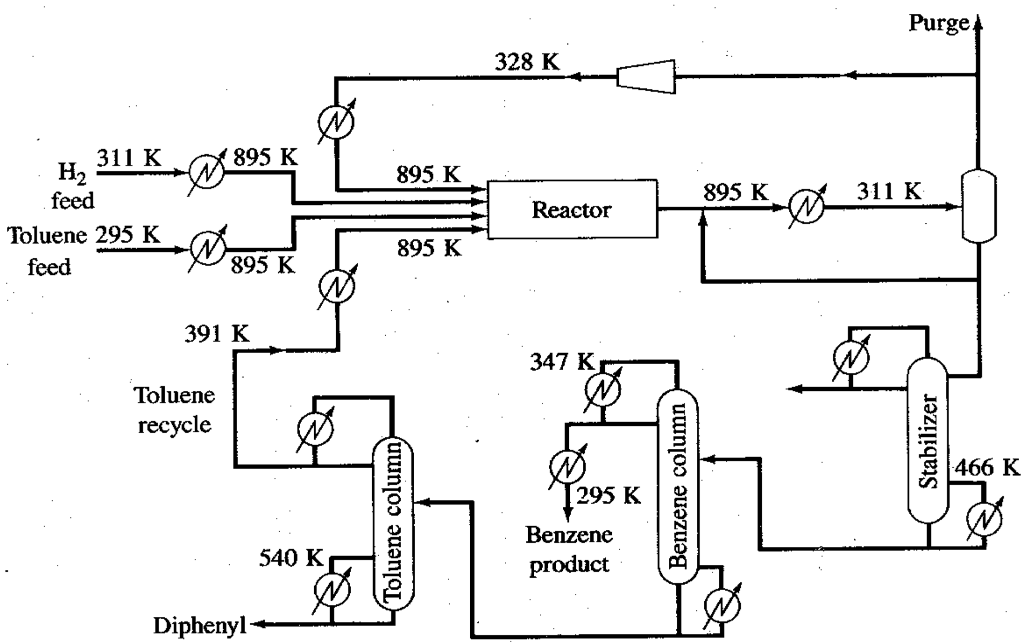
Chapter 4

Heat Exchanger Network Design

4.1 Basic Understanding of the HEN Problem

In a process flow diagram(PFD), a number of streams must be heated and other streams must be cooled. For example, consider a Toluene Hydrodealkylation Process.

$$\begin{aligned} \text{Toluene} + \text{H2} &\rightarrow \text{Benzene} + \text{CH4} \\ \text{2Benzene} &\leftrightarrow \text{Diphenyl} + \text{H2} \end{aligned}$$

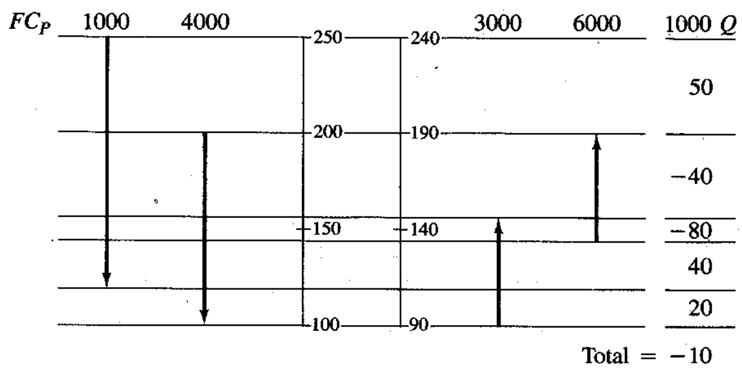


Assume that we have two hot streams (that should be cooled) and two cold streams (that should be heated). Under different optimization criteria, we will design HENs where the hot streams and cold streams exchanges necessary heat between them and also with hot and cold utilities. The stream information is as follows:

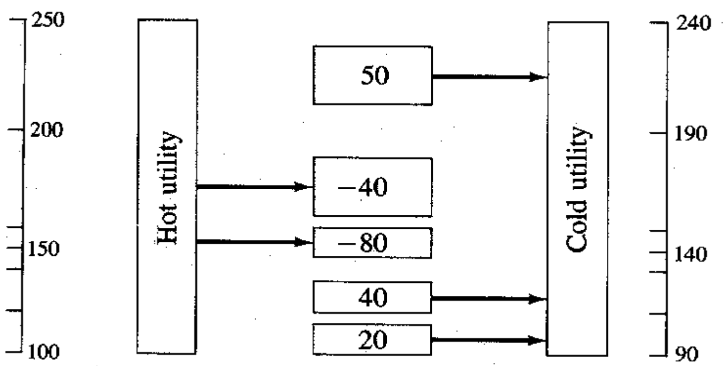
	Fc_p(Btu/min C)	T_{in}(C)	T_{out} (C)	Qavailable 10^3 (Btu/min)
H1	1000	250	120	130
H2	4000	200	100	400
C1	3000	90	150	-180
C2	6000	130	190	-360
Steam (Hot Utility)				
CW (Cold Utility)				

Minimum Utility Cost

Under the minimum temperature approach of 10^{o}C, the net heat exchange at each temperature interval can be depicted as

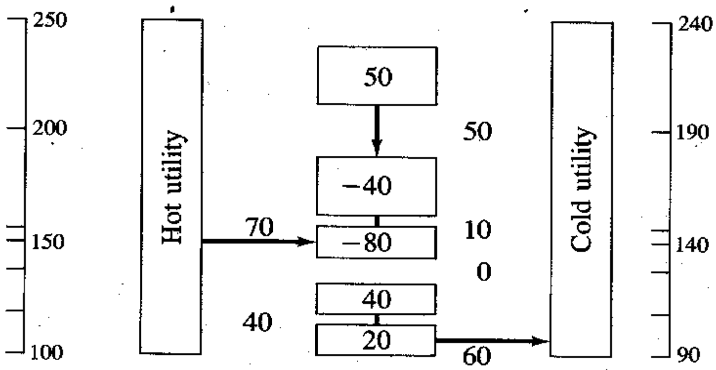


At an extreme, assume that the surplus/deficient heat after exchanging between the hot and cold streams at each temperature interval is dispensed/supplemented to/from the utilities. The situation can be expressed as



Total utility cost becomes $50 + 40 + 80 + 40 + 20 = 260 \times 10^3$ (Btu/hr).

From the fact that heat flows from high to low temperatures, the excess heat at high temperatures can be used to make up the heat demand at low temperatures.

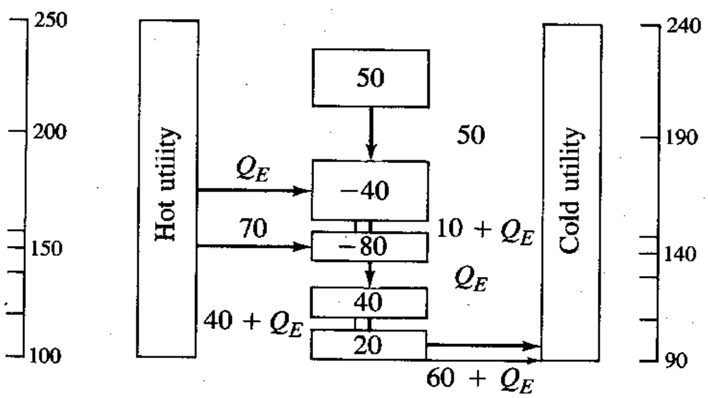


From this simple consideration on the first law of thermodynamics, we can see that the minimum heating requirement is 70×10^3 Btu/hr and the minimum cooling requirement is 60×10^3 Btu/hr.

