order unstable and delay integrating processes. A closed-loop time constant (λ) guidelines has been recommend cover a wide range of θ/τ ratio. Simulation study has been performed to show the superiority of the proposed method for both the nominal and perturbed processes.

Design Procedure

The IMC controller has been shown to be a powerful method for control system synthesis Morari and Zafiriou [1]. However, for unstable processes the IMC structure cannot be implemented, since any input d will make y grow without bound if G_p is unstable. Nevertheless, as discussed in Morari and Zafiriou [1], we could still use IMC approach to design a closed loop feed controller for an unstable process, if only the (i) q stable; (ii) $G_p q$ stable; (iii) $(1-G_p q)G_p$ stable.

Since the IMC controller q is designed as $q = p_m^{-1} f$ in which p_m^{-1} includes the inverse of the model portion, the controller satisfies the first condition. The second condition could be satisfied through the design of the IMC filter f. For this, the filter is designed as

$$f = \frac{\sum_{i=1}^{m}\alpha_i s^i + 1}{(\lambda s+1)^r} \tag{2}$$

where r is the number of poles to be canceled; α_i are determined by Eq. (3) to cancel the unstable poles in G_D; r is selected large enough to make the IMC controller proper.

$$1-\tilde{G}_p q\Big|_{s=dup_1,\, dup_2, \ldots,\, dup_m} = \left|1-\frac{p_m^{-1}(\sum_{i=1}^{m}\alpha_i s^i+1)}{(\lambda s+1)^r}\right|_{s=dup_1,\, dup_2, \ldots,\, dup_m} = 0 \tag{3}$$

Then, the IMC controller comes to be

$$q = p_m^{-1}\frac{(\sum_{i=1}^{m}\alpha_i s^i+1)}{(\lambda s+1)^r} \tag{4}$$

The numerator expression $\left(\sum_{i=1}^{m}\alpha_i s^i+1\right)$ in Eq. (4) causes an unreasonable overshoot in the servo response, which can be eradicated by adding the setpoint filter f_R as $f_R = 1\big/\left(\sum_{i=1}^{m}\alpha_i s^i+1\right)$. The feedback controller G_c which is equivalent to the IMC controller q is represented by $G_c = q\big/\left(1-\tilde{G}_p q\right)$. The resulting ideal feedback controller obtained as

$$G_c = \frac{p_m^{-1}\dfrac{(\sum_{i=1}^{m}\alpha_i s^i+1)}{(\lambda s+1)^r}}{1-\dfrac{p_A\left(\sum_{i=1}^{m}\alpha_i s^i+1\right)}{(\lambda s+1)^r}} \tag{5}$$

The aforementioned resulting controller in Eq. (5) does not have a standard PID controller configuration. The remaining objective is to design the PID controller cascaded with first order filter that resemble the equivalent feedback controller most closely and is discussed in the next section.

Proposed Tuning Rule

The first order delay unstable process (FODUP) is the typical representative model which is commonly utilized in the chemical process industries.

<u>*First-Order Delay Unstable Process (FODUP)*</u>

$$G_P = G_D = \frac{Ke^{-\theta s}}{\tau s-1} \tag{6}$$

where K is the gain, τ the time constant and θ is time delay. The IMC filter structure exploited is given as $f = \alpha s+1\big/(\lambda s+1)^3$. The resulting IMC controller can be obtained $q=(\tau s-1)(\alpha s+1)\big/K(\lambda s+1)^3$. The ideal feedback controller equivalent to the IMC controller is

$$G_c = \frac{(\tau s-1)(\alpha s+1)}{K\left[(\lambda s+1)^3 - e^{-\theta s}(\alpha s+1)\right]} \tag{7}$$

Approximating the dead time $e^{-\theta s}$ with a 1/2 Pade expansion $e^{-\theta s} = (6-2\theta s)\big/\left(6+4\theta s+\theta^2 s^2\right)$. The 1/2 Pade approximation is precise enough to convert the ideal feedback controller into a PID cascaded first order filter with barely any loss of accuracy as well as retain the desired controller form.

$$G_c = \frac{(\tau s-1)(\alpha s+1)\left(6+4\theta s+\theta^2 s^2\right)}{K\left[(\lambda s+1)^3\left(6+4\theta s+\theta^2 s^2\right)-(\alpha s+1)(6-2\theta s)\right]} \tag{8}$$

