Ch 02
Fundamentals of rheology

Types of flow

Shear flow

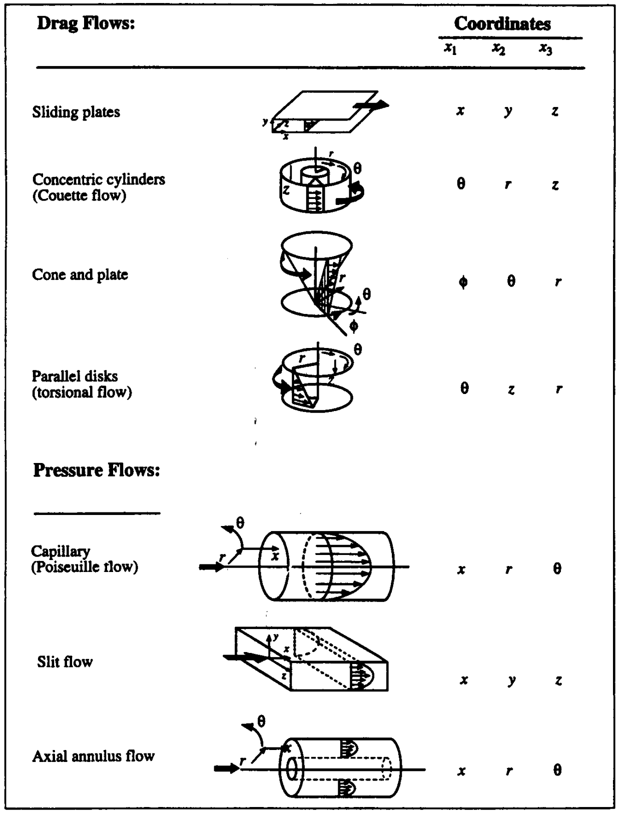


Figure 1.5 Geometries for producing shearing flows. (Adapted from Macosko, *Rheology Principles, Measurements, and Applications*, Copyright © 1994. Reprinted by permission from John Wiley & Sons.)

Extensional flow

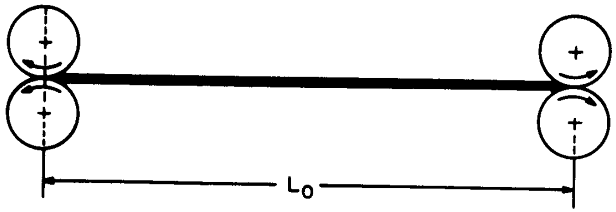


Figure 1.12 Rotating clamp device used by Meissner to impose a uniaxial extensional strain on a cylindrical filament of polymer of length L_0. Leaf springs in one of the sets of rotating clamps allow the extensional stress to be measured. (From Meissner 1971, reprinted with permission from Steinkopff Publishers.)

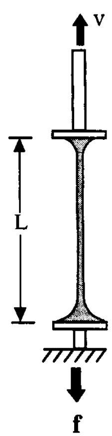


Figure 1.13 The stretching of a filament of viscoelastic liquid (shaded) sticking to two flat plates, one moving and the other attached to a force transducer. (From Macosko, *Rheology Principles, Measurements, and Applications*, Copyright © 1994. Reprinted by permission from John Wiley & Sons.)

