3. Two- and three-dimensional systems

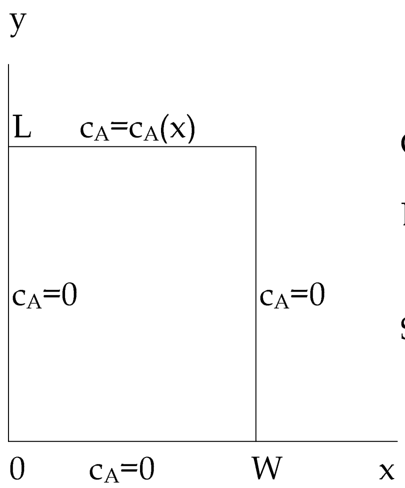
Systems involving irregular boundaries (2 or 3 dimensional)

Concentration profiles for 2 or 3 spatial coordinates

(1) Analytical solution

2-dimensional system & no net bulk motion within the system

$N_A = -N_B$



GE: $\frac{\partial^2 c_A}{\partial x^2} + \frac{\partial^2 c_A}{\partial y^2} = 0$: Laplace equation

BC.'s: $c_A = 0$ at x=0, x=W & y=0

$c_A = c(x)$ at y=L

Solution by separation of variables

$$c_A = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{W} \sinh\frac{n\pi y}{L}$$

$$\vec{J_A} = -D_{AB}\nabla c_A$$

(2) Graphical solution by flux plotting

Potential field plot of mass-flow and constant-concentration lines

Direction of mass flow

Mass flow per tube : $N_{A,x}\Delta x = D_{AB}\Delta c_A$

(3) Analogical solution

Similar to electrical potential distribution

Analog field plotter

(4) Numerical solution

Finite differences of partial differentials

$$\frac{\partial^2 c_A}{\partial x^2} = \frac{c_{A,i+1,j} - 2c_{A,i,j} + c_{A.i-1,j}}{(\Delta x)^2}, \quad \frac{\partial^2 c_A}{\partial y^2} = \frac{c_{A,i,j+1} - 2c_{A,i,j} + c_{A.i,j-1}}{(\Delta y)^2}$$

$$\frac{c_{A,i+1,j} - 2c_{A,i,j} + c_{A.i-1,j}}{(\Delta x)^2} + \frac{c_{A,i,j+1} - 2c_{A,i,j} + c_{A.i,j-1}}{(\Delta y)^2} = 0$$

For $\Delta x = \Delta y$, $c_{A,i+1,j} + c_{A.i-1,j} + c_{A,i,j+1} + c_{A.i,j-1} - 4c_{A,i,j} = 0$

4. Simultaneous momentum, heat, and mass transfer

simultaneous transfer of mass, energy and momentum

drying of a wet surface, by a hot, dry gas

(1) Simultaneous heat and mass transfer

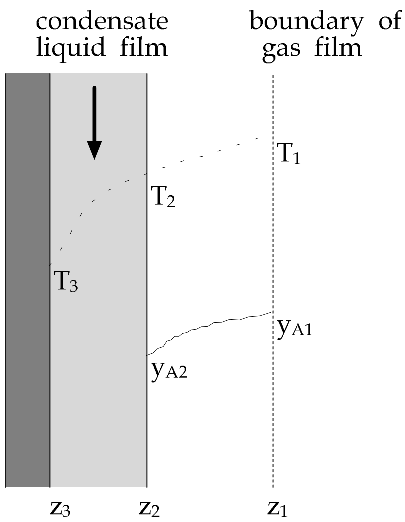
Heat flux by energy transfer: $\frac{q_D}{A} = \sum_{i=1}^{n} N_i \overline{H}_i$ - the heat flux due to the diffusion of mass,

where $\overline{H}_i$ is the partial molar enthalpy of species i.