Expanding and rearranging the above Eq. (8)

$$G_c = \frac{(-\tau s+1)\left(6+4\theta s+\theta^2 s^2\right)(\alpha s+1)}{-K(6\theta-18\lambda+6\beta)s\left[1+\dfrac{\left(2\alpha\theta+\theta^2+12\lambda\theta+18\lambda^2\right)}{(6\theta-18\lambda+6\alpha)}s+\dfrac{\left(3\lambda\theta^2+12\lambda^2\theta+6\lambda^3\right)}{(6\theta-18\lambda+6\alpha)}s^2+\dfrac{\left(3\lambda^2\theta^2+4\lambda^3\theta\right)}{(6\theta-18\lambda+6\alpha)}s^3\dfrac{\lambda^3\theta^2}{(6\theta-18\lambda+6\alpha)}s^4\right]} \tag{9}$$

The analytical PID formula can be obtained by rearranging the above Eq. (9) and presented as

$$K_C = -\frac{4\theta}{K(6\theta+18\lambda-6\alpha)} \qquad \tau_I = 2\theta/3 \qquad \tau_D = \theta/4 \qquad a=\alpha \tag{10}$$

The parameters b in filter can be obtained by equating the remaining part of the denominator of Eq. (9) with the process pole and filter $(bs+1)$. Since the remaining part of the denominator of Eq. (9) contains the factor of the process pole, filter $(bs+1)$ and a high order polynomial terms in s. The high order polynomial term in s has barely any impact because it is not in control relevant frequency range.

$$b = \frac{\left(2\alpha\theta+\theta^2+12\lambda\theta+18\lambda^2\right)}{(6\theta-18\lambda+6\alpha)}+\tau \tag{11}$$

The value of the extra degree of freedom α is selected so that it cancels out the open-loop unstable pole at $s=1/\tau$. This means certainly adopt α so that $\left[1-(\alpha s+1)e^{-\theta s}/(\lambda s+1)^3\right]_{s=1/\tau}=0$. The value of α is obtained after some simplification

$$\alpha=\tau\left[\left(1+\frac{\lambda}{\tau}\right)^3 e^{\theta/\tau}-1\right] \tag{12}$$

3.2. Delayed Integrating Process (DIP)

$$G_p=G_D=\frac{Ke^{-\theta s}}{s}=\frac{\psi Ke^{-\theta s}}{(\psi s-1)} \tag{13}$$

The DIP can be modeled by considering the integrator as an unstable pole near zero and the consequently PID tuning rules are obtained follows:

$$K_C=-\frac{4\theta}{K\psi(6\theta+18\lambda-6\alpha)};\quad \tau_I=2\theta/3;\quad \tau_D=\theta/4;\quad a=\alpha;\quad b=\frac{(2\alpha\theta+\theta^2+12\lambda\theta+18\lambda^2)}{(6\theta-18\lambda+6\alpha)}+\psi;\quad \alpha=\psi\left[\left(1+\frac{\lambda}{\psi}\right)^3 e^{\theta/\psi}-1\right] \tag{14}$$

Simulation Results

4.1. Example 1. FODUP

A widely published example of a FODUP has been considered for the comparisons [2,5,6] is

$$G_p=G_D=\frac{1e^{-0.4s}}{1s-1} \tag{15}$$

Recently Liu *et al*. [6] had already demonstrated its superiority over many widely accepted previous approaches e.g., Tan *et al*. [4]. The proposed method is compared with the Lee *et al*. [2] and Liu *et al.* [6]. The three controller parameters for Liu *et al.* [6] method were taken as $K_c=2$, $C(s)=(s+1)/(0.4s+1)$, for $\lambda=0.5$ the disturbance estimator $F(s)=2.634+\frac{1}{0.9566s}+0.4058s$. For the fair comparison λ has been adjusted to obtained $Ms=3.03$ for each tuning rule with same as Liu *et al.* [6]. For the proposed method $\lambda=0.20$ gives the $Ms=3.03$ and tuning parameters are $K_c=0.4615$, $\tau_I=0.2667$, $\tau_D=0.1$, $a=1.5779, b=0.1053$ and $f_R=1.5779$. The tuning parameters for the Lee *et al.* [2] method is identical with Liu *et al.* [6] disturbance estimator for the same Ms value. The disturbance estimator design of the Liu *et al.* [6] method is identical with the Lee *et al.* [2], but the setpoint response is different in both cases, because both of them have different approach. Lee *et al.* [2] PID setting are $K_c=2.634$, $\tau_I=2.5197$, $\tau_D=0.1541$ and $f_R=2.3566$. Figs. 1(a & b) show the comparison of the proposed method with Liu *et al* [6] and Lee *et al.* [2], by introducing a unit step change in the both setpoint and load disturbance. For the servo response the setpoint filter is used for both the proposed and Lee *et al.* [2] methods whereas three control element structure is used for the Liu *et al* [6].