We can see that there is a temperature across which no energy transfer is made. This temperature is called the *"pinch temperature"*.

- ◇ 140^{o}C for hot streams
- ◇ 130^{o}C for cold streams

What if additional heat is provided above the pinch ? As can be recognized from the figure, the heat supplied above the pinch is removed to the cold utility below the pinch. "Net Cost Increase !!"

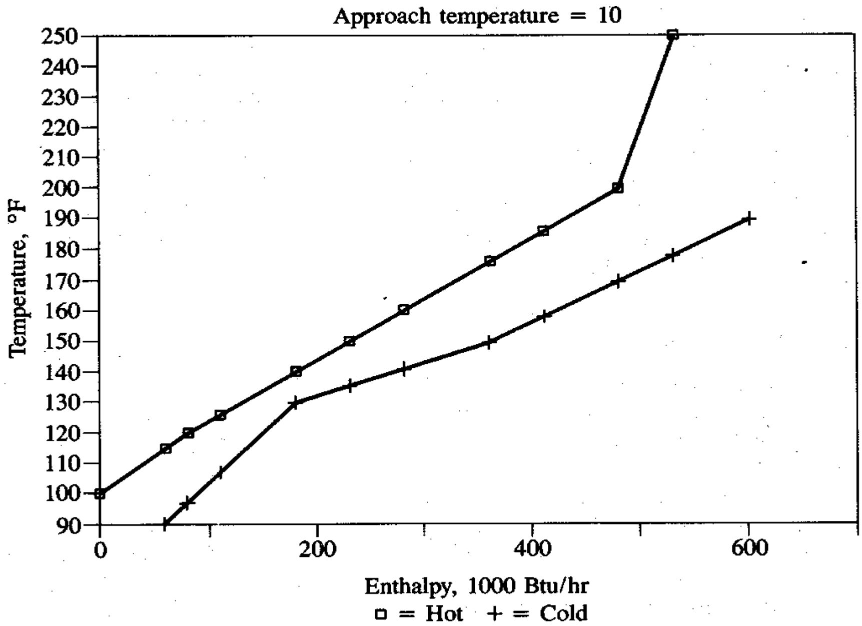


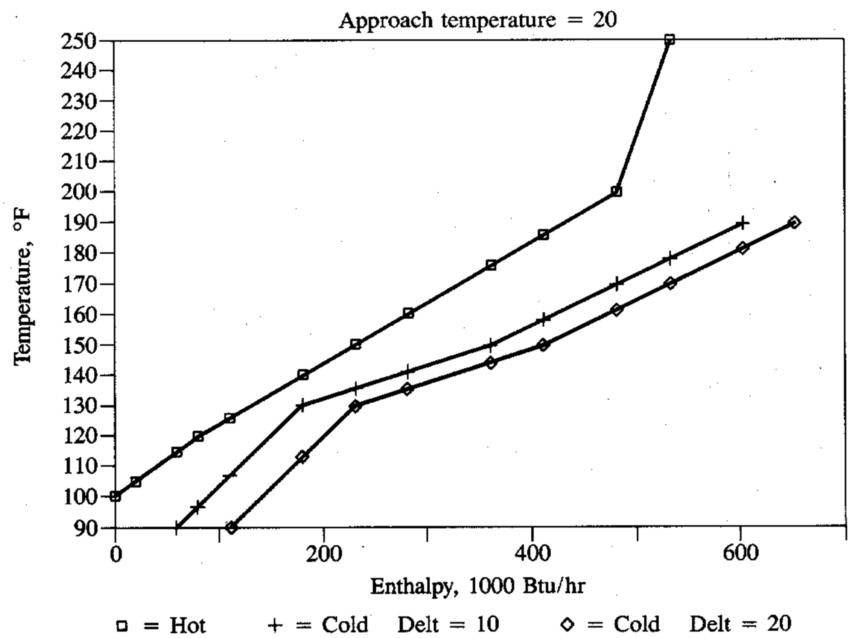
In conclusion,

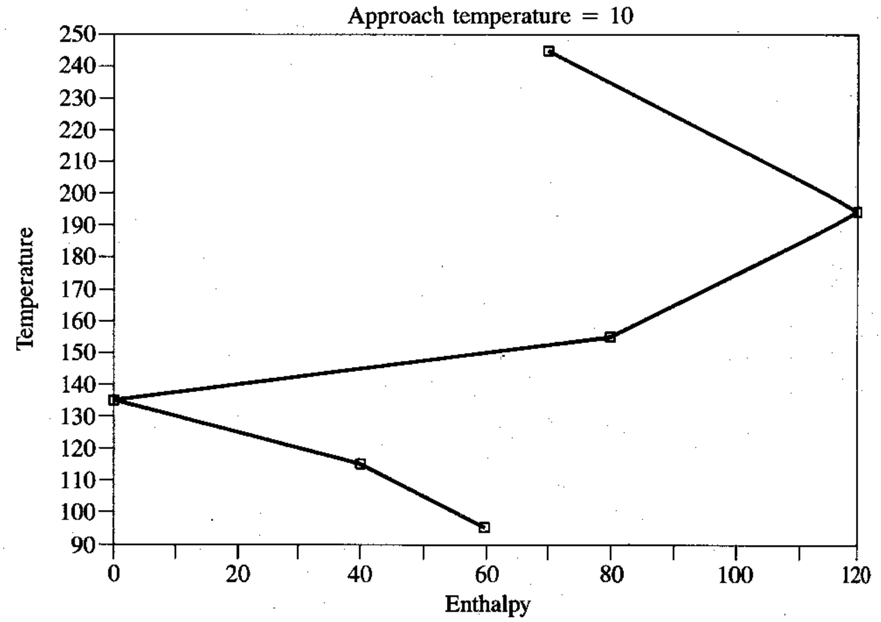
> Above pinch, we only supply heat from the hot utility.
> Below pinch, we only reject heat to the cold utility.
> More importantly, "Do not transfer heat across the pinch!", or equivalently design separate HENs above and below the pinch temperature.

Temperature-Enthalpy Diagram

An alternative graphical display to the cascade diagram which can easily accommodate the effect of change in the minimum temperature approach.

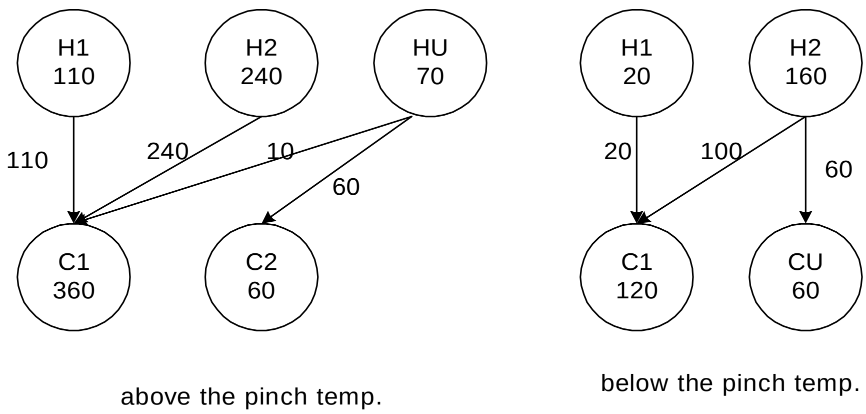






Minimum Number of Matches

Let us consider the minimum number of heat exchangers that are required for the necessary heat exchange. From the simple heat distribution rule shown in the figure, it can be seen that the minimum number of matches is equal to the number of process streams plus utility minus one, or simply $N - 1$.



