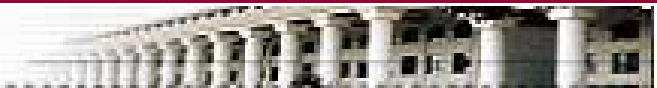
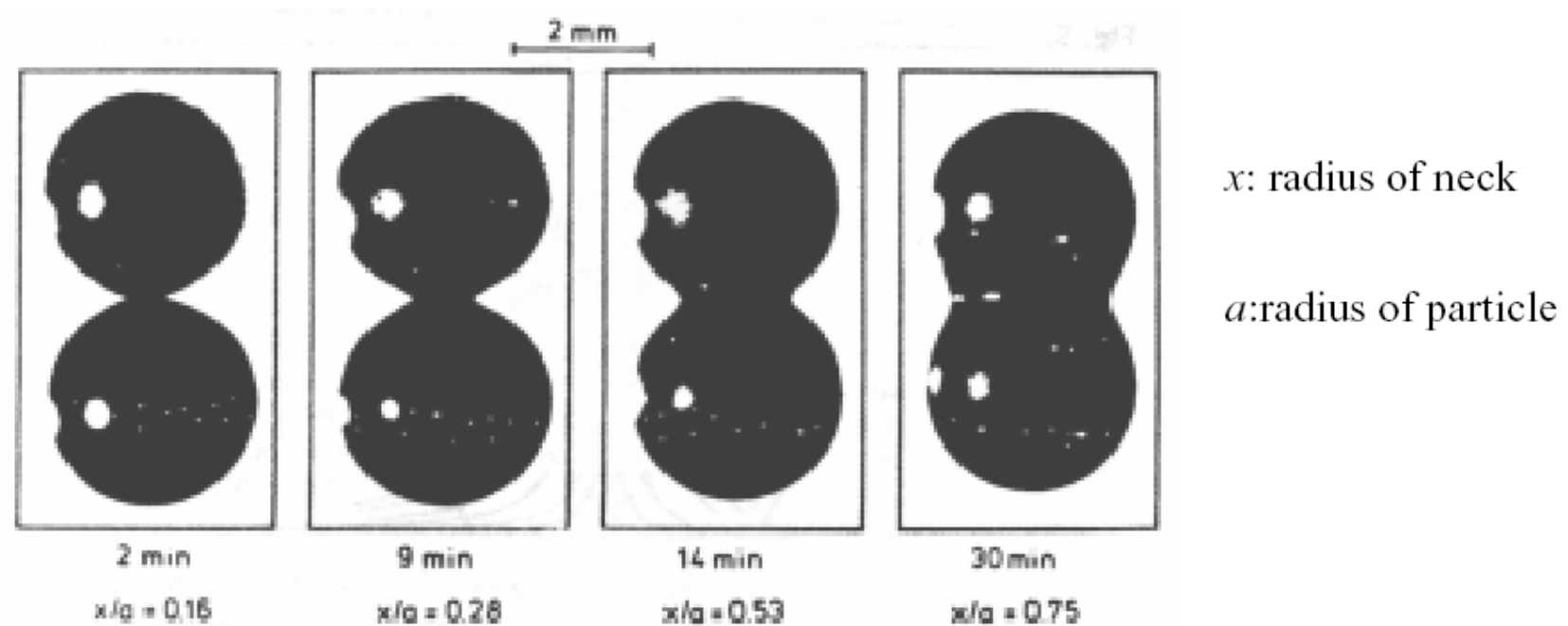
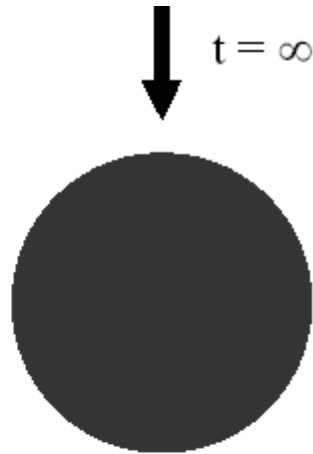


## IX. Sintering

### - General Phenomena

Sintering is a process of densification driven by interfacial energy. Sintering is a thermodynamically spontaneous process.