It is clear from the Fig. (1), the proposed method results in the improved load disturbance response. For the servo response the Liu *et al*. (2005) has better performance but the settling time of the Liu *et al.* [6] and the proposed method is similar. In Fig. (1a) Lee *et al.* [2] response is slow and it requires long settling time. It is important to note that in the well known modified IMC structure has theoretical advantage of eliminating the time delay from the characteristic equation. Unfortunately, this advantage is lost if the process model is inaccurate. Besides, there usually exists the process unmodeled dynamics in real process plant that inevitably tends to deteriorate the control system performance severely. For the disturbance rejection the proposed methods has big advantage and the Liu *et al.* [6] and Lee *et al.* [2] method have identical response.

The robustness of the controller by inserting a perturbation uncertainty of 10% in all three parameters simultaneously to obtain the worst case model mismatch, i.e., $G_p=G_D=1.1e^{-0.44s}/(0.9s-1)$. The simulation results for the proposed and other tuning methods are presented in Fig. (2) for both the set-point and the disturbance rejection. It is clear form the Fig. (2) that the proposed controller tuning method has the best setpoint as well as load response while the modified IMC controller structure which contains the three-element controller of the Liu *et al.*(2005) method has worst response for the model mismatch for the setpoint.

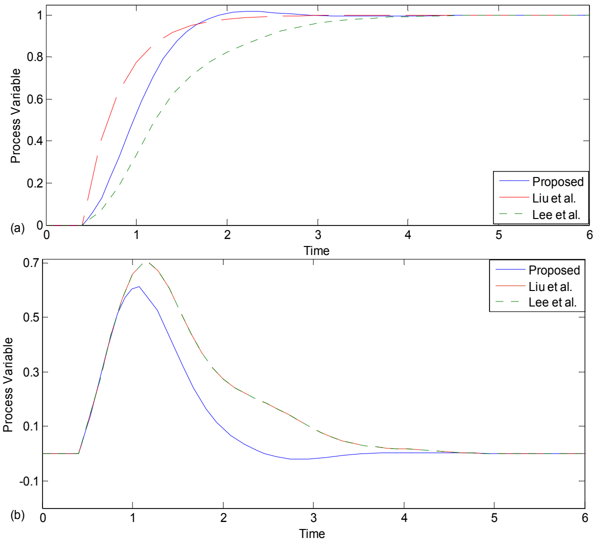

Fig. 1. Simulation results for Example 1 (FODUP)

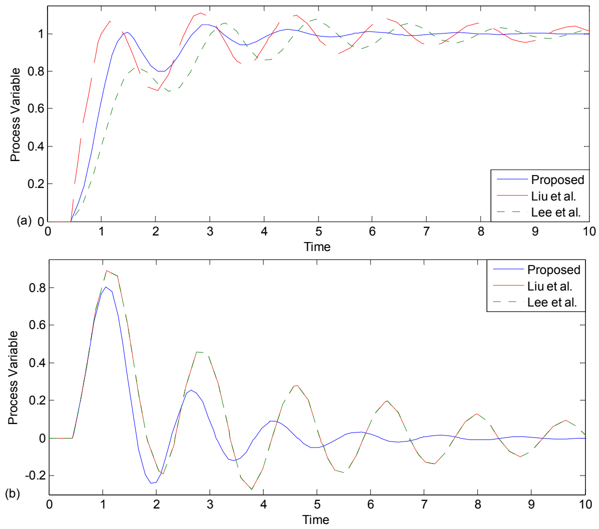

Fig. 2. Perturbed response for Example 1 (FODUP)

4.3. Closed-loop time constant λ guidelines

The closed-loop time constant λ is an only one user-defined tuning parameter in the proposed tuning rule. Based on extensive simulation studies, it is observed that the starting value of λ can be considered to be half of the process time delay, which can give robust control performance. If not, the value should be increased carefully until both the nominal and robust control performances are achieved.

Conclusions

A simple design method of the analytical PID cascaded filter tuning method has been proposed based on the IMC principle. The proposed method has excellent improvement in both setpoint and disturbance rejection for the FODUP process. The simulation has been conducted for the fair comparison when the various controllers were tuned to have the same degree of robustness by the measure of Ms value. The robustness study has been conducted by inserting a perturbation uncertainty in all parameters, where proposed study has clear advantage. The closed-loop time constant λ guideline was also proposed.

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