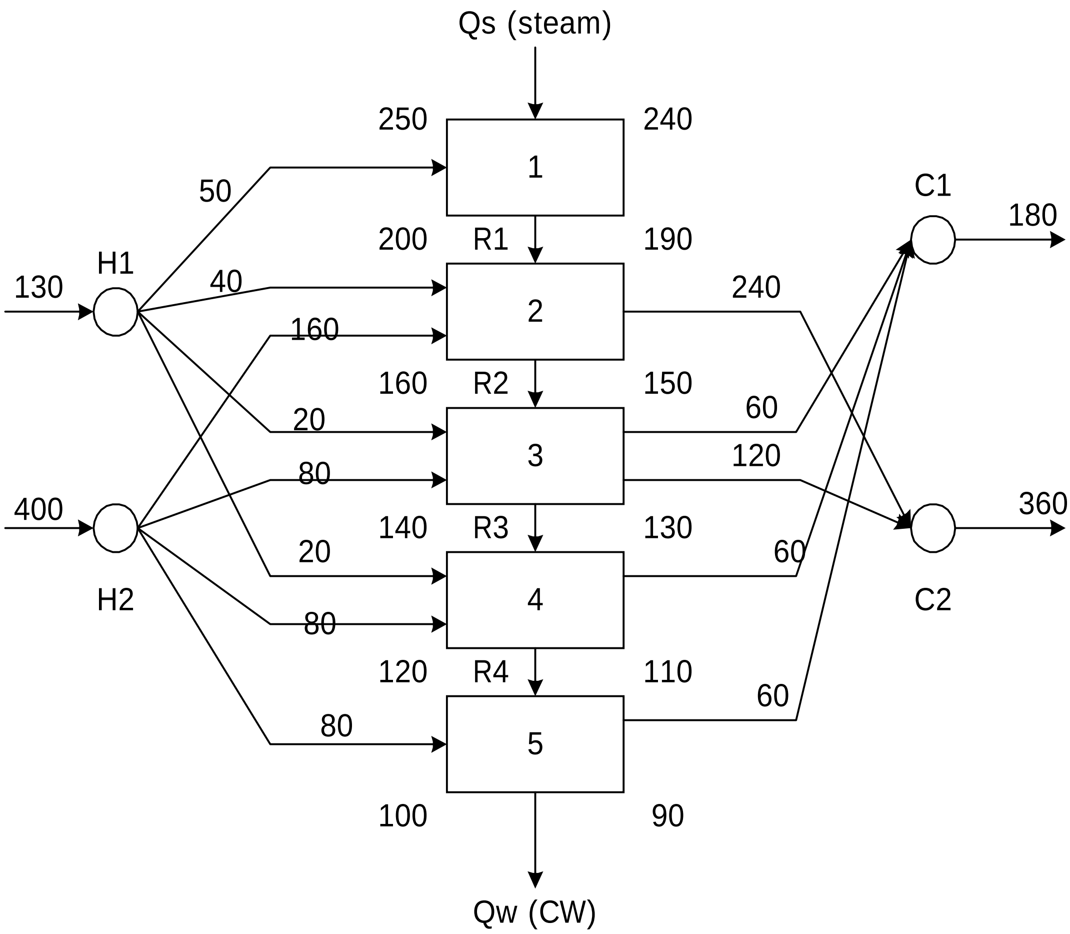
4.2 Optimization Formulation of the HEN Synthesis

4.2.1 Minimum Utility Cost

The figure shows another representation of the cascade diagram. This representation tells that the HEN problem can be regarded as a transshipment problem where

- Heat is the commodity that must be transferred from the sources to the destinations through some intermediate warehouses;
- H1 are H2 are sources;
- C1 and C2 are destinations;
- temperature interval boxes are intermediate warehouses.

The transshipment problem can be formulated as a standard LP problem.



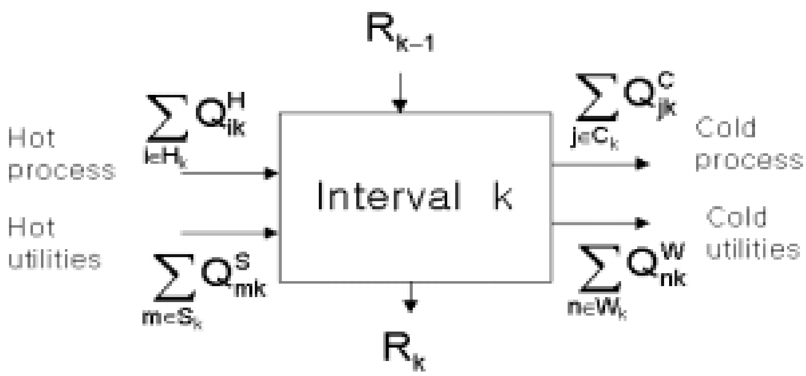
The minimum utility cost problem can be written as

$$
\begin{aligned}
\min_{Q_s,Q_w,R_i} \quad & Z = Q_s + Q_w \\
\text{subject to} \quad & R_1 = Q_s + 50 \\
& R_2 = R_1 + 40 + 160 - 240 = R_1 - 40 = Q_s + 10 \\
& R_3 = R_2 + 20 + 80 - 60 - 120 = R_2 - 80 = Q_s - 70 \\
& R_4 = R_3 + 20 + 80 - 60 = R_3 + 40 = Q_s - 30 \\
& Q_w = R_4 + 80 - 60 = R_4 + 20 = Q_s - 10 \\
& R_i, Q_s, Q_w \geq 0
\end{aligned}
$$

From the last expression of each constraint equation, it is obvious that $Q_s = 70$ (and accordingly $Q_w = 60$) is the minimum utility required, which is equivalent to the previous analysis.

For generalization of the transshipment model to the case of multiple utilities, let us define

$$
\begin{aligned}
H_k &= \{i|\text{hot stream } i \text{ supplies heat to interval } k\} \\
C_k &= \{j|\text{cold stream } j \text{ demands heat from interval } k\} \\
S_k &= \{m|\text{hot utility } m \text{ supplies heat to interval } k\} \\
W_k &= \{n|\text{cold utility } n \text{ extracts heat from interval } k\}
\end{aligned}
$$



