Analytical Design of Multi-loop PI Controllers Based on the Direct Synthesis for Multi-delay Processes

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1. Introduction

The multivariable control design and analysis is currently important topic in the advanced process control. Hence, there are many different approaches that exist in literature [1-8] for designing high performance multi-loop control systems. It should be noted that the multiple time delays are the main cause of the strong interactions that can give extremely complicated loop dynamics. According to the characterization of non-minimum phase zeros and unavoidable time delays, the high performance of control system can be achieved while design procedure is ensuring the actual multi-loop control system performance.

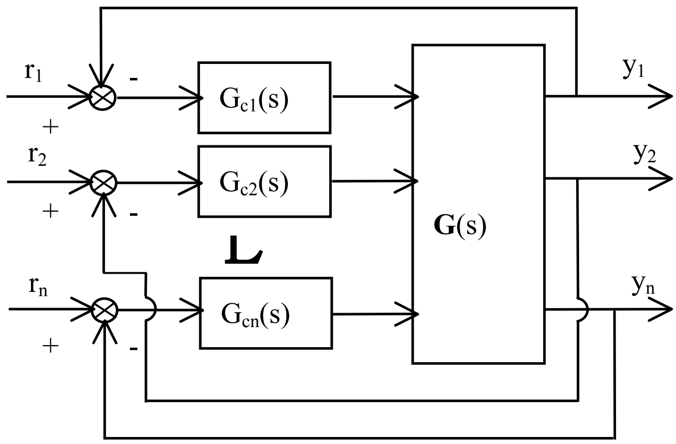

Figure.1. Multi-loop control system.

2. Proposed control system design

2.1. The multi-loop direct synthesis based on set-point responses

Consider a standard block diagram of multivariable feed-back control in Fig.1, the closed-loop transfer function matrix can be written as

$$\frac{\mathbf{y}(s)}{\mathbf{r}(s)} = \mathbf{H}(s) = \mathbf{G}(s)\tilde{\mathbf{G}}_c(s)\left(\mathbf{I} + \mathbf{G}(s)\tilde{\mathbf{G}}_c(s)\right)^{-1} \tag{1}$$

The multi-loop feedback controller can be obtained by rearranging the Eq. 1 as follows

$$\tilde{G}_C(s) = G^{-1}(s)H(s)\left[I - H(s)\right]^{-1} \tag{2}$$

The new design method is proposed to design multi-loop PID controller simply as follows: firstly, according to the direct synthesis approach, the diagonal desired closed-loop response is specified as a closed-loop transfer function for set-point changes. Secondly, consider to correlation between a square matrix and a diagonal matrix, each elements of the controller can be derived as

$$g_{cii}(s) = \frac{\mathbf{G}^{ii}}{\det(\mathbf{G})} diag\left(\frac{h_{ii}}{1-h_{ii}}\right)_{nxn} \tag{3}$$

By using the relative gain array (RGA) that is presented by Bristol [2], Eq. 3 can be rewritten as

$$g_{cii}(s) = \lambda_{ii}(s) g_{ii}^{-1}(s) diag\left(\frac{h_{ii}(s)}{1-h_{ii}(s)}\right)_{nxn} \tag{4}$$

where g_{ii} is the (i,i) *th* element of process model G, $\lambda_{ii} = g_{ii}\frac{G^{ii}}{|G|}$ is relative gain array for diagonal elements, and h_{ii} is each diagonal element of closed-loop transfer function matrix.

<u>2.2. Design the models of desired closed-loop transfer function.</u>

Consider to H_2 optimal performance objective of IMC control system, the realizable closed-loop transfer function can be obtained as

$$h_{ii}(s) = e^{-\theta_i s} f_i(s) \prod_{k=1}^{q_i} \frac{z_k - s}{z_k^* + s}, \qquad i = 1,2,\mathrm{K}\ , m \tag{5}$$

where $f_i(s)$ is the i*th* loop IMC filter and chosen simply as

$$f_i(s) = \frac{1}{(\tau_i s + 1)^r} \tag{6}$$

where τ_i is the tuning parameter for each loop to achieve the desirable response performance and robustness. Furthermore, τ_i is also the closed-loop time constant; z_k is the RHP zeros of det (G) and z_k^* is the complex conjugate of RHP zeros.