$\gamma_{SV}$  : surface tension of  
solid-vapor interface

At  $t=0$ , we have two balls with  $r_i = a_0$  and the surface energy is

$$G_0 = 2\gamma_{SV}(4\pi a_0^2) = 8\pi a_0^2 \gamma_{SV}$$

At  $t=\infty$ , we have one ball with  $r_f = 2^{1/3} a_0$ , and

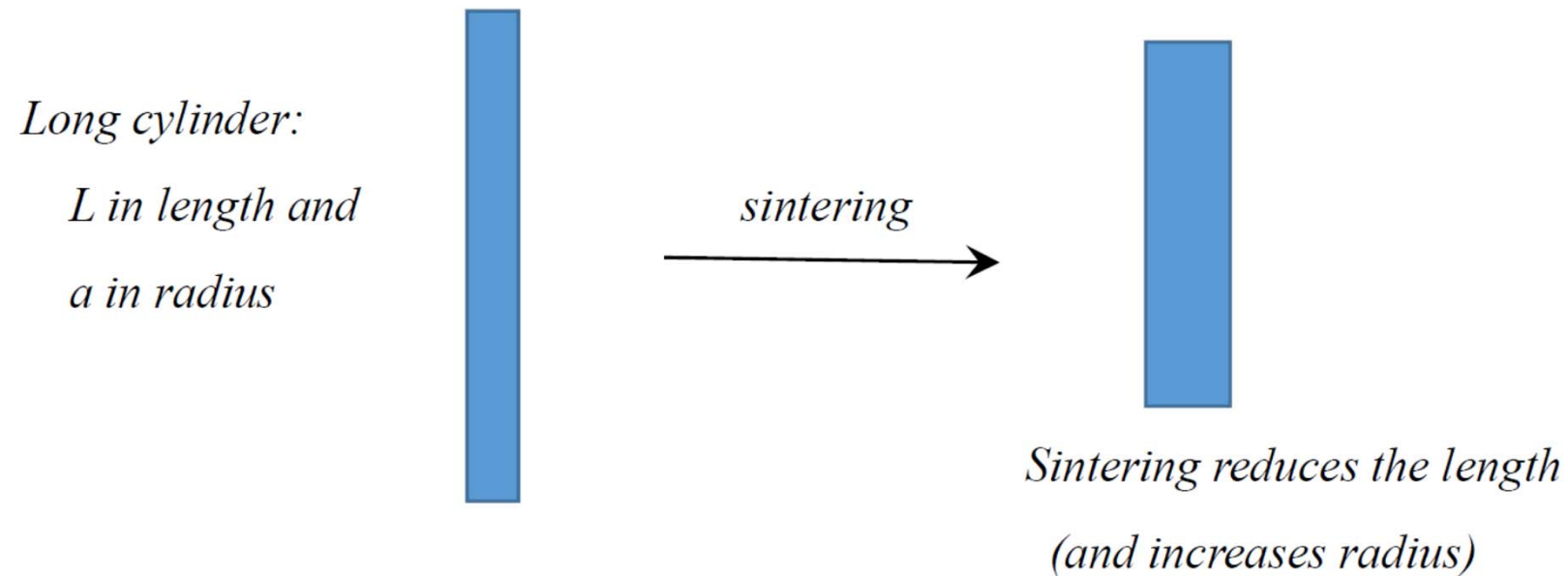
$$G_f = \gamma_{SV}(4\pi r_f^2) = 4 \times 2^{2/3} \pi a_0^2 \gamma_{SV}$$

$$\Rightarrow \Delta G = G_f - G_0 = 4(2^{2/3} - 2)\pi a_0^2 \gamma_{SV} < 0$$



## - Viscous Sintering (Noncrystalline Materials)

### Energy Balance and Sintering Rate



The rate of dissipation of energy during the movement of material is:

$$E_f = (3\mu\varepsilon_z^2)(\pi a^2 L)$$

where  $\mu$ : viscosity of solid

$\varepsilon_z$ : strain rate in L direction

$$\varepsilon_z = \left(\frac{1}{L}\right)\left(\frac{dL}{dt}\right)$$

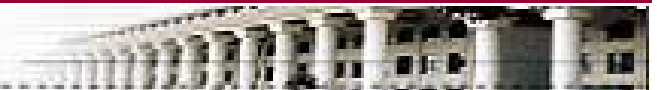
The rate of the energy change due to the reduction in surface area is:

$$E_s = \gamma_{sv} \left(\frac{dS}{dt}\right)$$

Neglecting the end surfaces, we have  $S = 2\pi aL$

$$\frac{dS}{dt} = 2\pi aL \left[ \left(\frac{1}{a}\right)\left(\frac{da}{dt}\right) + \left(\frac{1}{L}\right)\left(\frac{dL}{dt}\right) \right] = 2\pi aL[\varepsilon_r + \varepsilon_z] = \pi aL\varepsilon_z$$

(Note: for incompressible cylinder,  $\varepsilon_r = -\varepsilon_z/2$ )



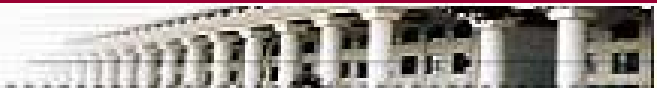
Energy balance gives

$$E_f + E_s = 0$$

$$\varepsilon_z = -\gamma_{sv} / 3a\mu$$

Sintering rate  $\uparrow$  as

particle or pore size  $\downarrow$   
surface energy  $\uparrow$   
viscosity  $\downarrow$



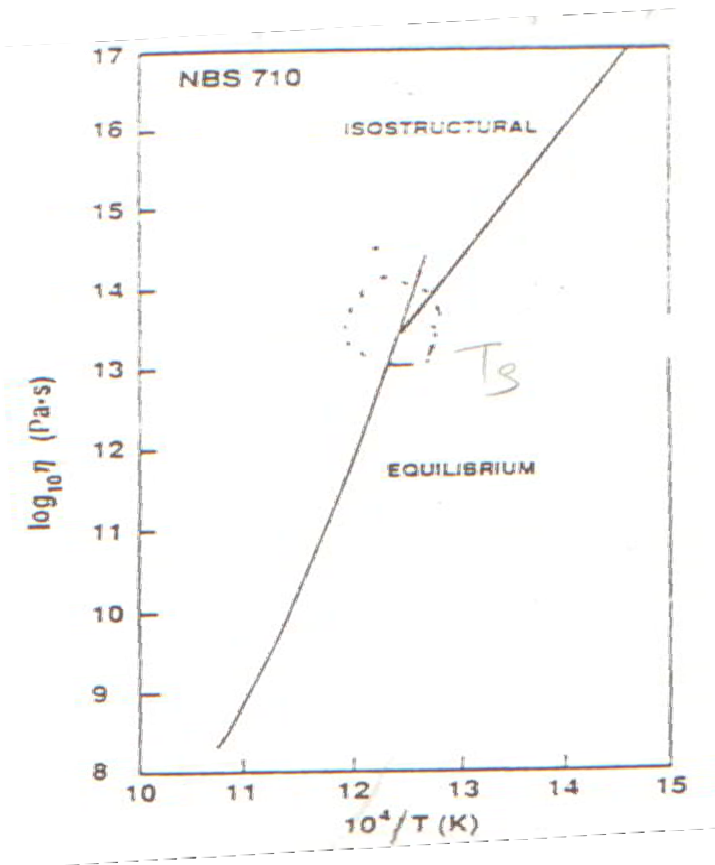
## Temperature Dependence of Viscosity and Surface Tension

Viscosity decreases by an order of magnitude for every 20-40°C increase in temperature.