Known parameters: Q_{ik}^H, Q_{jk}^C, and unit costs of utilities

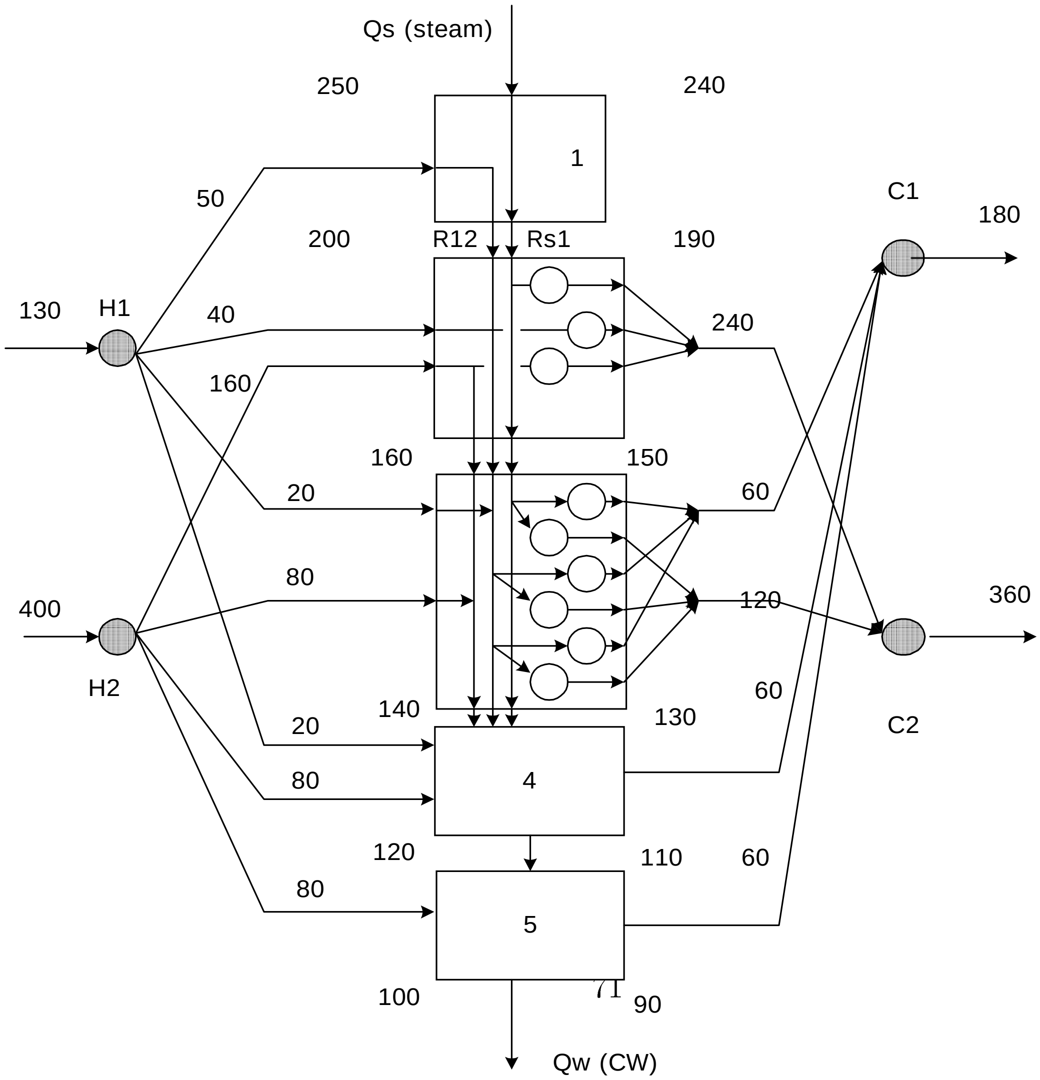
Variables to determine: Q_{mk}^S and Q_{nk}^W, and R_k

Then the minimum utility cost problem can be formulated as

$$
\begin{aligned}
\min \quad & Z = \sum_k \sum_{m \in S_k} c_m Q_{mk}^S + \sum_k \sum_{n \in W_k} c_n Q_{nk}^W \\
\text{s.t.} \quad & R_k - R_{k-1} - \sum_{m \in S_k} Q_{mk}^S + \sum_{n \in W_k} Q_{nk}^W = \sum_{i \in H_k} Q_{ik}^H - \sum_{j \in C_k} Q_{jk}^C, \quad k = 1, \cdots, K \\
& Q_{mk}^S, Q_{nk}^W, R_k \geq 0, \quad R_0 = R_K = 0
\end{aligned}
$$

The above is an LP problem. Other constraints such as maximum utility supplies can be imposed.

4.2.2 Minimum Utility Cost with Constrained Matches



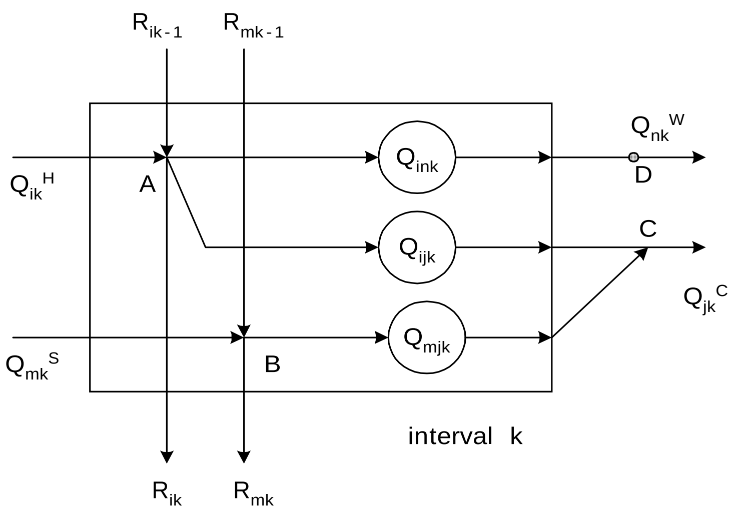
In practice, additional constraints may be imposed. For example,

- due to geometric limitation or safety reason or others, it may not be possible to exchange heat between certain pairs of hot and cold streams
- minimum or maximum amount of heat exchanged between a certain pair of streams may be limited.

In order to accommodate all those information in the problem formulation, it is necessary to expand the transshipment model such that the detailed heat exchange paths are specified.

The following figure presents the idea how the heat exchange can be made. The circles in a box represent the heat exchangers that may be used. From all the possible heat exchangers as listed in the figure, we select the relevant ones through optimization.

The following figure shows an exemplary heat exchanger network in a temperature interval with notation conventions. It is assumed that heat is supplied from a hot stream i and hot utility m. The hot stream sources are two: the residual heat from the upper interval $R_{i,k-1}$ and the heat in the k^{th} interval Q_{ik}^H. These two hot steams are separated only conceptually. They are in fact in one physical stream. The diagram for the hot utility is interpreted similarly.



Now we define the necessary notations in detail as follows:

$$
\begin{aligned}
Q_{ijk} &= \{\text{exchange of heat of hot stream } i \text{ and cold stream } j \text{ at interval } k\} \\
Q_{mjk} &= \{\text{exchange of heat of hot utility } m \text{ and cold steam } j \text{ at interval } k\} \\
Q_{ink} &= \{\text{exchange of heat of hot stream } i \text{ and cold utility } n \text{ at interval } k\} \\
R_{ik} &= \{\text{heat residual of hot stream } i \text{ existing at interval } k\} \\
R_{mk} &= \{\text{heat residual of hot utility } m \text{ existing at interval } k\} \\
H_k &= \{i|\text{hot stream } i \text{ supplies heat to interval } k\} \\
C_k &= \{j|\text{cold stream } j \text{ demands heat from interval } k\} \\
S_k &= \{m|\text{hot utility } m \text{ supplies heat to interval } k\} \\
W_k &= \{n|\text{cold utility } n \text{ extracts heat from interval } k\}
\end{aligned}
$$

