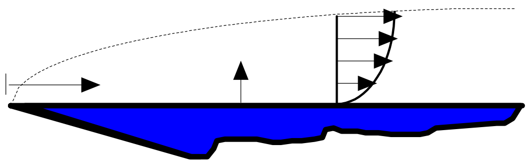
4. Exact analysis of the laminar concentration boundary layer

Blasius flow : laminar flow parallel to a flat plate



(1) Boundary layer equations in steady-state, two-dimensional flow

· continuity equation for incompressible fluids : $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$

· equation of motion in x-direction : $v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$

· mass transfer equation : $v_x \frac{\partial c_A}{\partial x} + v_y \frac{\partial c_A}{\partial y} = D_{AB} \frac{\partial^2 c_A}{\partial y^2}$

(2) Boundary conditions

$\frac{v_x}{v_\infty} = 0$ at y=0 $\qquad \frac{v_x}{v_\infty} = 1$ at y=∞

↓

$\frac{v_x - v_s}{v_\infty - v_s} = 0$ at y=0 $\qquad \frac{v_x - v_s}{v_\infty - v_s} = 1$ at y=∞

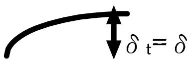
$\frac{C_A - C_{AS}}{C_\infty - C_s} = 0$ at y=0 $\qquad \frac{C_A - C_{AS}}{C_\infty - C_s} = 1$ at y=∞

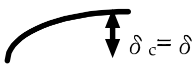
(3) Solution

· assumption: similarity between thermal and solutal transfers

Pr(= υ / α }=1, Sc(υ /D$_{AB}$)=1

운동량/열 운동량/물질





· dimensionless forms: $f = 2\frac{v_x - v_{x,s}}{v_\infty - v_{x,s}} = 2\frac{C_x - C_{A,s}}{C_{A,\infty} - C_{A,s}}$

· similarity variable $\mu = \frac{y}{2}\sqrt{\frac{v_\infty}{v_x}} = \frac{y}{2x}\sqrt{Re_x}$

$$\frac{df}{d\eta}\Big|_{\eta=0} = f'(0) = \frac{d[2(v_x/v_\infty)]}{d[(\eta/2x)\sqrt{Re_x}}\Big|_{\eta=0} = 1.328$$

$$= \frac{d[2(\frac{C_A - C_{AS}}{C_{A\infty} - C_S})}{d[(y/2x)\sqrt{Re_x}} = 1.328$$

· 경계면에서 물질수지 고려

$$N_{A,y} = -D_{AB}\frac{dC_A}{dy}\Big|_{y=0} = -D_{AB}\left[\frac{0.332\ Re_x^{\frac{1}{2}}}{x}\right](C_{A\infty} - C_{AS})$$

$$\frac{dC_A}{dy}\Big|_{y=0} = (C_{A\infty} - C_A)\left[\frac{0.332}{x}\ Re_x^{\frac{1}{2}}\right]$$

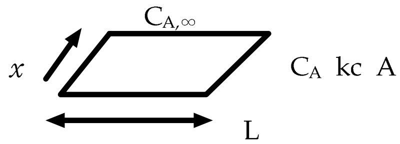
↓

· mass flux(확산} ⇒ 대류 $N_{A,y} = k_c(C_{AS} - C_{A\infty})$

$$\because \frac{k_c}{D_{AB}} = Sh_x = 0.332\ Re_x^{\frac{1}{2}}$$

$$\rightarrow \frac{\delta}{\delta_c} = Sc^{\frac{1}{3}} \qquad Sh_x = 0.332\ Re_x^{\frac{1}{2}} \cdot Sc^{\frac{1}{3}}$$

· total mass flux



$$W_A = \overline{k_c}A(C_{AS} - C_{A\infty})$$

$$= \int_A k_c(C_{AS} - C_{A\infty})dA$$

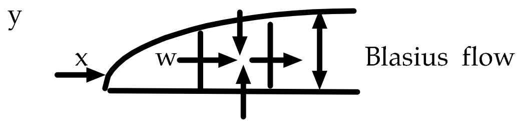
$$= (C_{AS} - C_{A\infty})\int_A \frac{0.332\ D_{AB}\ Re_x^{\frac{1}{2}}\ Sc^{\frac{1}{3}}}{x}dA$$

$$\frac{k_c}{D_{AB}} = Sh_{LAB} = 0.664\ Re_L^{\frac{1}{2}}\ Sc^{\frac{1}{3}}$$

↓sherwood 수 $Sh_r = 2\ Sh_x\Big|_L$

Ex. 1 (b) $W_{A2} = \int_0^{\delta_c} C_A\ v_x dy\Big|_{x+\Delta x}$

5. Approximate analysis



W_A : molar rate of mass transfer (A)