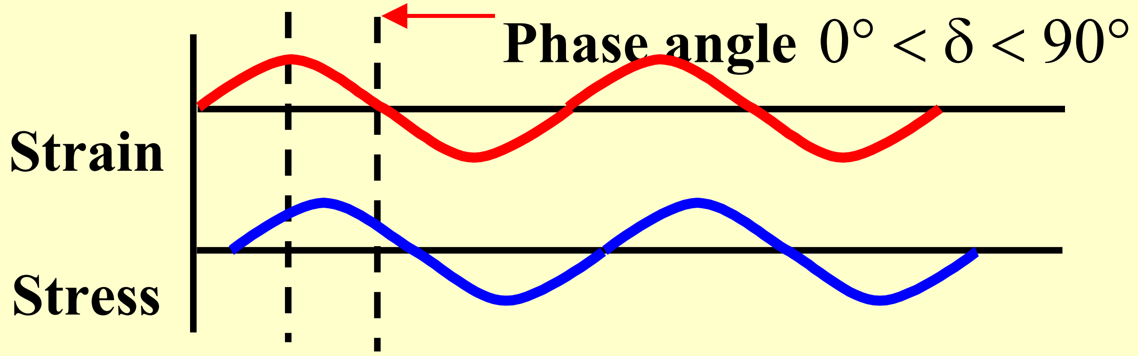
각 flow의 특징을 기술할 수 있어야 함

Shear flow

Steady shear viscosity $\eta = \sigma / \dot{\gamma}$

Transient shear viscosity $\eta^{+}(\dot{\gamma}, t) = \sigma(\dot{\gamma}, t) / \dot{\gamma}$

Storage and loss moduli $\sigma(t) = \gamma_0 [G'(\omega)\sin(\omega t) + G''(\omega)\cos(\omega t)]$



Dynamic properties

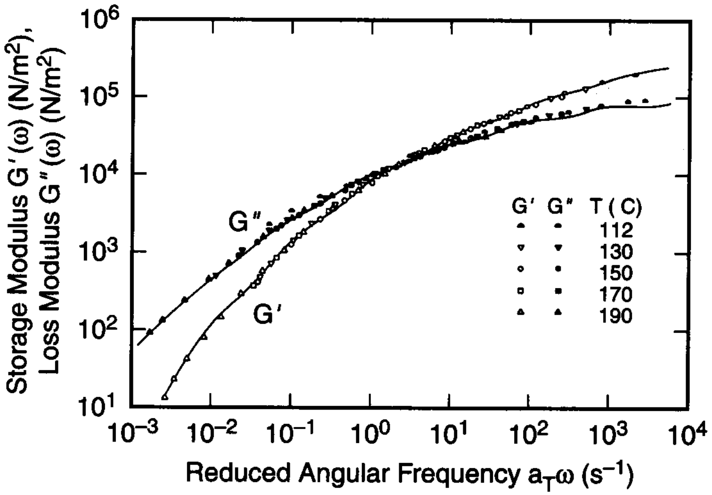


Figure 1.11 Storage and loss moduli for a low density polyethylene "Melt I." (These data were measured at several temperatures and shifted along the frequency axis by a "shift factor" a_T to form collapsed curves; see Section 3.5.2). The lines are empirical fits of Eqs. (3-25a) and (3-25b) to the data. (From Laun 1978, reprinted with permission from Steinkopff Publishers.)

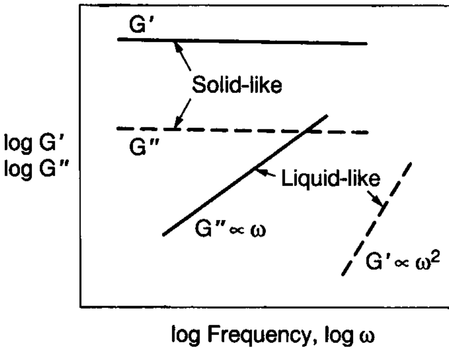


Figure 1.8 Illustrations of frequency-dependent storage and loss moduli G' and G'' for prototypical "liquid-like" and "solid-like" materials.