The expanded LP transshipment model can be formulated as

$$
\begin{aligned}
\min \quad & Z = \sum_k \sum_{m \in S_k} c_m Q^S_{mk} + \sum_k \sum_{n \in W_k} c_n Q^W_{nk} \\
\text{s.t.} \quad & R_{ik} - R_{i,k-1} + \sum_{j \in C_k} Q_{ijk} + \sum_{n \in W_k} Q_{ink} = Q^H_{ik}, \quad i \in H_k \quad \text{balance for hot stream} \\
& R_{mk} - R_{m,k-1} + \sum_{j \in C_k} Q_{mjk} - Q^S_{mk} = 0, \quad m \in S_k \quad \text{balance for hot utility} \\
& \sum_{i \in H_k} Q_{ijk} + \sum_{m \in S_k} Q_{mjk} = Q^C_{jk}, \quad j \in C_k \quad \text{balance for cold stream} \\
& \sum_{i \in H_k} Q_{ink} - Q^W_{nk} = 0, \quad n \in W_k, \quad k = 1, \cdots, K \quad \text{balance for sold utility} \\
& R_{ik}, R_{mk}, \ Q_{ijk}, \ Q_{mjk}, \ Q_{ink}, \ Q^S_{mk}, \ Q^W_{nk} \geq 0
\end{aligned}
$$

On the above LP model, we can impose various constraints flexibly. For example, we ma impose

$$Q_{ijk} = 0$$

if we want to forbid a match btn hot i and cold j streams for all intervals k, etc.

4.2.3 Prediction of Matches for Minimizing the Number of Units

Once the minimum utility cost problem is solved, we have optimum Q^S_{mk} and Q^W_{nk}. The LP solution for other variables such as Q_{ijk}, R_{ik}, etc is not unique in general.

Now with optimum Q^S_{mk} and Q^W_{nk}, we can solve the problem for minimum number of units. For this, we define binary variables for each subnetwork

$$y^q_{ij} = \begin{cases} 1 & \text{if hot stream } i \text{ and cold stream } j \text{ exchanges heat} \\ 0 & \text{if hot stream } i \text{ and cold stream } j \text{ doesn't exchanges heat} \end{cases}$$

where the superscript q represents a subnetwork such as the above- or below-pinch region.

Since Q^S_{mk} and Q^W_{nk} are no longer unknowns, we can treat them as hot and cold streams and share the common index i and j.

Using such conventions, the minimum number of units problem (under minimum utility cost) can be described as, for each subnetwork,

$$\begin{aligned} \min \quad & \sum_{i \in H} \sum_{j \in C} y^q_{ij} \\ \text{s.t.} \quad & R_{ik} - R_{i,k-1} + \sum_{j \in C_k} Q_{ijk} = Q^H_{ik}, \quad i \in H_k, \quad k = 1, \cdots, K_q \\ & \sum_{i \in H_k} Q_{ijk} = Q^C_{jk}, \quad j \in C_k \\ & R_{ik}, \; Q_{ijk} \geq 0 \\ & \sum_{k=1}^{K_q} Q_{ijk} - U_{ij} y^q_{ij} \leq 0 \end{aligned}$$

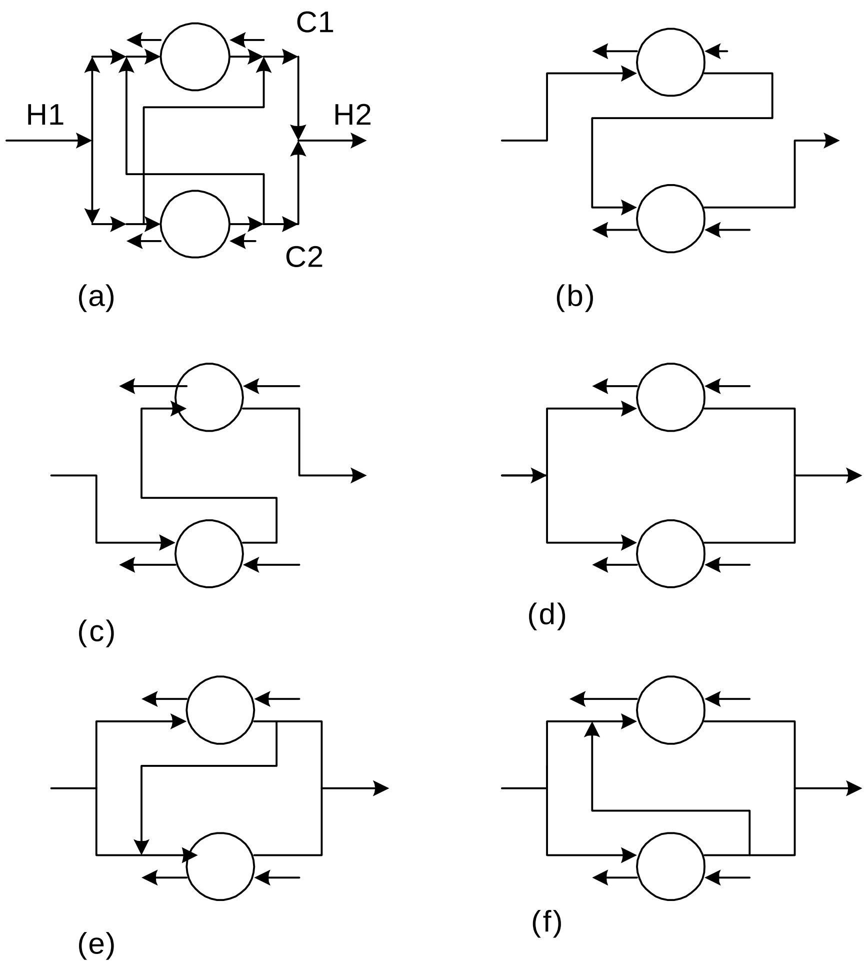
where U_{ij} is the smallest of the heat contents of the two streams. If the hot stream has 1,000 and the cold stream has 2,000 heat units, U_{ij} is set to 1,000 units. The last inequality defines if the heat exchange will be made btn i and j in the q subnetwork.

We can see that the problem of the minimum number of units is described as a MILP (mixed integer linear programming) problem.

4.2.4 Automatic Derivation of Network Structure using the Concept of Superstructure

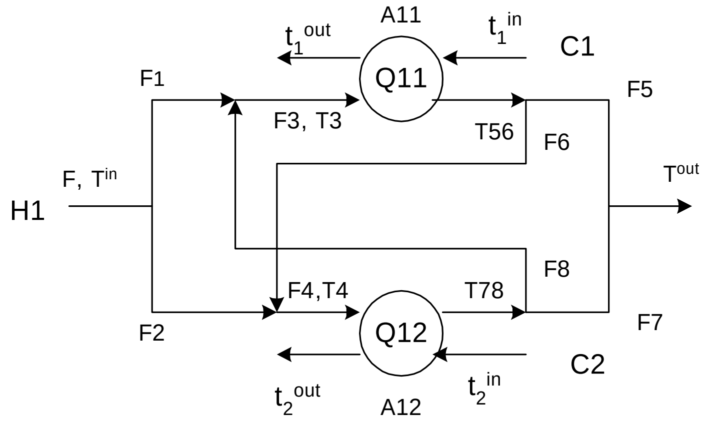
Using the information provided by the MILP transshipment model, the heat exchanger network configuration can be automatically derived from the so-called superstructure.

A superstructure for HEN among some hot and cold streams is one that contains all the possible heat exchanges among the streams. An example of such a superstructure is given in the figure below for the case of one hot and two cold streams, H1 and {C1, C2}. Figs (b) through (f) represent the all the possible heat exchange schemes which are included in Fig (a) as a superstructure.



