Lecture 18. MSMPR Crystallization Model

- MSMPR Crystallization Model
- Crystal-Population Balance
	- Number of crystals
	- Cumulative number of crystals
	- Predominant crystal size
	- Growth rate

MSMPR Crystallization Model

- Mixed-suspension, mixed-product-removal (MSMPR) model is useful for the design and analysis of draft-tube, baffled crystallizer
- Assumptions
	- (1) Continuous, steay-flow, steay-state operation
	- (2) Perfect mixing of the magma
	- (3) No classification of crystals
	- (4) Uniform degree of supersaturation for the magma
	- (5) Crystal growth rate independent of crystal size
	- (6) No crystals in the feed, but seeds are added initially
	- (7) No crystal breakage
	- (8) Uniform temperature
	- (9) Mother liquor in product magma in equilibrium with the crystals
	- (10) Nucleation rate is constant, uniform, and due to secondary nucleation by crystal contact
	- (11) Crystal-size distribution is uniform in the crystallizer and equal to that in the magma
	- (12) All crystals have the same shape

Crystal-Population Balance (1)

- The crystal-size distribution can be estimated as a function of the rpm of the draft-tube propeller and external circulation rate by a crystal-population balance in the MSMPR model **Constrained Allen Constrained Constrained Constrained Cons** e crystal-size distribution can be estimated a

m of the draft-tube propeller and external circ

stal-population balance in the MSMPR model

e number of crystals per unit size per unit volume
 $\boldsymbol{n} = \frac{d(N/V_{ML})}{dL} = \frac{1}{V_{$ **Population Balance (1**

listribution can be estimated as a function

ube propeller and external circulation rate is

v dalance in the MSMPR model

v stals per unit size per unit volume
 $\frac{1}{V_{ML}} \frac{dN}{dL}$ $\frac{L \cdot \text{characteristic crystal$
- The number of crystals per unit size per unit volume

$$
n = \frac{d(N/V_{ML})}{dL} = \frac{1}{V_{ML}} \frac{dN}{dL}
$$

L : characteristic crystal size (e.g. from a screen analysis) $\frac{dN}{N}$: cumulative number of crystals of size L and smaller in the magma in the crystallizer

 dL *V_{ML}* dL *V_{ML}* : volume of the mother liquor in the crystallizer magma

• For a constant, crystal-size growth rate independent of crystal size

$$
G=dL/dt
$$

$$
\mathbf{M} \qquad \mathbf{\Delta} \mathsf{L} \text{ law of McCabe} \qquad \frac{1}{2} \qquad \frac{1}{2}
$$

 t_L : residence time in the magma in the $\overline{}$ crystallizer for crystals of size L

Crystal-Population Balance (2)

• Number of crystals in the size range dL

**Crystal — Population Balance
** *M* e^{t} *dN* = $nV_{ML}dL$
 $dN = nV_{ML}dL$
 dN the perfect-mixing assumption for the magma,
 $\frac{number_liquer volume with drawn}{m}$ • From the perfect-mixing assumption for the magma, **number of crystals in the size range dl.**

N = $nV_{ML}dL$

the perfect-mixing assumption for the magma,

number of crystals withdrawn

number of crystals in crystallizer

number of crystals in crystallizer

number of crys **Crystal — Population Balance (2)**

ther of crystals in the size range dL
 $dN = nV_{mt}dL$

in the perfect—mixing assumption for the magma,

number of crystals withdrawn

mother - liquor volume in crystallizer

mother - liqu **number of crystals in the size range dL**
 $N = nV_{ML}dL$

the perfect-mixing assumption for the magma,

number of crystals withdrawn

number of crystals in crystallizer

other - liquor volume in crystallizer

number of crys **Crystal — Population Balance**

ther of crystals in the size range dL
 $dN = nV_{ML}dL$

m the perfect-mixing assumption for the magma,

number of crystals withdrawn

mother - liquor volume withdrawn

mother - liquor volume i **number of crystals in the size range dL**
 $N = nV_{ML}dL$

