Implementation of Constrained Optimal Level Control Using a Conventional Proportional-Integral Controller

Nguyen Viet Ha, Joonho Shin[1], Moonyong Lee[*]
School of Display and Chemical Engineering, Yeungnam University,
[1]Corporate R&D, LG Chem
(mynlee@yu.ac.kr[*])

1. Introduction

The industrial importance of liquid level loops has led to extensive research interest to achieve the enhanced control performance of the level loop.

The level controller is required to provide non-aggressive and smooth control action as well minimizing the deviation of the level. Furthermore, a level loop normally has two important requirements: (1) the rate of change of the outlet flow should be kept below a specified allowable limit; (2) the deviation of the level should also be within a specified allowable limit.

In this study, we developed an analytical design method for a conventional PI controller that enables the constrained optimal control of the liquid level loop. The constrained optimal level control problem is firstly formulated and then converted into a simple form with two independent variables by using a proper variable transformation. The Lagrangian multiplier method is applied to handle the constraint optimization problem and the optimal PI tuning rule is finally found from the analysis of the global optimum condition. The proposed method is shown to deal with the two major control specifications in level loops explicitly, while minimizing the optimal control performance measure.

2. Constrained optimal control

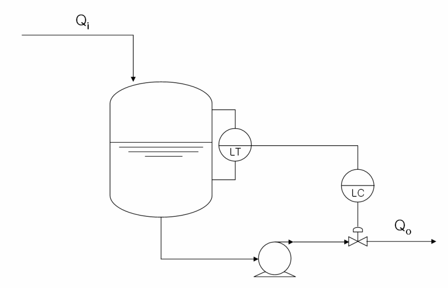

Fig.1. Schematic of a level control loop featuring manipulation of the outlet stream

The liquid level control system presented in Figure 1 is simply described as

$$H(s) = \frac{1}{As}Q_i(s) - \frac{1}{As}Q_o(s) \tag{1}$$

where $Q_o(s) = K_L(1+\frac{1}{\tau_I s})(H(s) - H^{set}(s))$ and $K_L = K_c \frac{Q_{o\max}}{\Delta H}$

The closed-loop transfer functions for the level control system are:

$$H(\mathrm{s}) = \frac{\tau_H \tau_I}{A} \frac{s}{\tau_H \tau_I s^2 + \tau_I s + 1} Q_i(s) + \frac{\tau_I s + 1}{\tau_H \tau_I s^2 + \tau_I s + 1} H^{set}(s) \tag{2}$$

$$Q_o(s) = \frac{\tau_I s + 1}{\tau_H \tau_I s^2 + \tau_I s + 1} Q_i(s) - \frac{K_L \tau_H s(\tau_I s + 1)}{\tau_H \tau_I s^2 + \tau_I s + 1} H^{set}(s) \quad (3)$$

Where $\tau_H = \frac{A}{K_L} = \frac{\tau_V}{K_c}$; $\tau_V = \frac{(\Delta H)A}{Q_{o\max}}$ and the damping factor is expressed as $\zeta = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau_H}}$ (4)

The control objective is to minimize : the rate of change of the outlet flow and the deviation of the level, while keeping (1) the rate of change of the outlet flow within a specified allowable limit and (2) the deviation of the level within a specified allowable limit for a given load variation. The constrained optimal control problem can be defined as finding the controller parameters that minimize the performance measure in Equation (5a) for a given step change in Q_i, subject to the constraints in Equations (5b~c):

$$\min \Phi = \omega \int_0^\infty [\frac{H(t)}{\Delta H}]^2 dt + (1-\omega)\int_0^\infty [\frac{Q'_o(t)}{Q'_{omax}}]^2 dt \quad (5a)$$

Subject to $|Q'_o(t)| \le Q'_{omax}$ and $|H(t)| \le H_{\max}$ (5b~c)