Superstructure for H1 - C1/C2 HEN (a) and all the possible interconnections embedded (b-f)

Through optimization, one of the heat exchange schemes will be selected. For optimization formulation, we define the associated variables as in the following figure:



This time, we consider the problem to minimize the capital investment cost. Total capital investment is represented by $c_1 A_{11}^{\beta} + c_2 A_{12}^{\beta}$. Here the heat transfer area is a function of the heat duty and the log-mean temperature difference such that $Q = UA\Delta T_{log}$. The log-mean temperature difference is approximated as

$$\Delta T_{log} = \frac{\theta_2 - \theta_1}{\log \theta_2/\theta_1} \approx [\theta_1\theta_2(\theta_2 + \theta_1)/2)^{1/3}$$

The optimization problem can be stated as, given Q_{11}, Q_{12}, F, T^{in}, T^{out}, t_i^{in}, and t_i^{out},

$$\min \quad \text{Cost} = c_1 \left[\frac{Q_{11}}{U_{11}[\theta_1^1\theta_2^1(\theta_2^1 + \theta_1^1)/2)^{1/3}}\right]^{\beta} + c_2 \left[\frac{Q_{12}}{U_{12}[\theta_1^2\theta_2^2(\theta_2^2 + \theta_1^2)/2)^{1/3}}\right]^{\beta}$$

subject to

- Mass balance for the initial splitter

$$F_1 + F_2 = F$$

- Mass and heat balances for mixers at the initial two units

$$F_1 + F_8 - F_3 = 0$$
$$F_1 T^{in} + F_8 T_{78} - F_3 T_3 = 0$$
$$F_2 + F_6 - F_4 = 0$$

$$F_2T^{in} + F_6T_{56} - F_4T_4 = 0$$

- Mass balance for splitters at outlets of exchangers

$$F_3 - F_6 - F_5 = 0$$
$$F_4 - F_7 - F_8 = 0$$

- Heat balances in exchangers

$$Q_{11} - F_3(T_3 - T_{56}) = 0$$
$$Q_{12} - F_4(T_4 - T_{78}) = 0$$

- Definitions of temperature differences

$$\theta_1^1 \;=\; T_3 - t_1^{out}$$
$$\text{etc}$$

- Feasibility constraints for temperature differences

$$\theta_1^1,\ \theta_2^1,\ \cdots\ \geq \Delta T_{min}$$

- Nonnegativity conditions of the heat capacity flows

$$F_j \geq 0$$

The minimum capital cost problem is described as a NLP problem.

4.3 Simultaneous Optimum HEN Synthesis

What we have employed for HEN design was a sequential design approach such that

$$\begin{array}{ll} \min & \text{Capital Cost} \\ & \text{s.t. } \min \ \text{Number of Units} \\ & \qquad \text{s.t. } \min \ \text{Minimum Utility Cost} \end{array}$$

What should be optimized is in fact

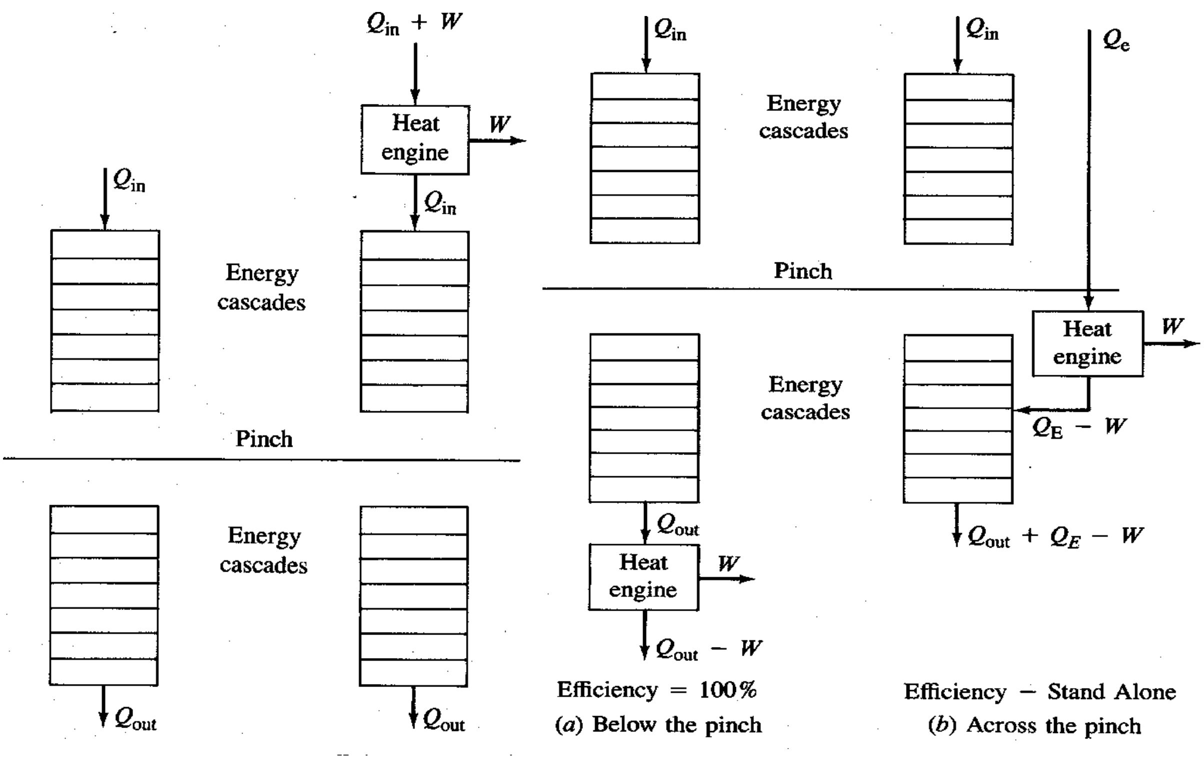
$$\min \{\text{Total Cost} = \text{Capital Cost} + \text{Operating Cost}\}$$

Optimization formulation for the latter problem (simultaneous optimum HEN design) leads to a MINLP (Mixed Integer Nonlinear Programming) problem.

The simultaneous HEN synthesis gives us a more reasonable result but the problem size becomes much larger than the sequential design approach.