the perfect-mixing assumption for the magma,
 number of crystals withdrawn
 number of crystals in crystallizer
 number of crystals withdrawn
 number of crysta hber of crystals in the size range dL
 $dN = nV_{ML}dL$

m the perfect-mixing assumption for the magma
 number of crystals withdrawn
 mother - liquor volume withdrawn
 number of crystals in crystallizer
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number of crystals in crystallizer

number of crystals in crystallizer

number of crystals withdrawn

number of crystals in crystallizer

number of crystals in crystallizer

nother - liquor v **umber of crystals withdrawn**
 ther - liquor volume withdrawn
 umber of crystals in crystallizer
 ther - liquor volume in crystallizer
 mber of crystals withdrawn
 nder - liquor volume withdrawn
 ther - liquor

number of crystals in crystallizer
mother - liquor volume in crystallizer

mother-liquor volume withdrawn
mother-liquor volume in crystallizer

number of crystals withdrawn	
mother - liquidor volume withdrawn	
number of crystals in crystallizer	
mother - liquidor volume in crystallizer	
number of crystals withdrawn	
number of crystals in crystallizer	
mother - liquidor volume withdrawn	
mother - liquidor volume in crystallizer	
$-\frac{\Delta ndL}{ndL} = -\frac{\Delta n}{n} = \frac{Q_{ML}\Delta t}{V_{ML}}$	Q_{ML} : volumetic flow rate of mother liquidor in the withdrawal product magna

in the withdrawn product magma

Crystal-Population Balance (3)

Crystal–Population Balance (3)
\n
$$
\frac{\Delta L = G \Delta t}{-\frac{\Delta n}{n} = \frac{Q_{ML} \Delta t}{V_{ML}}} \Bigg\} - \frac{\Delta n}{\Delta L} = \frac{Q_{ML} n}{G V_{ML}} \xrightarrow{\text{Inkining} \atop \text{the limit}} -\frac{dn}{dL} = \frac{Q_{ML} n}{G V_{ML}}
$$
\n
$$
\text{Petermination time of mother liquid or in the crystallizer, } \tau = V_{ML}/Q_{ML}
$$
\n
$$
-\frac{dn}{n} = \frac{dL}{G\tau} \qquad \text{In=} \quad n^{\circ} \exp(-L/G\tau)
$$
\n
$$
\text{Number of crystals per unit volume of mother liquid or below size L}
$$
\n
$$
\frac{N}{V} = \int_0^L ndL
$$

$$
-\frac{dn}{n} = \frac{dL}{G\tau}
$$

• Number of crystals per unit volume of mother liquor below size L

$$
\frac{N}{V_{ML}} = \int_0^L ndL
$$

• Number of crystals per unit volume of mother liquor

$$
\frac{N_T}{V_{ML}} = \int_0^\infty n dL
$$

Crystal-Population Balance (4)

• Cumulative number of crystals of size smaller than L, as a function of the total

Crystal–Population Balance (4)
\nmultative number of crystals of size smaller than L, as a
\nction of the total
\n
$$
x_n = \frac{\int_0^L n^o e^{-L/G\tau} dL}{\int_0^\infty n^o e^{-L/G\tau} dL} = 1 - \exp(-L/G\tau) = 1 - e^{-z} \frac{z(-L/G\tau) \cdot \text{dimensionless
\ncrystal size
\nCumulative distribution
\nCountative crystal population) \nDifferential distribution
$$

Crystal-Population Balance (5) **Crystal – Population Balance (5)**

• Moment equation for a relation $n = f(z)$
 $x_k = \int_0^z n z^k dz$
 $\int_0^\infty n z^k dz$ **Also of the moment and** \mathbf{a} **d n**
*nz^kdz k***: order of the moment
Distribution basis | Cumulative | Difference | Distribution basis | Cumulative | Diference | Diference | Diference | Diference | Diference | Diference | Diferenc**

$$
x_k = \frac{\int_0^z nz^k dz}{\int_0^{\infty} nz^k dz}
$$
 k: order of the moment