Shear viscosity and normal stress difference

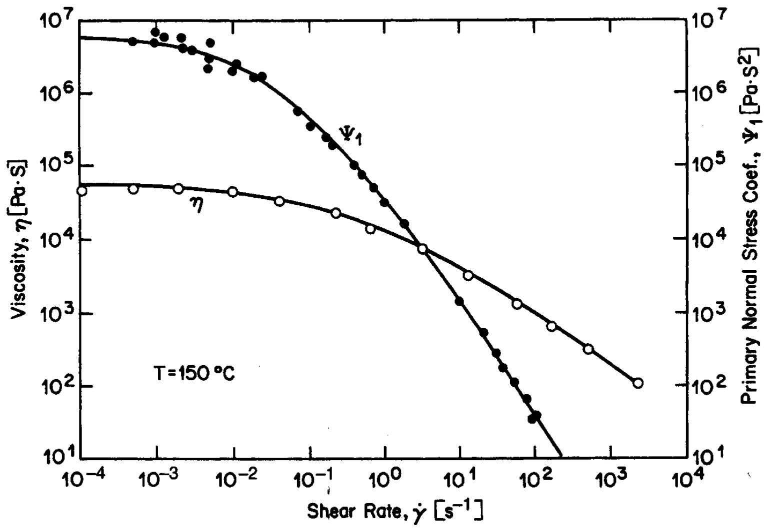


Figure 1.9 Steady-state shear viscosity η and first normal stress coefficient Ψ_1 versus shear rate for a low-density polyethylene melt, "Melt I." (From Laun 1978, reprinted with permission from Steinkopff Publishers.)

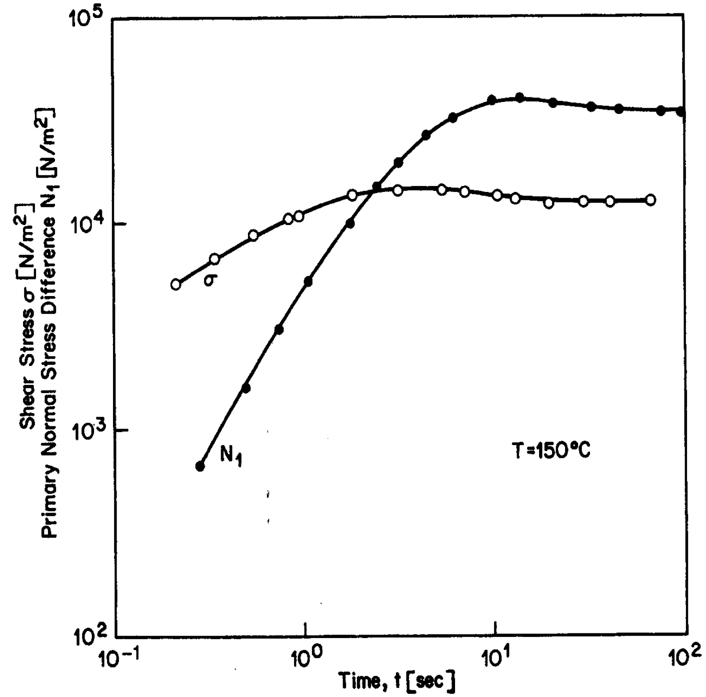


Figure 1.10 Transient shear stress σ and first normal stress difference N_1 after start-up of steady shearing for a low-density polyethylene melt, "Melt I," at a shear rate $\dot{\gamma} = 1$ sec^{-1}. (From Laun 1978, reprinted with permission from Steinkopff Publishers.)

