2. Physical Properties

2.1 Introduction

Two most important fluid properties, required in the treatment of the fluid flow, are the density*(ρ) and the viscosity*(η).

2.2 Units

Length (L), Mass (M), Time (Θ)

Velocity: $L\Theta^{-1}$

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* Please also see the Navier-Stokes eq′ns in chapter 7.

Unit systems:

cgs (centimeter-gram-second)
English (pound-foot-second)
SI (système international)
meter (m), kilogram (kg), second (s)

Derived units:

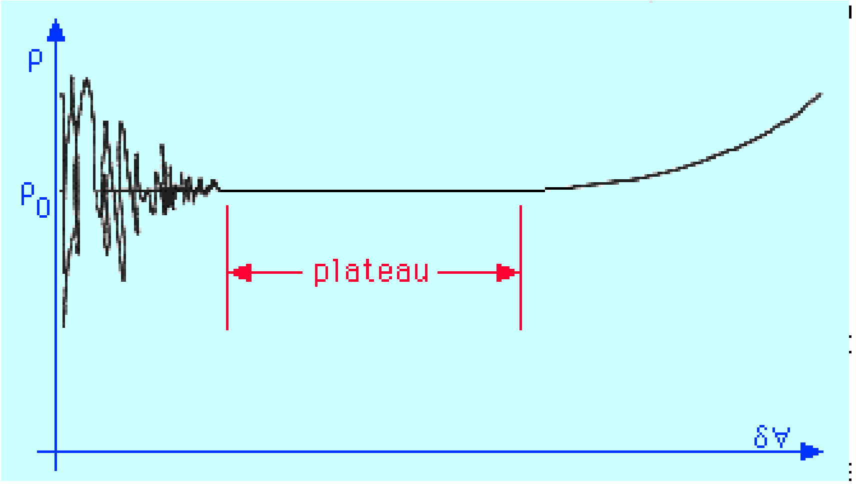
Force	newton (N)	= $kg \cdot m/s^2$
Pressure, stress	pascal (Pa)	= N/m^2
Work, energy	joule (J)	= $N \cdot m$
Power	watt (W)	= J/s

2.3 Continum Hypothesis

The continum hypothesis assumes that mathematical limits for volumes tending to zero are reached over a scale that remains large compared to molecular dimensions.

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta M}{\Delta V}$$

$$M = \int \rho(x,y,z)\,dV$$



2.4 Viscosity

Operational Definition:

(Figure 2-1)

Shear stress: $\tau_s = \frac{F}{A}$ Shear rate: $\Gamma_s = \frac{U}{H}$

$$\tau_s = \tau_s(\Gamma_s)$$

$$\frac{d\tau_s}{d\Gamma_s} > 0$$ monotonic increase

$$\eta = \frac{\tau_s}{\Gamma_s}$$: viscosity

- Unit: Pa·s

  In cgs unit, poise (p) (1 p = 0.1 Pa·s)

- The viscosity of water is about 1 cp (=10^{-3} Pa·s)