• Predominant crystal size, L_{pd} in terms of the mass distribution: corresponding to the peak of the differential-mass distribution

\n
$$
x_{a} = 1 - (1 + z + \frac{z^{2}}{2})e^{-z}
$$
\n
$$
dx_{a}/dz = \frac{z^{2}}{2}e^{-z}
$$
\n

\n\n
$$
x_{m} = 1 - (1 + z + \frac{z^{2}}{2} + \frac{z^{3}}{6})e^{-z}
$$
\n
$$
dx_{m}/dz = \frac{z^{3}}{6}e^{-z}
$$
\n

\n\n domain at crystal size, L_{pd} in terms of the mass distribution: $d\left(\frac{dx_{m}}{dz}\right) = 0 = \frac{3z^{2}e^{-z}}{6} - \frac{z^{3}e^{-z}}{6}$ \n $z = 3 = \frac{L}{G\tau}$ \n $\therefore L_{pd} = 3G\tau$ \n

Crystal-Population Balance (6)

• Growth rate (G) depends on the supersaturation and degree of agitation, and residence time (τ) depends on crystallizer design and operation **Crystal — Population Balancy**
th rate (G) depends on the supersaturation and
tion, and residence time (τ) depends on crystalli
pperation
 $\frac{1}{\sqrt{n}u} \frac{dN}{dt} = \frac{1}{V_{ML}} \frac{dN}{dL} \left(\frac{dL}{dt}\right)$
 $\lim \frac{1}{dN} \frac{dN}{dt} = B^{\circ}$ th rate (G) depends on the superation, and residence time (τ) depends
operation
 $\frac{1}{M} \frac{dN}{dt} = \frac{1}{V_{ML}} \frac{dN}{dL} \left(\frac{dL}{dt}\right)$
 $\lim \frac{1}{M} \frac{dN}{dt} = B^{\circ}$ **ystal - Population Balance (6)**

rate (G) depends on the supersaturation and degree

n, and residence time (τ) depends on crystallizer des

ration
 $\frac{dN}{dt} = \frac{1}{V_{ML}} \frac{dN}{dL} \left(\frac{dL}{dt}\right)$
 $\frac{1}{dN} \frac{dN}{dt} = B^{\circ}$ **Crystal - Population Balance (6)**
wth rate (G) depends on the supersaturation and degree
ation, and residence time (τ) depends on crystallizer desi
operation
 $\frac{1}{V_{ML}} \frac{dN}{dt} = \frac{1}{V_{ML}} \frac{dN}{dL} \left(\frac{dL}{dt}\right)$
 $\lim_{L \to$ **all — Population Balance (6)**

(G) depends on the supersaturation and degree of

nd residence time (τ) depends on crystallizer design
 $=\frac{1}{V_{ML}}\frac{dN}{dL}\left(\frac{dL}{dt}\right)$
 $\frac{dN}{dt}=B^o$

$$
\frac{1}{V_{ML}}\frac{dN}{dt} = \frac{1}{V_{ML}}\frac{dN}{dL}\left(\frac{dL}{dt}\right)
$$

Crystal–Population Balance (6)
\nwith rate (G) depends on the supersaturation and degree of
\nation, and residence time (τ) depends on crystallizer design
\noperation
\n
$$
\frac{1}{V_{ML}} \frac{dN}{dt} = \frac{1}{V_{ML}} \frac{dN}{dL} \left(\frac{dL}{dt}\right)
$$
\n
$$
\lim_{L \to 0} \frac{1}{V_{ML}} \frac{dN}{dt} = B^o
$$
\n
$$
\frac{dL}{dt} = G
$$
\n
$$
\lim_{L \to 0} \frac{1}{V_{ML}} \frac{dN}{dL} = n^o
$$
\n
$$
B^o = Gn^o
$$
\n
$$
n = n^o \exp(-L/G\tau) = \frac{B^o}{G} \exp(-L/G\tau)
$$
\nThis equation can be used to obtain
\nourleation and growth rates
\nper-law function for the effect of operating conditions on B°
\n
$$
B^o = k'_N G^i M'_T N'
$$