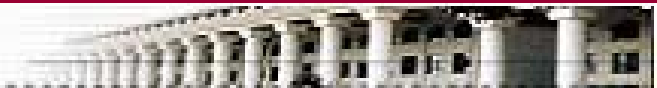
$$\mu = \mu_0 \exp[A/(T - T_0)]$$

Or over a narrow range of temperature

$$\mu = \mu_0 \exp(-E/RT)$$



Surface energy of glass is weakly dependent on temperature.



**- Cylindrical Model for Microstructure Gel**

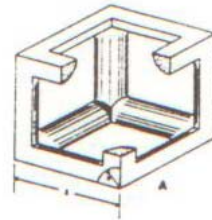
$\rho$ : density of porous gel

$\rho_0$ : green density of gel (sub "0" is for gel before sintering)

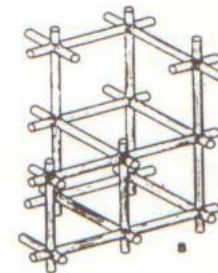
$\rho_s$ : theoretical density of gel after complete densification

$\rho / \rho_s$ : relative density

Unit cell of gel



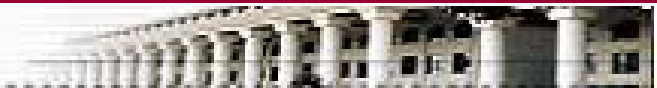
Relative density=0.1



Relative density=0.5



a - radius of cylinder  
 l - cell length  
 $x = a/l$

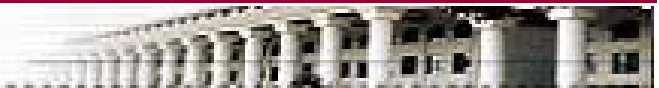
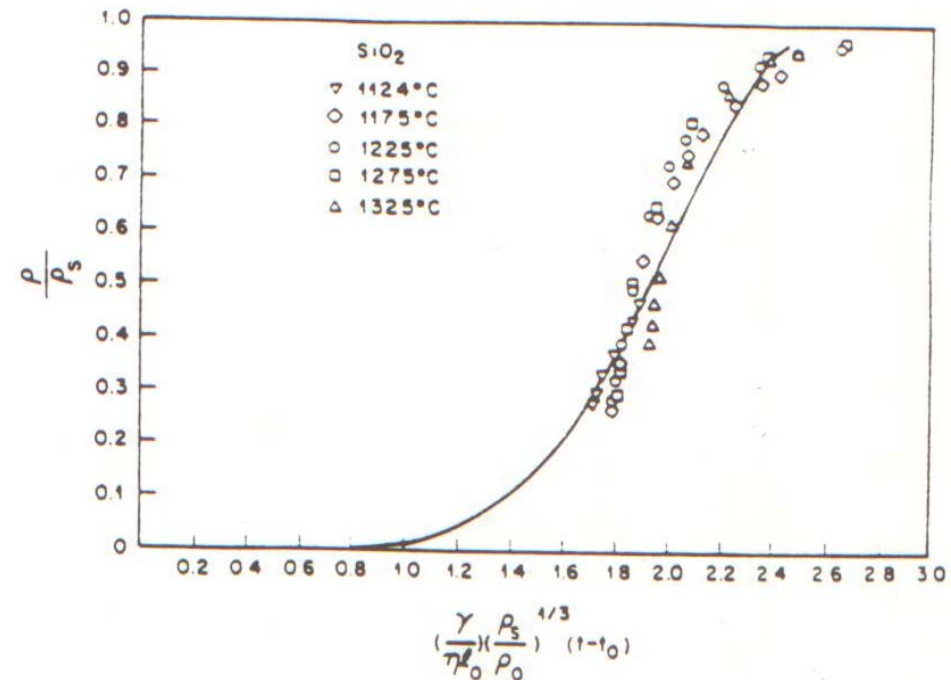


For the cylindrical model, the relative density can be correlated to the size of cylinder by:

$$\rho / \rho_s = x^2 (3\pi - 8\sqrt{2x})$$

and  $x$  is a function of sintering time as:

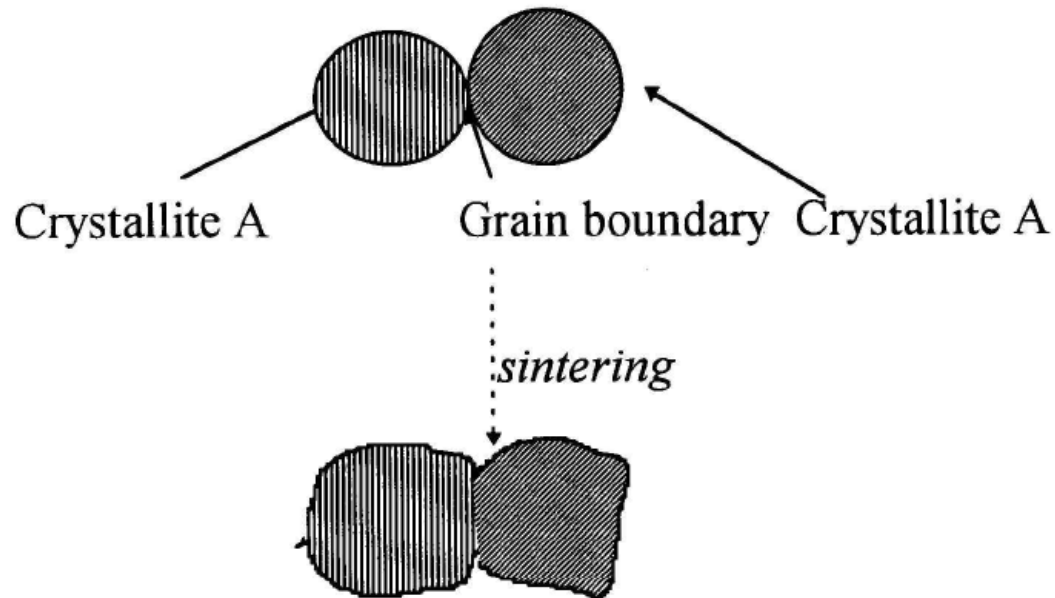
$$\int_{x_0}^x \frac{2dx}{(3\pi - 8\sqrt{2x})^{1/3} x^{2/3}} = \frac{\gamma_{sv} \rho_s^{1/3} t}{\mu l_0 \rho_0^{1/3}}$$