Mass Balance : $W_{A2} = W_{A1} + W_{A3} + W_{A4}$

$$W_{A1} = \int_0^{\delta_c} C_A\ v_x dy\Big|_x$$

$$W_{AC} = C_{A\infty}\left[\frac{\partial}{\partial x}\int_0^{\delta_c} v_x dy\right]\Delta x \qquad W_{A4} = k_c(C_{AS} - C_{A\infty})\Delta x$$

$$\frac{d}{dx}\int_0^{\delta_c}(C_A - C_{A\infty})v_x dy = k_c(C_A - C_{A\infty})$$

$$C_A = C_A(y), \qquad V_x = V_x(y)$$

$$\delta = \delta_c \quad \frac{v_x}{v_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3, \quad \frac{C_A - C_{A\infty}}{C_{A\infty} - C_{AS}} = \frac{3}{2}\left(\frac{y}{\delta_c}\right) - \frac{1}{2}\left(\frac{y}{\delta_c}\right)^3$$

$\frac{d}{dx}\int_0^{\delta_c}(C_A - C_{A\infty})v_x dy = k_c(C_A - C_{A\infty})$ 식에 대입

$$Sh_x = 0.36\ Re^{\frac{1}{2}}\ Sc^{\frac{1}{3}} \Rightarrow Sh_L = \frac{\overline{k_c}L}{D_{AB}} = 2\ Sh_x\ |_{x=L}\ Sh_t = 0.72\ Re_L^{\frac{1}{2}}\ Sc^{\frac{1}{3}}$$

6. Mass, energy and momentum transfer

(1) Reynolds Analogy

Momentum & Analogy

$$\frac{q_c}{v_\infty} = \frac{c_f}{2} \rightarrow \frac{h}{\rho\ v_\infty\ c_p} = c_f$$

$$\frac{k_c}{v_\infty} = \frac{h}{\rho\ v_\infty\ c_p}$$

(2) Turbulence Momentum

eddy에 의한 bulk mixing

$\tau = \rho[\upsilon + \varepsilon_M]\frac{d\overline{v}_x}{dy}$ (ε_M은 eddy momentum)

mass : $N_{A,y} = -(D_{AB} + \varepsilon_M)\frac{\overline{c_A}}{dy}$

thermal : $\frac{q}{A} = -\rho\ c_p(\alpha + \varepsilon_H)\frac{d\overline{T}}{dy}$

$c_A = c_A + c'_A$ (c'_A 는 infinitesimally small)

(3) Prandtl & von Karman Analogy

Prandtl analogy (convection)

- heat & momentum
- mass & heat

$$c_f = \frac{\tau_s}{\rho\left(\frac{v_\infty}{2}\right)}\ \frac{k_c}{v_\infty} = \frac{c_f}{2} = \frac{\tau_s}{\rho\ v_\infty^2} \Rightarrow N_{uL} = \frac{\frac{c_f}{2}\cdot Re\cdot Sc}{1+5\sqrt{\frac{c_f}{2}}(Sc-1)}$$

(4) Chilton - Colburn Analogy

(j - factor analogy) $j_D = \frac{k_c}{v_\infty}(Sc)^{2/3} = \frac{c_f}{2}$

Ex) Blasius flow

$$\frac{N_{ux,AB}}{Re_x\cdot Sc^{1/3}} = \frac{N_{ux,AB}}{Re_x\cdot sc}\ Sc^{2/3} = \frac{c_f}{2}$$

$$j_N = j_D = \frac{c_f}{2}\ \frac{h}{\rho\ v_\infty\ c_p}\ (\mathrm{Pr})^{2/3} = \frac{k}{v_D}\ (Sc)^{2/3}$$

7. Models

(1) Film model : steady-state model

$$k_c = \frac{D_{AB}}{\delta} \frac{P}{P_{B,lm}}$$

$$k_c^0 = \frac{D_{AB}}{\delta}$$

(2) Penetration model : unsteady-state model (disappearance of solute)

$$N_A = \sqrt{\frac{D_{AB}}{\pi \ t_{\exp}}} \ (c_{A,s} - c_{A,\infty}), \qquad k_c \propto \sqrt{D_{AB}}$$

(3) Surface-renewal model : time-period for fresh solvent surface

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