The constrained optimal control problem can be expressed in terms of τ_H and ζ as follows:

$$\min \Phi(\tau_H, \zeta) = \alpha \tau_H{}^3 \zeta^2 + \beta \cdot \frac{1}{\tau_H}\left(1 + \frac{1}{4\zeta^2}\right) \quad (6a)$$

subject to $\tau_H \ge \gamma_L h(\zeta)$ and $\tau_H \le \frac{\gamma_U}{g(\zeta)}$ (6b~c)

where $\alpha = 2\omega\left(\frac{\Delta Q_i}{A\Delta H}\right)^2$ $\beta = \left(\frac{1-\omega}{2}\right)\left(\frac{\Delta Q_i}{Q'_{o\max}}\right)^2$; $\gamma_U = \frac{AH_{\max}}{\Delta Q_i}$, $\gamma_L = \frac{\Delta Q_i}{Q'_{o\max}}$,

$$h(\zeta) = \frac{\sqrt{1+x^2}}{2}\exp\left(-\frac{3\tan^{-1}x - \pi}{x}\right) \quad for \quad 0 < \zeta < 0.5 \quad ; \quad h(\zeta) = 1 \quad for \quad 0.5 \le \zeta$$

$$g(\zeta) = \frac{2}{\sqrt{1+x^2}}\exp\left(-\frac{\tan^{-1}x}{x}\right) \quad for \quad 0 < \zeta < 1 \quad ; g(\zeta) = \frac{2}{\sqrt{1-x^2}}\exp\left(-\frac{\tanh^{-1}x}{x}\right) \quad for \quad 1 > \zeta$$

where $x = \frac{\sqrt{1-\zeta^2}}{\zeta}$ $for \quad 0 < \zeta < 1$; $x = \frac{\sqrt{\zeta^2 - 1}}{\zeta}$ $for \quad \zeta > 1$

It is preferable for a level controller to allow smaller $Q'_{o\max}$ and $H_{\max}$ specifications. However, if smaller $Q'_{o\max}$ and/or $H_{\max}$ specifications are applied, the feasible region bounded by these constraints also becomes smaller and eventually disappears. For a given $Q'_{o\max}$ (or $H_{\max}$) specification, the tightest $H_{\max}$ (or $Q'_{o\max}$) specification occurs at $\zeta^t = 0.4040$ and thus is calculated by

$$H_{\max} Q'_{o\max} = h(\zeta^t) g(\zeta^t)(\Delta Q_i^2 / A) = 0.5206(\Delta Q_i^2 / A) \quad (7)$$

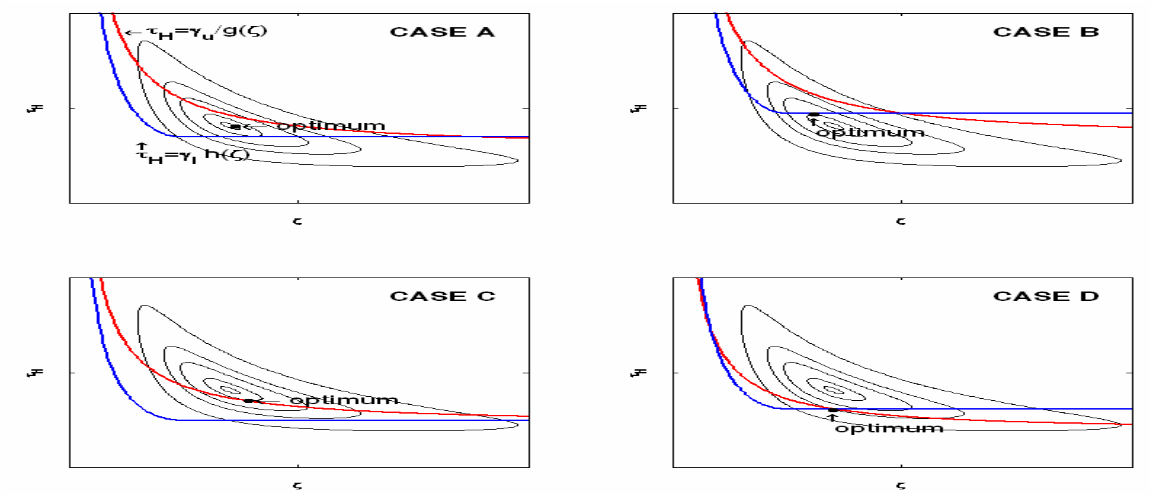

Fig.2. Typical contours and constraints with four possible cases of optimum location.