· total energy transport by conduction and diffusion: $\frac{q}{A} = -k\nabla T + \sum_{i=1}^{n} N_i \overline{H}_i$

· total energy transport by convection and diffusion: $\frac{q}{A} = h\Delta T + \sum_{i=1}^{n} N_i \overline{H}_i$

Example: condensation of moist vapor on a cold window plane



· Heat transfer by natural convection

$$Nu_L = 0.68 + \frac{0.670 Ra_L^{1/4}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{4/9}}$$

· Mass transfer in the gas phase : Fick's law

$$N_{A,z} = \frac{-cD_{AB}}{1-y_A}\frac{dy_A}{dz} = \frac{(cD_{AB})_{avg}(y_{A1}-y_{A2})}{(z_2-z_1)y_{B,lm}}$$

· Total energy flux through the liquid film

$$\frac{q_z}{A} = h_{liquid}(T_2 - T_3) = h_c(T_1 - T_2) + N_{A,z}M_A(H_1 - H_2)$$

· Solution by trial and error method

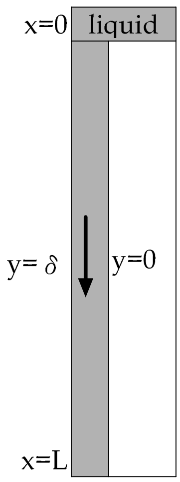
Guess the temperature of the liquid surface,

→ T_2, h_c and $(cD_{AB})_{avg}$ → y_{A2} from $y_{A2} = x_A P_A / P$

→ $N_{A,z}$ → h_{liquid} from Eq. 21-20 → Check Eq. 26-70

(2) Simultaneous momentum and mass transfer

absorption: wetted wall column: width W, $0 \le y \langle = \delta$, & $0 \le x \le L$



· Falling film velocity profile in fully developed region: $v_x = 2v_{\max}\left(\frac{y}{\delta} - \frac{1}{2}(\frac{y}{\delta})^2\right)$

· Molar flux: $N_{A,x} = x_A(N_{A,x} + N_{B,x}) = c_A v_x$ & $N_{A,y} = -D_{AB}\frac{\partial c_A}{\partial y}$

G.E.: $v_x \frac{\partial c_A}{\partial x} - D_{AB}\frac{\partial^2 c_A}{\partial y^2} = 0$ $\quad\because\quad \frac{\partial N_{A,x}}{\partial x} + \frac{\partial N_{A,y}}{\partial y} = 0$

B.C.'s: $c_A = 0$ at $x = 0$, $\frac{\partial c_A}{\partial y} = 0$ at $y = 0$, & $c_A = c_{A0}$ at $y = \delta$

Solution by Johnstone and Pigford (1942),

$$\frac{c_A|_{x=L} - c_A|_{y=\delta}}{c_A|_{x=0} - c_A|_{y=\delta}} = 0.7857e^{-5.1213n} + 0.01e^{-39.318n} + 0.035e^{-105.64n} + \cdots$$

where $n = D_{AB}L/\delta^2 v_{\max}$

Example: Penetration model

"solute A penetrates only a short distance, a slow rate of diffusion or short time"

G.E. : $v_{max}\dfrac{\partial c_A}{\partial x}=D_{AB}\dfrac{\partial^2 c_A}{\partial y^2}$;

B.C.'s : $c_A=0$ at $x=0$, $c_A=c_{A0}$ at $y=\delta$, & $c_A=0$ at $y=-\infty$

Solution by using Laplace transforms:

$$c_A(x,\xi)=c_{A0}\left(1-erf\left(\frac{\xi}{\frac{4D_{AB}x}{v_{max}}}\right)\right)=c_{A0}\left(1-erf\left(\frac{\xi}{4D_{AB}t_{exp}}\right)\right)$$

where $t_{exp}(=x/v_{max})$ is the time of explosure.

Unidirectional mass flux: $N_{A,y}\big|_{y=\delta}=c_{A0}\sqrt{\dfrac{D_{AB}v_{max}}{\pi x}}$ or $c_{A0}\sqrt{\dfrac{D_{AB}}{\pi t_{exp}}}$

rapid chemical disappearance of the diffusing component,

mass flux $\propto\sqrt{D_{AB}}$ ~ penetration model : unsteady-state model

cf.) mass flux $\propto D_{AB}$ ~ film model : steady-state model