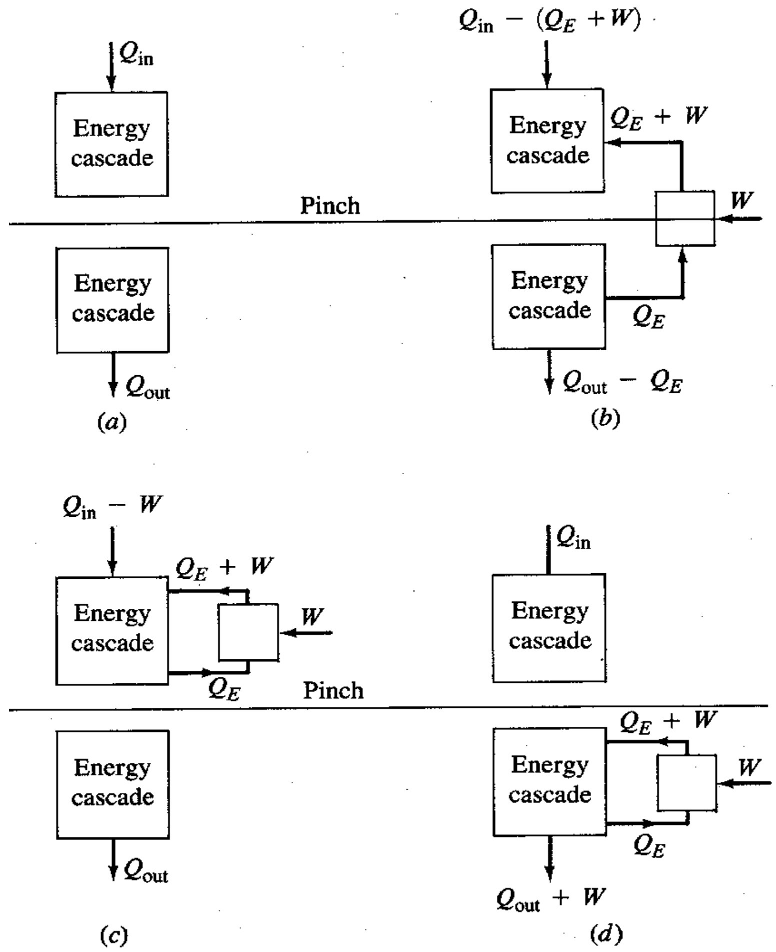
4.4 Heat and Power Integration

Use of Heat Engines



Therefore, place the heat engine either above or below the pinch, not across the pinch!

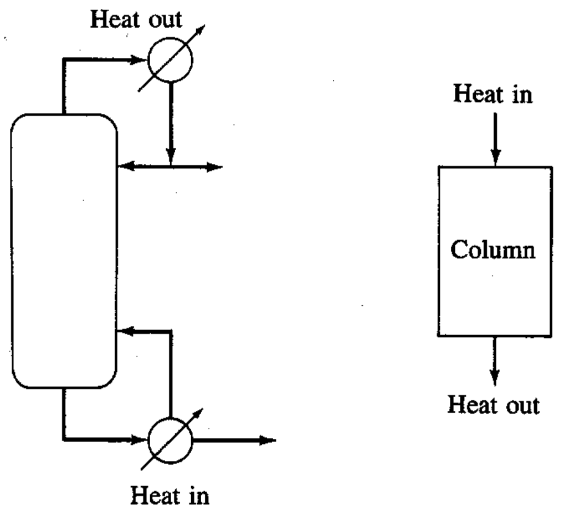
Use of Heat Pumps



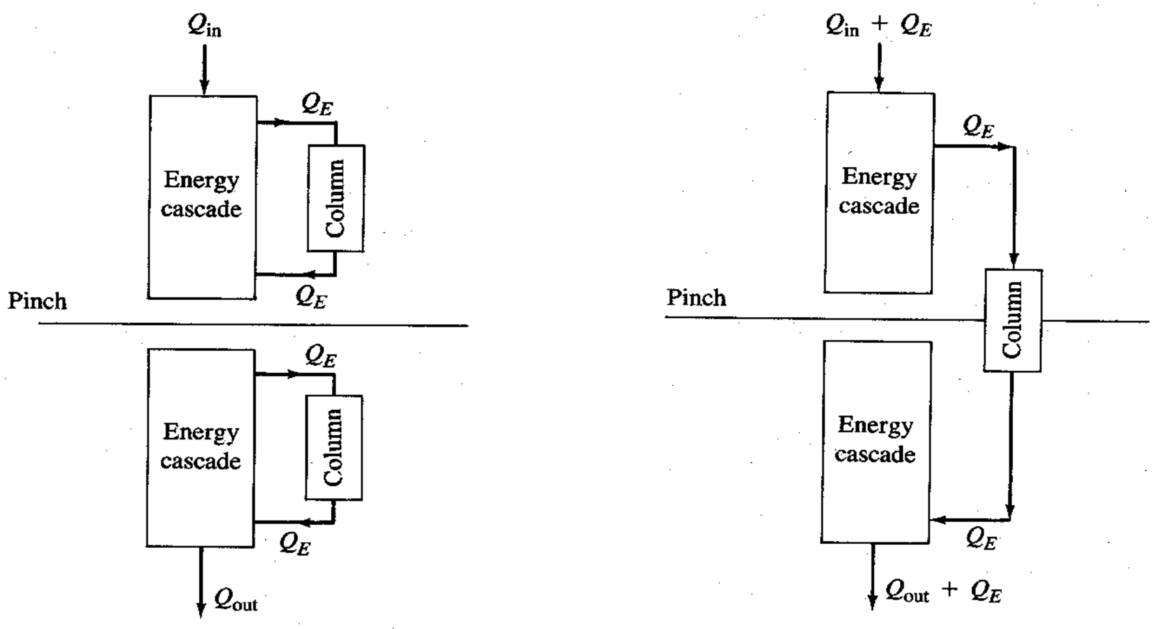
Therefore, place the heat pump across the pinch !

4.5 Heat and Distillation

Distillation process can be considered a heat engine

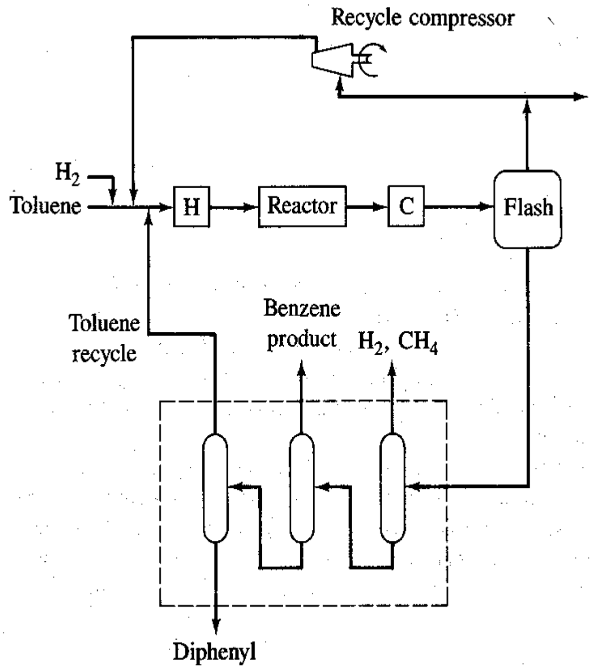


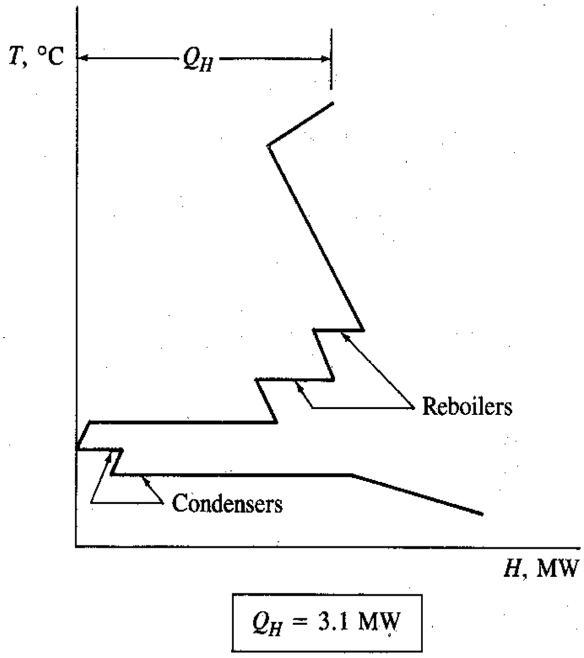
As for the heat engine, “place dstillation columns either above or below the pinch”.