Extensional flow

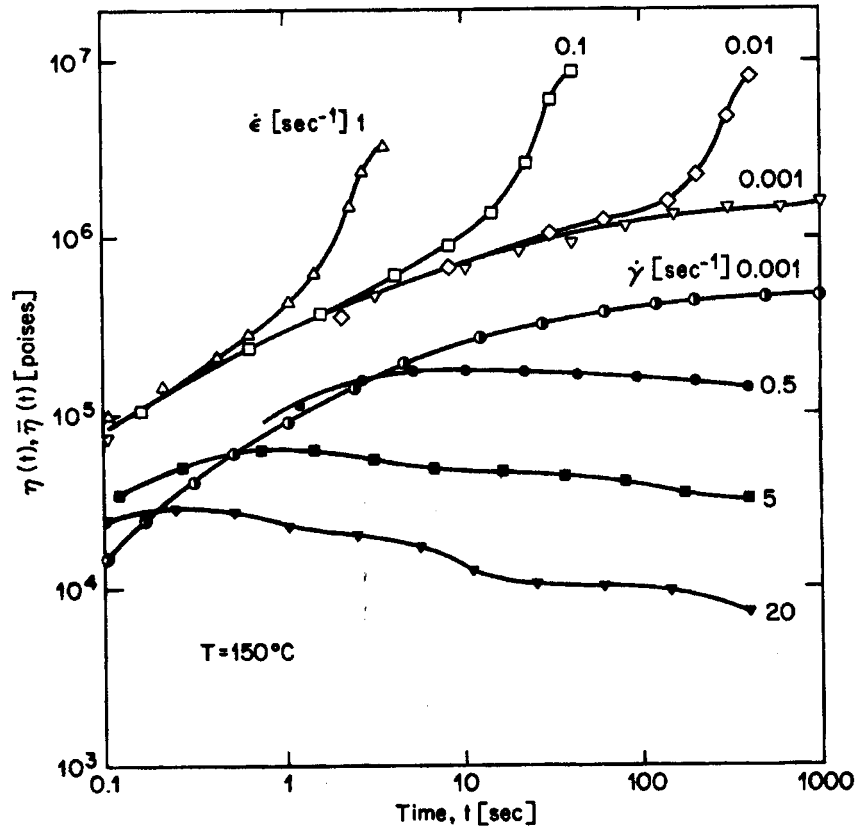


Figure 1.14 Uniaxial extensional viscosity $\bar{\eta}$ (open symbols) and shear viscosity η (closed and half-closed symbols) as functions of time after start-up of steady uniaxial extension or steady shearing for "Melt I." (From Meissner, J. Appl. Polym. Sci. 16:2877, Copyright © 1972. Reprinted by permission of John Wiley & Sons, Inc.)

Kinematics

Velocity gradient tensor

$$\nabla \mathbf{v} \equiv \begin{pmatrix} \frac{\partial \upsilon_1}{\partial x_1} & \frac{\partial \upsilon_2}{\partial x_1} & \frac{\partial \upsilon_3}{\partial x_1} \\ \frac{\partial \upsilon_1}{\partial x_2} & \frac{\partial \upsilon_2}{\partial x_2} & \frac{\partial \upsilon_3}{\partial x_2} \\ \frac{\partial \upsilon_1}{\partial x_3} & \frac{\partial \upsilon_2}{\partial x_3} & \frac{\partial \upsilon_3}{\partial x_3} \end{pmatrix}$$

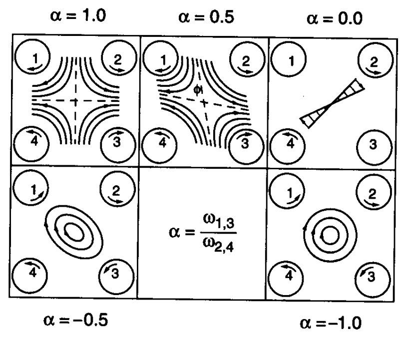


Figure 1.15 Flow fields produced by the four-roll mill. $\omega_{1,3}$ is the rotational velocity of rollers 1 and 3, and $\omega_{2,4}$ is that of rollers 2 and 4; α is the flow-type parameter [see Eq. (1-11)]. (From Fuller and Leal 1980, reprinted with permission from Steinkopff Publishers.)

Shear flow | Extensional flow | Uniaxial flow | Mixed flow

$$\nabla \mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \\ \dot{\gamma} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\nabla \mathbf{v} = \begin{pmatrix} \dot{\varepsilon}_1 & 0 & 0 \\ 0 & \dot{\varepsilon}_2 & 0 \\ 0 & 0 & \dot{\varepsilon}_3 \end{pmatrix}$$

$$\nabla \mathbf{v} = \begin{pmatrix} \dot{\varepsilon} & 0 & 0 \\ 0 & -\dot{\varepsilon}/2 & 0 \\ 0 & 0 & -\dot{\varepsilon}/2 \end{pmatrix}$$

$$\nabla \mathbf{v} = \frac{1}{2} G \begin{pmatrix} 1+\alpha & 1-\alpha \\ -(1-\alpha) & -(1+\alpha) \end{pmatrix}$$