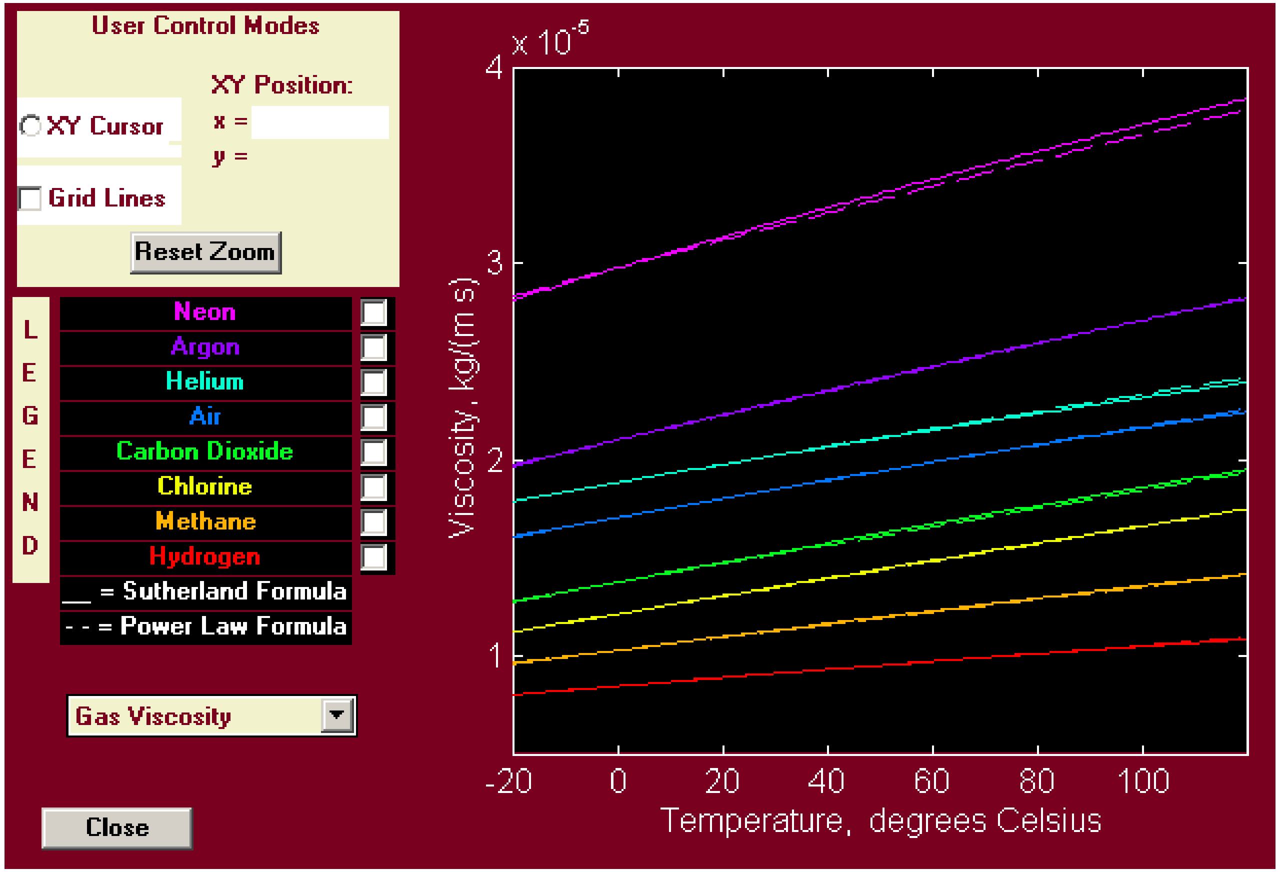
Newtonian fluids:

The viscosity is independent of the shear rate and depends only on temperature and, to a much lesser extent, on pressure.

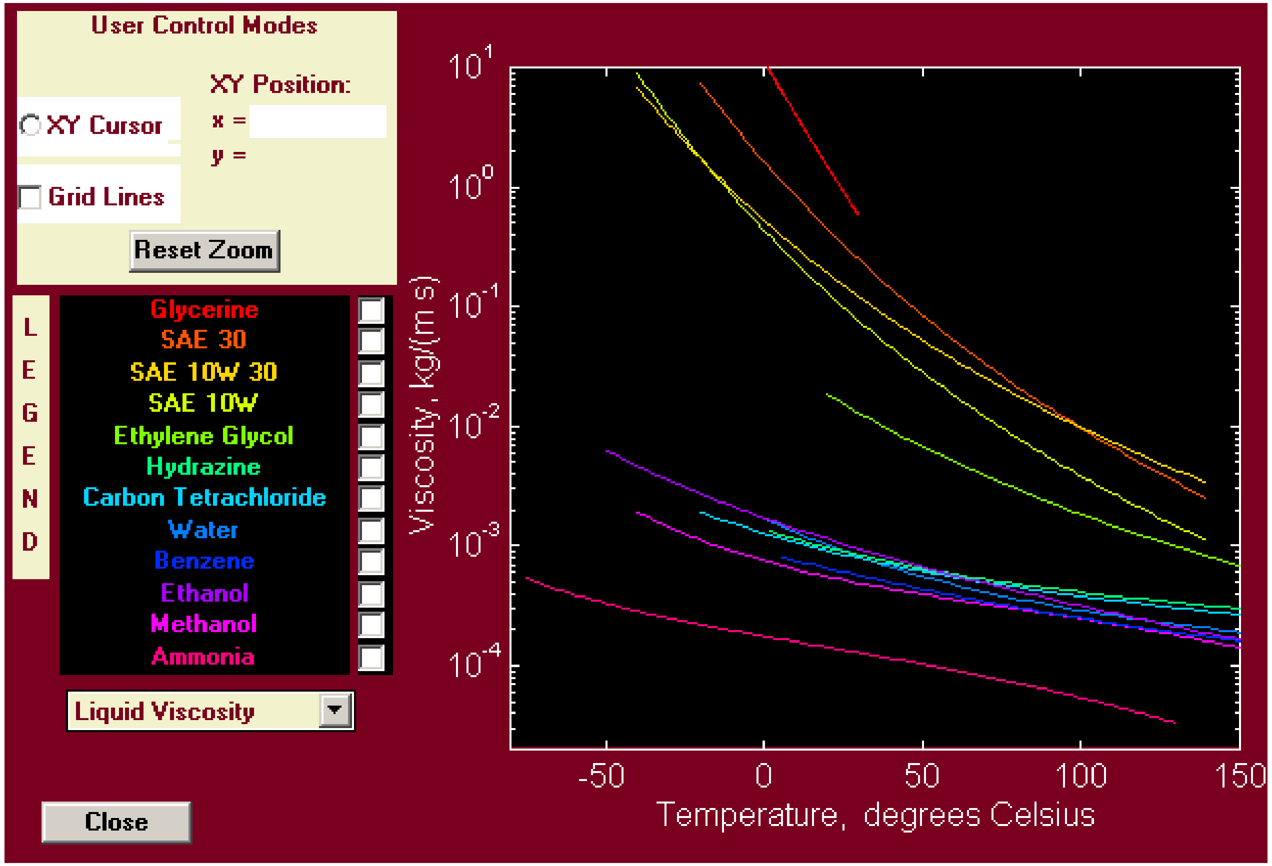
Low-molecular-weight liquids and all gases
(Figures 2-2, 2-3)