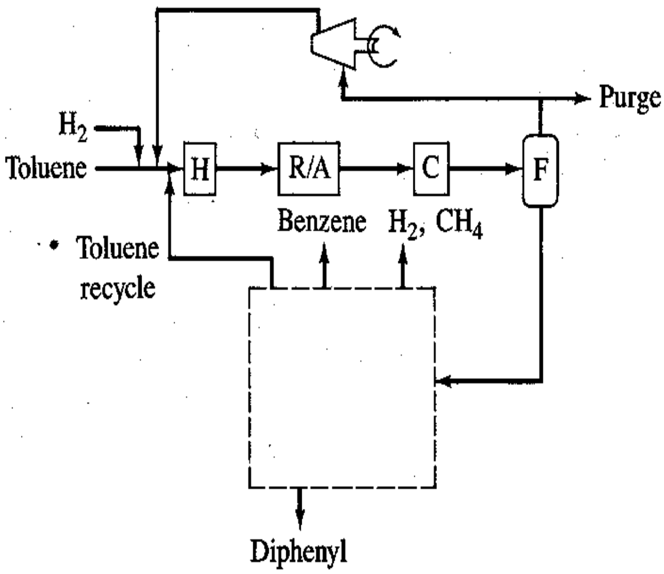
4.5.1 Application to HDA Process

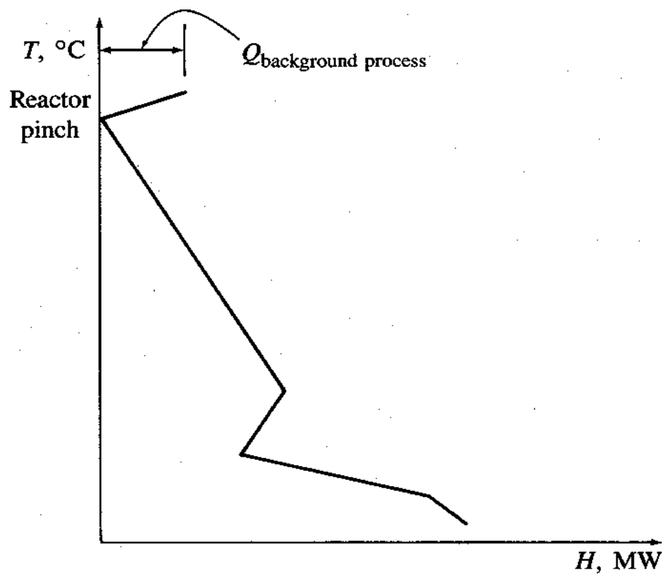
- T-H Plot for the Total Flowsheet



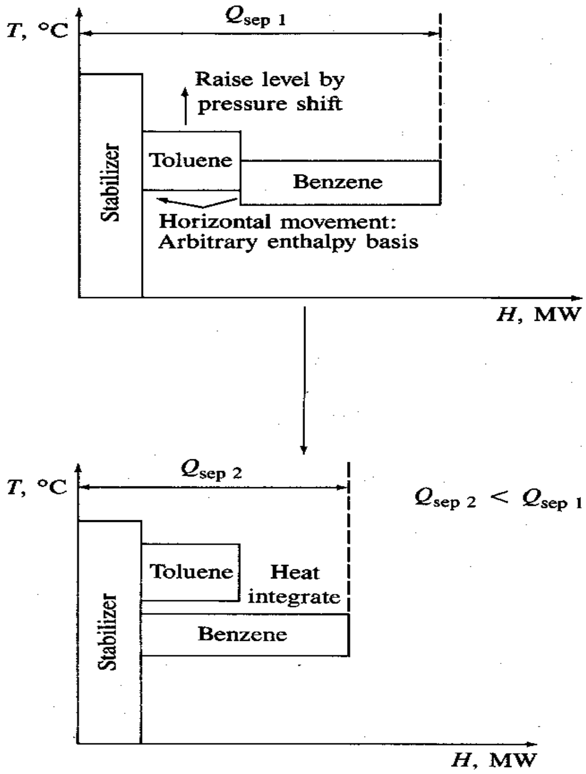


- T-H Plot of the Background Process

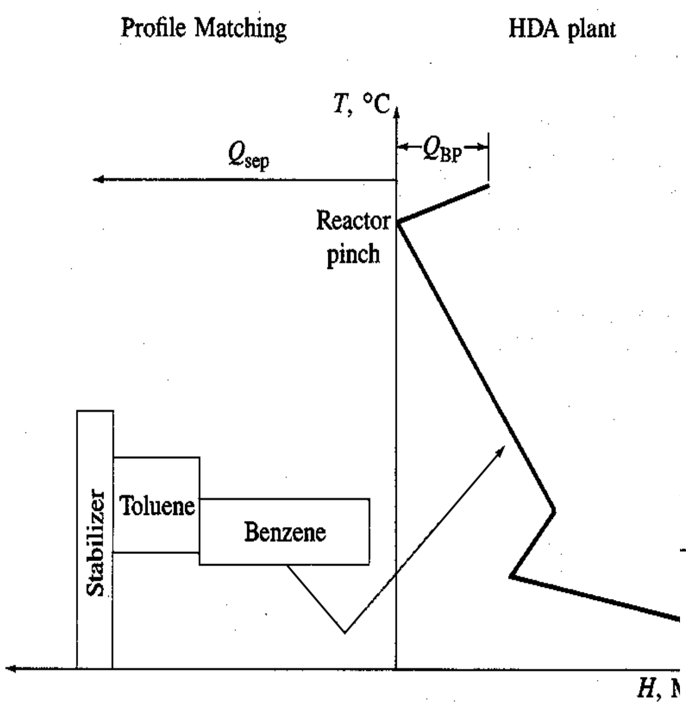


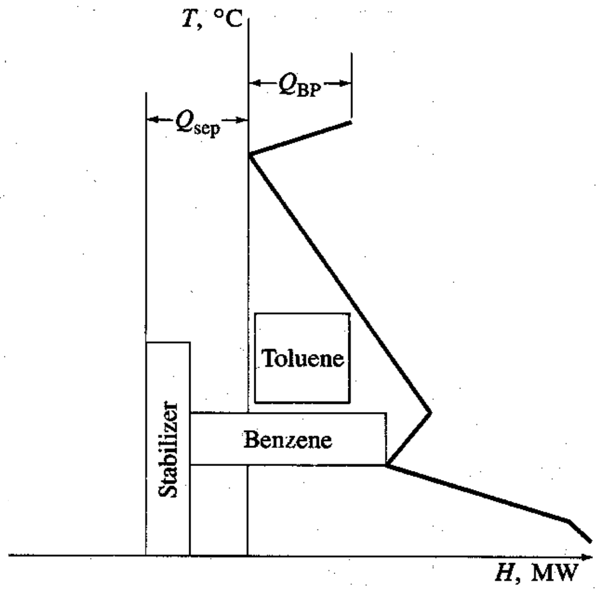


• Integration of the Liquid Separation System



• Integration of the Distillation System with the Process





$Q_{sep} + Q_{BP} = 1.6 + 0.9$ MW
$= 2.5$ MW
Cf. base case of 3.1 MW