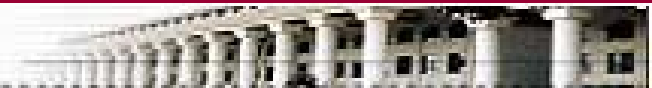


# SINTERING OF CRYSTALLINE MATERIALS

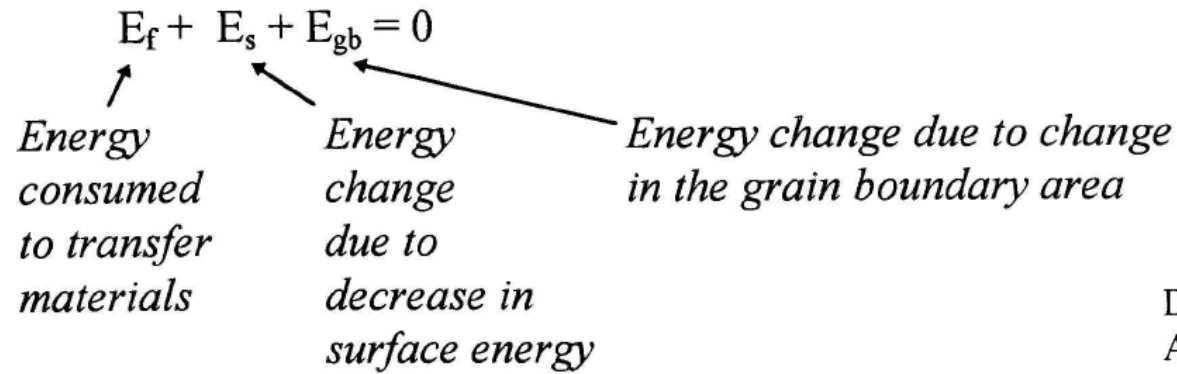
## - General Phenomena



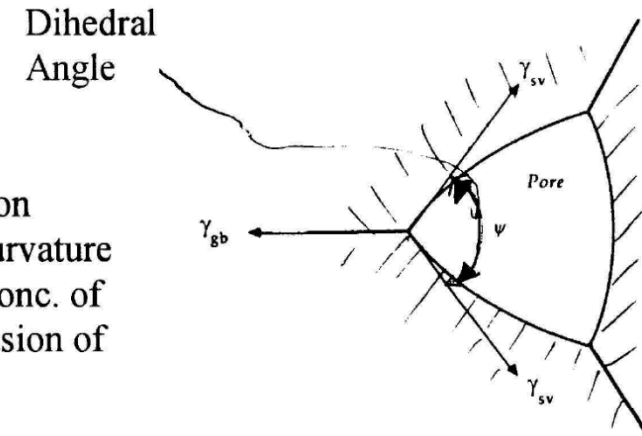
Driving force is reduction of total surface energy; There is an additional interfacial energy  $\gamma_{gb}$  at the grain boundary; During sintering grains grow, surface/volume ratio decreases; interfacial energy decreases.



- Energy Balance:

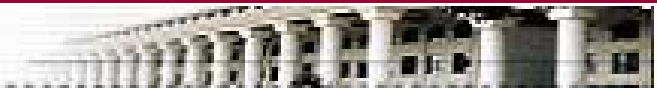
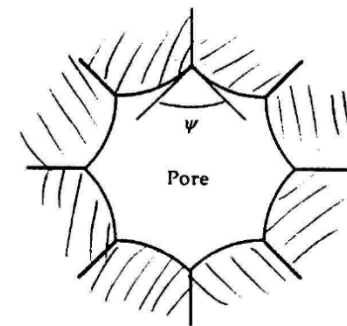


- . 3 grains => pore formation
- . negative V/S interface curvature
- . tension creating higher conc. of vacancies, favoring diffusion of solid => pore shrinks



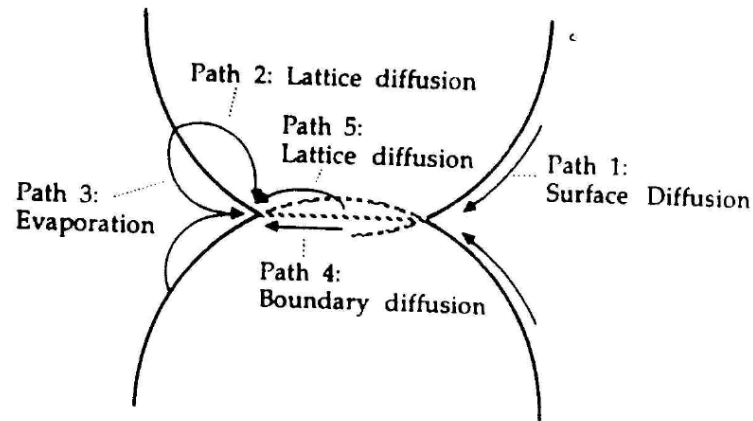
$$\gamma_{gb} = 2 \gamma_{sv} \cos (\psi / 2)$$

- . more than 3 grains
- . positive V/S interface curvature
- . diffusion of solid from pores
- . pore grows



**- Sintering Paths**

Crystallite A



Crystallite B

Five Paths for Solid Transport

Path 1	Surface diffusion, $D_s$	} Coarsen the body by making the necks, pores may grow
Path 2	Lattice diffusion, $D_{L(I)}$	
Path 3	Evaporation	
Path 4	Boundary diffusion, $D_{gb}$	} Grain boundary changes, decrease in distance between particles, pores shrink
Path 5	Lattice diffusion II, $D_{L(II)}$ (volume diffusion)	