Figure 2 shows four cases are possible with respect to the global optimum location: (1) the global optimum is in the interior of the constraint set, denoted by $(\zeta^{\dagger}, \tau_H^{\dagger})$; (2) the global optimum is on the constraint $\tau_H = \gamma_L h(\zeta)$,denoted by (ζ^{*}, τ_H^{*}) ; (3) the global optimum is on the constraint $\tau_H = \dfrac{\gamma_U}{g(\zeta)}$, denoted by $(\zeta^{**}, \tau_H^{**})$; (4) the global optimum is located on the right vertex point formed by the two constraints, denoted by (ζ^{v}, τ_H^{v}) .

The procedure for finding the global optimum of (ζ, τ_H) is as follows:

(i) Calculate the unconstrained extreme point $(\zeta^{\dagger}, \tau_H^{\dagger})$

If $\gamma_L h(\zeta^{\dagger}) \le \tau_H^{\dagger} \le \dfrac{\gamma_U}{g(\zeta^{\dagger})}$, then $(\zeta^{\dagger}, \tau_H^{\dagger})$ is the global optimum.

(ii) If $\tau_H^{\dagger} < \gamma_L h(\zeta^{\dagger})$, then calculate (ζ^{*}, τ_H^{*}) and (ζ^{v}, τ_H^{v}) with If $\zeta^{*} \le \zeta^{v}$, then (ζ^{*}, τ_H^{*}) is the global optimum ; If $\zeta^{*} \ge \zeta^{v}$, then (ζ^{v}, τ_H^{v}) is the global optimum.

(iii) If $\tau_H^{\dagger} > \dfrac{\gamma_U}{g(\zeta^{\dagger})}$, then calculate $(\zeta^{**}, \tau_H^{**})$ and (ζ^{v}, τ_H^{v}) . If $\zeta^{**} \le \zeta^{v}$, then $(\zeta^{**}, \tau_H^{**})$ is the global optimum ; If $\zeta^{**} \ge \zeta^{v}$, then (ζ^{v}, τ_H^{v}) is the global optimum.

If the feasible region is unbounded, no vertex point formed by the two constraints exists for $\zeta > 0.4040$, and accordingly case (4) does not exist. Also, the inequality conditions $\zeta^{*} \le \zeta^{v}$ and $\zeta^{**} \le \zeta^{v}$ do not need to be evaluated for cases (2) and (3), respectively.

Once the global optimum is obtained in terms of ζ and τ_H, the corresponding optimal PI parameters can be directly calculated as: $K_C = \dfrac{(\Delta H)A}{Q_{o\max}\tau_H}$; $\tau_I = 4\zeta^2\tau_H$ (8)

3. Simulation

The liquid level of a drum with: A=1 m^2 and a working volume $A\Delta H$ of 2 m^3 is controlled by a PI controller. The $Q_{o\max}$ is 4 m^3/min. The initial steady-state level is 50% and the Q_i *and* Q_o are both 1 m^3/min. The ΔQ_i is 1 m^3/min. The weighting factor for optimal control is set to $\omega = 0.8$.