The viscosity is a decreasing function of temperature for the liquids and an increasing function of temperature for the gases.
(Figures 2-5, 2-6)

Viscosity of gases



Viscosity of liquids



Non-Newtonian fluids:

The viscosity is a function of the shear rate.

- Shear-thinning (Figures 2-4, 2-7, 2-8)
- Shear-thickening (Figure 2-9)
- Bingham plastics (Figure 2-10)

High-molecular-weight liquids, slurries, and suspensions

$$\frac{d\tau_s}{d\Gamma_s} > 0 \quad \Rightarrow \quad \frac{d\ln\eta(\Gamma_s)}{d\ln\Gamma_s} > -1$$

Power-law fluids: $\eta(\Gamma_s) = K|\Gamma_s|^{n-1}$

K : consistency factor

n : power-law index

Time-dependent viscosity (structure change):

- Thixotropy
- Anti-thixotropy (Rheopexy)

Yield stress

Kinematic Viscosity:

$$\nu = \frac{\eta}{\rho}$$

Unit: m^2/s

In cgs unit, stoke (s) (= cm^2/s)

The kinematic viscosity of water is

about 1 centistoke (cs) (= 10^{-6} m^2/s).

Measurement:

Concentric cylinders with a small gap/radius ratio as an approximation to infinite parallel plates:

(Figures 2-11, 2-12)

The linear velocity, U, is $R\Omega$, and $\Gamma_s = \dfrac{R\Omega}{H}$.

The area, A, is $2\pi RL$.

The differential area $dA = RLd\theta$, and the differential force dF acting on dA is

$$dF = \tau_s dA = \eta \Gamma_s dA = \eta \left(\frac{R\Omega}{H}\right) RLd\theta$$

The differential torque dG is the product of the force and the lever arm:

$$dG = RdF = \eta \left(\frac{R^3 L\Omega}{H}\right) d\theta$$

And by integrating all the small contributions dG to obtain

$$G = \int dG = \int_0^{2\pi} \eta \left(\frac{R^3 L\Omega}{H}\right) d\theta = \eta \frac{2\pi R^3 L\Omega}{H}$$

or $$\eta = \frac{GH}{2\pi R^3 L\Omega}$$

2.5 Viscoelasticity

In Newtonian fluids, the time scale of initial transient is of the order of 10^{-12}s.

Any transient seen experimentally is an artifact which results from the inertia of the instrument, since moving parts cannot be set in motion instantaneously.

But, in polymeric systems, the initial transient can be quite long because of the need of the long polymer chains to rearrange.

The time scale of the transient can be shown to be a property of the material and not an artifact of a single experiment.

The finite transient time of the polymeric liquids is a consequence of a solidlike response of the material. (Figure 2-13)

Materials that show both fluid- and solidlike behavior, depending on the time scale of the process, are called viscoelastic. Viscoelastic liquids are characterized by a viscosity (η) and a relaxation time (λ).

2.6 Interfacial Tension

A property of a liquid-gas or liquid-liquid interface.

The differential change in free energy to create differentia interfacial area dA is equal to σdA where σ is the interfacial tension and has dimensions of energy per area or force per length.

The pressure difference caused by the interfacial tension across a static curved surface is equal to

$$\Delta P = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where, R_1 and R_2 are the principal radii of curvature of the surface.

Spherical bubble or droplet:

$$\Delta P = \frac{2\sigma}{R}$$

The pressure inside the bubble or droplet exceeds the pressure outside by the amount.

Interfacial tension effects can sometimes be quite important in
two-phase flows, and in certain
free surface flows.