# 시간지연을 가진 적분공정을 위한 IMC-PID 제어기 조율 방법

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## IMC Based PID Controller Tuning Method for the Integrating Process with Time Delay

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## Introduction

In the process control, more than 95% of the control loops are of the proportional-integral-derivative (PID) type. The main reason is its relatively simple structure, which can be easily understood and implemented in practice. A recent trend clearly shows that the PID tuning of time-delay first order integrating process, double integrating and integrating process with/without zero has been an active area of research in the literature [1-9]. The double integrating processes are mainly encountered in the aerospace control systems and industrial servo-motor such as spacecraft and high speed disk.

Due to its simplicity and performance, the analytically derived IMC-PID tuning methods [4,6,8,9] attracted the attention of industrial users. The IMC-PID tuning rule has only one user-defined tuning parameter, which is directly related to the closed-loop time constant. The IMC-PID controller provides a good set-point tracking but a sluggish disturbance response especially for the process with a small time-delay/time-constant ratio. However, for many process control applications, the disturbance rejection is much more important than the set-point tracking. Therefore, a controller design that emphasizes disturbance rejection rather than set-point tracking is an important design problem that has received renewed interest recently.

Therefore, the present study is devoted to develop the generalized IMC-PID design method for the first order integrating process with time delay, which can be extended for the double integrating and an integrator with negative and positive process.

#### **Controller Design Algorithm**

Figures 1 show the block diagram of IMC control and equivalent classical feedback control structures, where  $G_p$  is the process,  $\tilde{G}_p^0$  is the process model, and q is the IMC controller.

## IMC controller design steps

Step 1: A process model  $\mathcal{G}_p^{\circ}$  is factored into invertible and non invertible parts

$$G_P^0 = P_M P_A$$

where  $P_M$  is the portion of the model inverted by the controller;  $P_A$  is the portion of the model not inverted by the controller;  $P_A(0) = 1$ .

Step 2: The idealized IMC controller is the inverse of the invertible portion of the process model.  $q = P_M^{-1}$ (2)

To make the IMC controller proper, it is mandatory to add the filter. Thus, the IMC controller is designed as

$$q = q = P_M^{-1} f$$

(3)

To obtain a good response for processes with negative poles or poles near zero, the IMC controller q should be designed to satisfy the following conditions.

(1)

If the process  $G_p$  has poles near zero at  $z_1, z_2, L, z_m$ , then q should have zeros at  $z_1, z_2, L, z_m$  and also  $1-G_n q$  should have zeros at  $z_1, z_2, L, z_m$ .

Since the IMC controller q is designed as  $q = p_m^{-1} f$ , the first condition is satisfied automatically because  $p_m^{-1}$  is the inverse of the model portion with the poles near zero. The second condition can be fulfilled by designing the IMC filter  $f = (\sum_{i=1}^{m} \beta_i s^i + 1) / (\lambda^2 s^2 + 2\lambda \xi s + 1)^r$ , where  $\lambda$  is an adjustable parameter which controls the trade–off between the performance and robustness, r is selected to be large enough to make the IMC controller (semi–)proper, and  $\beta_i$  is determined by Eq. (4) to cancel the poles near zero in  $G_p$ . The value of  $\xi = 1$  is usually selected for the other processes except for the integrating process with strong lead term where  $\xi > 1$  gives the better performance and eliminate undershoot in disturbance rejection response.

$$1 - G_{p}q\Big|_{s=z_{1}, L-z_{m}} = \left|1 - \frac{p_{A}(\sum_{i=1}^{m} \beta_{i}s^{i}+1)}{\left(\lambda^{2}s^{2} + 2\lambda\xi s + 1\right)^{r}}\right|_{s=z_{1}, L, z_{m}} = 0$$
(4)

Then, the IMC controller is described as:

 $G_c$  to a standard PID controller is utilized here.

$$q = p_m^{-1} \frac{(\sum_{i=1}^m \beta_i s^i + 1)}{(\lambda^2 s^2 + 2\lambda\xi s + 1)^r}$$
(5)

From the above design procedure, a stable, closed–loop response can be achieved by using the IMC controller. The ideal feedback controller that is equivalent to the IMC controller can be expressed in terms of the internal model  $G_p^{\prime}$  and is  $G_c = q/(1-G_p^{\prime}q)$ . Since the resulting controller has not a standard PID controller form, the remaining issue is to design the PID controller that resembles the equivalent feedback controller most closely. Lee et al. [6] 's method for converting the ideal feedback controller

#### **Proposed Tuning Rule**

An integrating process involving time delay and time constant by the following transfer function is studied:

$$G_{p} = \frac{Ke^{-\theta s}}{s\left(\tau s + 1\right)} \tag{6}$$

The above process modeled can be considered as the second order plus dead time (SOPDT):

$$G_p = \frac{Ke^{-\theta s}}{(\tau s+1)(s+1/\phi)} = \frac{\phi Ke^{-\theta s}}{(\tau s+1)(\phi s+1)}$$

$$\tag{7}$$

where  $\phi$  is an arbitrary constant with a sufficiently large value and IMC filter  $f = (\beta_2 s^2 + \beta_1 s + 1)/(\lambda^2 s^2 + 2\lambda \xi s + 1)^2$  is utilized and the analytical PID formula can be obtained as:

$$k_{c} = \frac{\tau_{I}}{\phi K \left(4\lambda \xi + \theta - \beta_{1}\right)}, \quad \tau_{I} = \left(\phi + \tau + \beta_{1}\right) - \frac{\left(-\frac{\theta^{2}}{2} + \theta \beta_{1} - \beta_{2} + 2\lambda^{2} + 4\lambda^{2} \xi^{2}\right)}{\left(4\lambda \xi + \theta - \beta_{1}\right)}, \quad \tau_{D} = \frac{\left(\frac{\beta_{2} + (\tau + \phi)\beta_{1} + \phi\tau_{1}\right) - \frac{\left(-\frac{\theta^{2}}{2} + \theta \beta_{2} + 4\lambda^{2} \xi^{2}\right)}{\tau_{I}} - \frac{\left(-\frac{\theta^{2}}{2} + \theta \beta_{1} - \beta_{2} + 2\lambda^{2} + 4\lambda^{2} \xi^{2}\right)}{\left(4\lambda \xi + \theta - \beta_{1}\right)},$$

The values of  $\beta_1$  and  $\beta_2$  are selected to cancel out the poles at  $-1/\tau$  and  $-1/\phi$ . This requires  $[1-Gq]|_{s=-1/\tau, -1/\phi} = 0$  and thus  $\left[1-(\beta_2 s^2+\beta_1 s+1)e^{-\theta s}/(\lambda^2 s^2+2\lambda\xi s+1)^2\right]_{s=s=-\frac{1}{2}\tau, -\frac{1}{2}\phi} = 0$ . The values of  $\beta_1$  and  $\beta_2$  are obtained

$$\beta_{1} = \frac{r^{2} \left(\frac{\lambda^{2}}{r^{2}} - \frac{2\lambda\xi}{r} + 1\right)^{2} e^{-i\theta r} - \phi^{2} \left(\frac{\lambda^{2}}{\phi^{2}} - \frac{2\lambda\xi}{\phi} + 1\right)^{2} e^{-i\theta r} + (\phi^{2} - r^{2})}{(\phi - r)}, \qquad \beta_{2} = r^{2} \left[ \left(\frac{\lambda^{2}}{r^{2}} - \frac{2\lambda\xi}{r} + 1\right)^{2} e^{-i\theta r} - 1 \right] + \beta_{1} \tau$$

A set-point filter is introduced to enhance the servo response, which is given as  $f_{\rm R} = (\gamma \beta_1 s + 1)/(\beta_2 s^2 + \beta_1 s + 1)$ , where  $0 \le \gamma \le 1$ . The extreme case with  $\gamma = 0$  has no lead term in the set-

point filter which would cause a slow servo response. Note that  $\gamma$  can be adjusted online to obtain the desired speed of the set-point response.

## **Simulation Study**

Example 1: Stable FOPDT System with an Integrator

Consider the following integrating process (Wang and Cai [1]; Zhang et al. [2]):

$$G_{p} = \frac{1e^{-0.2s}}{s(s+1)}$$
(8)

The proposed controller was designed by considering the above process as  $G_p = 100e^{-0.2s}/(100s+1)(s+1)$ . The proposed method is compared with two other PID controllers: Wang and Cai [1], and Zhang et al [2]. In order to ensure a fair comparison, all controllers compared are tuned to have Ms = 1.65 by adjusting their respective  $\lambda$ .

A unit step change is introduced in both the set-point and load disturbance. Figure 2 compares the set-point and load disturbance responses by three compared controllers. The 2DOF control scheme was used in each method to enhance the set-point response. The proposed controller shows both smaller overshoot and faster disturbance rejection than those by the Wang and Cai [1] and Zhang et al. [2] methods. The robust performance is evaluated by simultaneously inserting a perturbation uncertainty of 20% in all three parameters in the worst direction and finding the actual process as  $G_p = 1.2e^{-0.24s}/s(0.8s+1)$ . The simulation results for the model mismatch demonstrate the superior

robust performance of the proposed controller as well.



Fig. 1. Block diagram of control system (a) classical feedback control structure (b) the IMC structure

# Example 2: Double Integrating Process with Dead Time

Consider the double integrating process studies by Liu et al. [3]

$$G_{p} = \frac{e^{-0.8s}}{s^{2}}$$
(9)

Fig. 2. Response of the nominal system for Example 1.

In the simulation study, we compare the proposed PID controller with the Liu et al. [3] and Skogestad [4] methods. For the proposed method,  $\lambda = 1.25$  has been selected for better disturbance rejection responses both in the nominal and model mismatch case. The PID setting of the Liu et al. [3] and Skogestad [4] methods have been obtained from the Liu et al.'s paper.

The proposed controller was designed by considering the above process as  $G_p = 10000e^{-0.8s}/(100s+1)(100s+1)$ . Figure 3 shows the closed-loop output response for a unit step change in both the set-point and load disturbance. The comparison of the output response confirms the superiority of the proposed method both in nominal and disturbance rejection.

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Fig. 3. Response of the nominal system for Example 2.

## Conclusions

In this article, we have discussed an IMC based simple design method for a PID controller for the first order integrating process with time delay. The results showed that both nominal and robust performances of the PID controller were significantly enhanced in the proposed method. The proposed controller consistently achieved superior performance for several process classes.

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