Deformation gradient tensor $\quad \delta\mathbf{x} = \delta\mathbf{x}' \cdot \mathbf{E}$

$$\mathbf{E}(t',t) \equiv \frac{\partial \mathbf{x}}{\partial \mathbf{x}'} = \begin{pmatrix} \frac{\partial x_1}{\partial x_1'} & \frac{\partial x_2}{\partial x_1'} & \frac{\partial x_3}{\partial x_1'} \\ \frac{\partial x_1}{\partial x_2'} & \frac{\partial x_2}{\partial x_2'} & \frac{\partial x_3}{\partial x_2'} \\ \frac{\partial x_1}{\partial x_3'} & \frac{\partial x_2}{\partial x_3'} & \frac{\partial x_3}{\partial x_3'} \end{pmatrix}$$

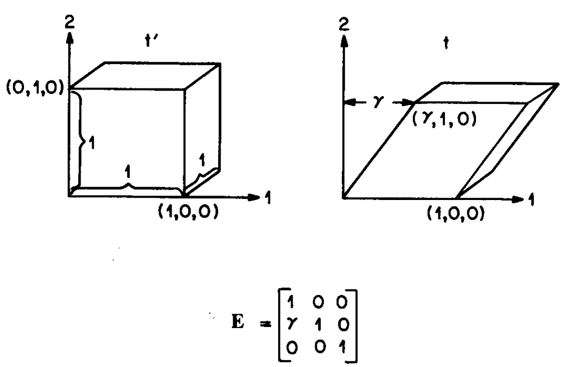


Figure 1.16 Definition of the deformation tensor **E** for the shearing deformation of a unit cube. (From Larson 1988, with permission.)

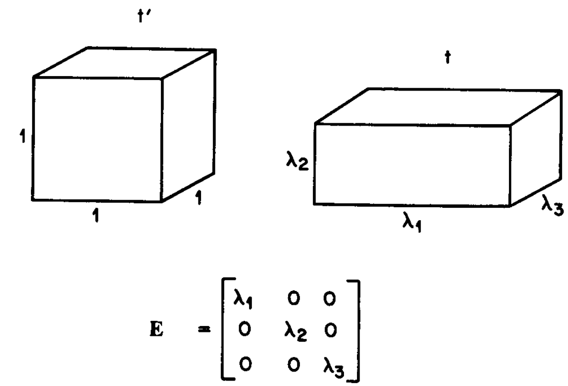


Figure 1.17 Definition of the deformation tensor **E** for a general extensional deformation of a unit cube. (From Larson 1988, with permission.)

Finger tensor $\mathbf{B} \equiv \mathbf{E}^T \cdot \mathbf{E}$

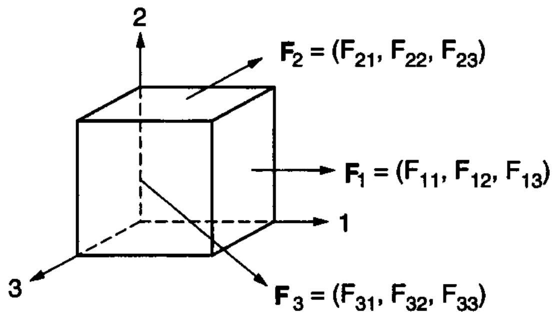
Shear flow

$$\mathbf{B} = \begin{pmatrix} 1+\gamma^2 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Extensional flow

$$\mathbf{B} = \begin{pmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_2^2 & 0 \\ 0 & 0 & \lambda_3^2 \end{pmatrix}$$

Stress tensor



$$\text{The Stress Tensor, } \mathbf{T} = \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}$$

Figure 1.18 The definition of the state-of-stress tensor in terms of force components acting on the faces of a cube. (From Larson 1988, with permission.)

$$\mathbf{T} = \boldsymbol{\sigma} - p\boldsymbol{\delta} \qquad\qquad \boldsymbol{\sigma} = 2\eta\mathbf{D}$$