• Power-law function for the effect of operating conditions on B°

Crystal-Population Balance (7) **Crystal — Population Balar**
 *n*ber of crystals per unit volume of mother lique
 $n_c = N_T/V_{ML} = \int_0^\infty n dL = n^\circ \tau G \int_0^\infty e^{-z} dz = n^\circ \tau G$

as of crystals per unit volume of mother liquor
 $m_a = \int_0^\infty m_a n dL$ m_b : mass of a partic $Population Balance (7)$
als per unit volume of mother liquor
 $=\int_0^\infty n dL = n^\circ \tau G \int_0^\infty e^{-z} dz = n^\circ \tau G$
per unit volume of mother liquor
 $H L$
 m_e : mass of a particle, $m_e = f_e L^2 \rho_e$
 f_e : volume shape factor *^z n G e dz n G* **Dulation Balance (7)**

int volume of mother liquor
 $=n^{\circ}\tau G\int_{0}^{\infty}e^{-z}dz=n^{\circ}\tau G$

volume of mother liquor

mass of a particle, $m_{\rho}=f_{\nu}\mathcal{L}\rho_{\rho}$ (f_{ν} : volume shape factor

• Number of crystals per unit volume of mother liquor

$$
n_c = N_T/V_{ML} = \int_0^\infty n dL = n^{\circ} \tau G \int_0^\infty e^{-z} dz = n^{\circ} \tau G
$$

• Mass of crystals per unit volume of mother liquor

Crystal—Population Balance (7)
\n
$$
n_{c} = N_{T}/V_{ML} = \int_{0}^{\infty} n dL = n^{\circ} \tau G \int_{0}^{\infty} e^{-z} dz = n^{\circ} \tau G
$$
\nso of crystals per unit volume of mother liquidor
\n
$$
m_{c} = \int_{0}^{\infty} m_{p} n dL
$$
\n
$$
m_{p} : \text{mass of a particle, } m_{p} = f_{r} L^{2} \rho_{p}
$$
\n
$$
f_{r} : \text{volume shape factor}
$$
\n
$$
m_{c} = 6 f_{v} \rho_{p} n^{\circ} (G \tau)^{4}
$$
\n
$$
m_{c} = 6 f_{v} \rho_{p} n^{\circ} (G \tau)^{4}
$$
\n
$$
m_{c} = \frac{1}{6 f_{v} \rho_{p} (G \tau)^{3}} = \frac{9}{2 f_{v} \rho_{p} L^{3}_{pd}}
$$
\n
$$
L_{pd} = 3 G \tau
$$
\n
$$
L_{p} = 3 G \tau
$$
\n
$$
B^{\circ} = \frac{n_{c} C}{m_{c} V_{ML}} = \frac{9 C}{2 f_{v} \rho_{p} V_{ML} L^{3}_{pd}}
$$
\n
$$
C : \text{mass rate of production of crystals}
$$

• Number of crystals per unit mass of crystals

$$
m_c = \int_0^{\infty} m_p r \, dx
$$

\n
$$
m_c = 6 f_v \rho_p n^{\circ} (G\tau)^4
$$

\n
$$
\frac{n_c}{m_c} = \frac{1}{6 f_v \rho_p (G\tau)^3} = \frac{9}{2 f_v \rho_p L_{pd}^3}
$$

\n
$$
L_{pd} = 3 G\tau
$$

\n
$$
B^{\circ} = \frac{n_c C}{m_c V_{ML}} = \frac{9 C}{2 f_v \rho_p V_{ML} L_{pd}^3}
$$

\n
$$
C: \text{ mass rate of product}
$$

• Nucleation rate

$$
B^{\text{o}} = \frac{n_c C}{m_c V_{ML}} = \frac{9C}{2 f_v \rho_p V_{ML} L_{pd}^3}
$$
 C: mass rate of production of crystals