Consider the following four cases:
Case 1 ($Q'_{o\max}$ is $3\,m^3/\text{min}^2$ and $H_{\max}$ is 1.0 m);**Case 2** ($Q'_{o\max}$ is $1.2\,m^3/\text{min}^2$ and $H_{\max}$ is 1.0 m) ; **Case 3** ($Q'_{o\max}$ is $4\,m^3/\text{min}^2$ and $H_{\max}$ is 0.2 m) ; **Case 4** ($Q'_{o\max}$ is $1.5\,m^3/\text{min}^2$ and $H_{\max}$ is 0.4 m)

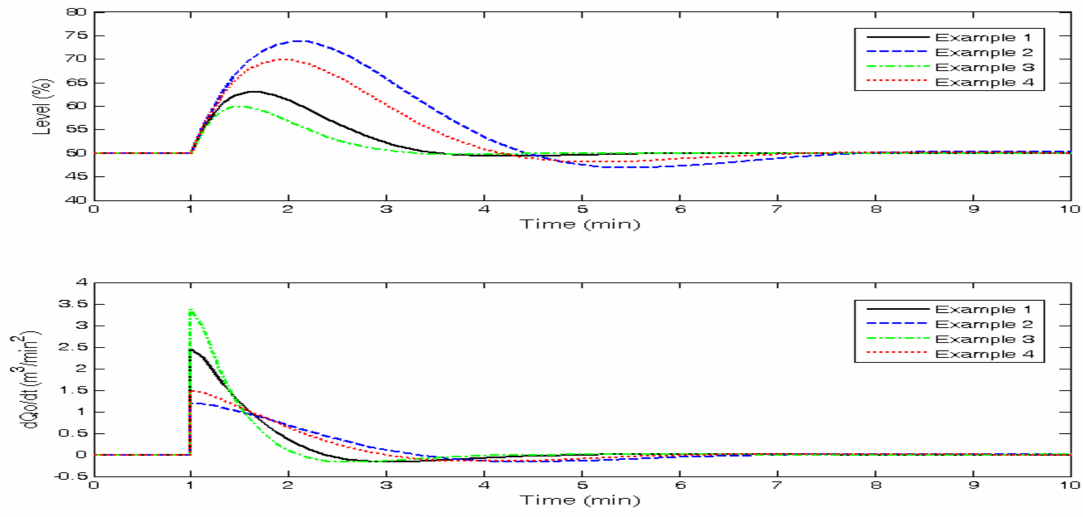

Fig. 3. Responses of level and rate of change of outlet flow for cases 1, 2, 3, and 4

Figure 3 compares the responses for the liquid level and the rate of change of the outlet flow rate for each case. A step change of $1\,m^3$/min is made in the inlet flow at t=0 in the simulation. As seen in the figure, the PI controllers designed by the proposed method give the optimal responses while strictly satisfying the given $H_{\max}$ and $Q'_{o\max}$ specifications.

4. Conclusions

A constrained optimal control problem for a liquid level system is formulated. The original constrained optimization problem is converted into a simple constrained problem with two independent variables. To obtain the analytical solution for the optimal PI parameters, the constrained optimization problem is further converted into the equivalent unconstrained problem using the classical Lagrangian multiplier method. It is shown that the proposed method enables a conventional PI controller to cope with all classes of level control (from tight level control to averaging level control) in the unified manner. The proposed method explicitly deals with the important control specifications as well as minimizes the optimal performance measure.

Acknowledgment

This research was supported by Yeungnam University research grants in 2008.

References

1. Luyben, W. and Buckley, P.S. (1977). A proportional-lag controller, *Instrum. Tech.*, **24**, 65-68.
2. MacDonald, K., McAvoy, T., and Tits, A. (1986). Optimal averaging level control, *AIChE J.*, **32**, 75-86.
3. Rivera, D.E., Morari, M., and Skogestad, S. (1986). Internal model control. 4. PID controller design, *Ind. Eng. Chem. Process Des. Dev.*, **25**, 252-265.
4. Wu, K., Yu, C., and Cheung, Y. (2001). A two degree of freedom level control, *J. of Process Control*, **11** 311-319.