Substituting Eqs. 5 and 6 into Eq. 4, the diagonal proposed controller can be obtained as

$$g_{cii}(s) = \lambda_{ii}(s) g_{ii}^{-1}(s) \left(\frac{e^{-\theta_i s} \prod_{k=1}^{q_i} \frac{z_k - s}{z_k^* + s}}{(\tau_i s + 1)^r - e^{-\theta_i s} \prod_{k=1}^{q_i} \frac{z_k - s}{z_k^* + s}} \right) \tag{7}$$

Hence, the controller form is not rational algebraic fraction. By using the Maclaurin expansion, the controller can be converted to the standard PID form within physically realized.

According to 2x2 systems with first-order plus delay time (FOPDT) model, and assume that no RHP zeros in the system, it is can be derived from Eq. 7 as

$$g_{cii}(s) = \lambda_{ii}(s) \frac{(T_{ii}s + 1)}{K_{ii}} \left(\frac{1}{(\tau_i s + 1)^r - e^{-\theta_i s}} \right) \tag{8}$$

where $\lambda_{ii}(s)$ is the dynamic relative gain. For 2x2 systems with FOPDT model, it is given by

$$\lambda(s) = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}} \frac{(T_{11}s+1)(T_{22}s+1)}{(T_{12}s+1)(T_{21}s+1)} e^{-\theta s}} \tag{9}$$

The multi-delays can be defined by

$$\theta = \theta_{12} + \theta_{21} - \theta_{11} - \theta_{22} \tag{10}$$

By using Maclaurin approximations, Eq. 8 can be converted to the standard form of multi-loop PI controller as

$$\tilde{G}_C(s) = \frac{1}{s}\left[\tilde{K}_C s + \tilde{K}_I\right] \tag{11}$$

This proposed design method can be applied not only for 2x2 systems with FOPDT model but also for other high order control systems.

3. Simulation study

Consider to the distillation column of Wood and Berry (Luyben, 1986) which can be represented as

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \tag{12}$$

The multi-loop PI controllers can be obtained by proposed method, biggest log modulus (BLT) tuning design method [4], and independent IMC tuning [5] as follows:

Proposed method : $G_C(s) = diag\left[0.260\left(1+\frac{1}{9.90s}\right), -0.104\left(1+\frac{1}{8.06s}\right)\right]$

BLT : $G_C(s) = diag\left[0.375\left(1+\frac{1}{8.29s}\right), -0.075\left(1+\frac{1}{23.6s}\right)\right]$

Ind. IMC tuning : $G_C(s) = diag\left[0.737\left(1+\frac{1}{17.2s}\right), -0.103\left(1+\frac{1}{15.9s}\right)\right]$

Figure 2 is shown the closed-loop time responses by several tuning methods. The magnitude of step set-point was made sequentially at 1^{st} and 2^{nd} loop as 1. r_i was selected as 1 for all loops. The closed-loop time constant $\tau_{1,2}$ was chosen arbitrary as 5, 5 for two loops. As can be seen in this figure, the proposed control system has better stability and robustness than those by BLT and independent IMC tuning. In addition, the total integral absolute error (IAE) value in proposed control system is also smaller than others.

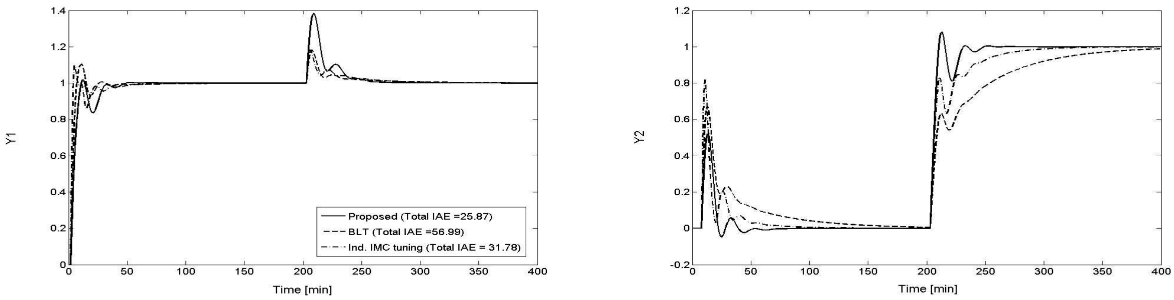

Fig. 2. Set-point responses for WB column.

4. Conclusions

In this paper, a new analytical design method is proposed for multi-delay processes. The proposed method is straightforward and easy to implement in MIMO control systems. Furthermore, the stability and robustness of proposed control systems can be enhanced by finding optimal closed-loop time constant. In this case, one can use weighted sum *Mp* criterion of T. N. Luan Vu, J. Lee, and M. Lee [8]. The time-domain simulation illustrates that the proposed PI control system provides well-balanced closed-loop time responses and it also guarantees for closed-loop stability and robustness.

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