CHE302 LECTURE XI
CONTROLLER DESIGN AND PID CONTOLLER TUNING

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Fall 2001

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CONTROLLER DESIGN

- **Performance criteria for closed-loop systems**
  - **Stable**
  - **Minimal effect of disturbance**
  - **Rapid, smooth response to set point change**
  - **No offset**
  - **No excessive control action**
  - **Robust to plant-model mismatch**

$$\min_{K_c, t_I, t_D} \int_0^\infty (w_1 e^2(t) + w_2 \Delta u^2(t))dt$$

- **Trade-offs in control problems**
  - **Set point tracking vs. disturbance rejection**
  - **Robustness vs. performance**

GUIDELINES FOR COMMON CONTROL LOOPS

- **Flow and liquid pressure control**
  - Fast response with no time delay
  - Usually with small high-frequency noise
  - PI controller with intermediate controller gain
- **Liquid level control**
  - Noisy due to splashing and turbulence
  - High gain PI controller for integrating process
  - Conservative setting for averaging control when it is used for damping the fluctuation of the inlet stream
- **Gas pressure control**
  - Usually fast and self regulating
  - PI controller with small integral action (large reset time)

- **Temperature control**
  - Wide variety of the process nature
  - Usually slow response with time delay
  - Use PID controller to speed up the response
- **Compositioncontrol**
  - Similar to temperature control usually with larger noise and more time delay
  - Effectiveness of derivative action is limited
  - Temperature and composition controls are the prime candidates for advance control strategies due to its importance and difficulty of control

TRIAL AND ERROR TUNING

- **Step1: With P-only controller**
  - Start with low K_c value and increase it until the response has a sustained oscillation (continuous cycling) for a small set point or load change. (K_{cu})
  - Set $K_c = K_{cu}$.
- **Step2: Add I mode**
  - Decrease the reset time until sustained oscillation occurs. (τ_{Iu})
  - Set $\tau_I = 3\tau_{Iu}$.
  - If a further improvement is required, proceed to Step 3.
- **Step3: Add D mode**
  - Decrease the reset time until sustained oscillation occurs. (τ_{Du})
  - Set $\tau_D = 3\tau_{Du}$.

(The sustained oscillation should not be cause by the controller saturation)

CONTINUOUS CYCLING METHOD

- **Also called as loop tuning or ultimate gain method**
  - Increase controller gain until sustained oscillation
  - Find ultimate gain (K_{CU}) and ultimate period (P_{CU})
- **Ziegler-Nichols controller setting**
  - ¼ decay ratio (too much oscillatory)

Controller	K_C	τ_I	τ_D
P	$0.5K_{CU}$	-	-
PI	$0.45K_{CU}$	$P_{CU}/1.2$	-
PID	$0.6K_{CU}$	$P_{CU}/2$	$0.5P_{CU}/8$

  - Modified Ziegler-Nichols setting

Controller	K_C	τ_I	τ_D
Original	$0.6K_{CU}$	$P_{CU}/2$	$P_{CU}/8$
Some overshoot	$0.33K_{CU}$	$P_{CU}/2$	$P_{CU}/3$
No overshoot	$0.2K_{CU}$	$P_{CU}/2$	$P_{CU}/3$

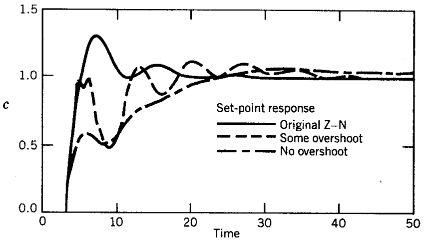
- **Examples**

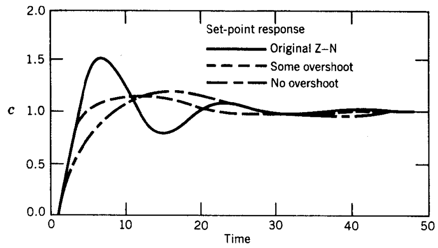
$$G_p(s) = \frac{4e^{-3.5s}}{7s+1} \qquad K_{CU} = 0.95 \qquad P_{CU} = 12$$

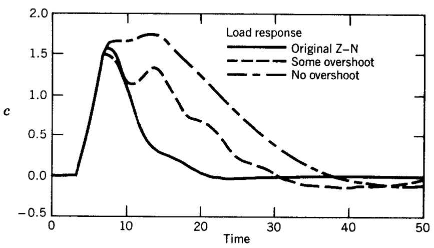
$$G_p(s) = \frac{2e^{-s}}{(10s+1)(5s+1)} \qquad K_{CU} = 7.88 \qquad P_{CU} = 11.6$$

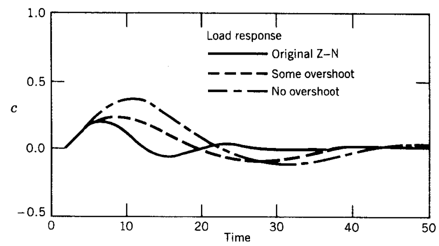
Controller	K_C	t_I	t_D
Original	0.57	6.0	1.5
Some overshoot	0.31	6.0	4.0
No overshoot	0.19	6.0	4.0

Controller	K_C	t_I	t_D
Original	4.73	5.8	1.45
Some overshoot	2.60	5.8	3.87
No overshoot	1.58	5.8	3.87









- **Advantages of continuous cycling method**
  - No a priori information on process required
  - Applicable to all stable processes

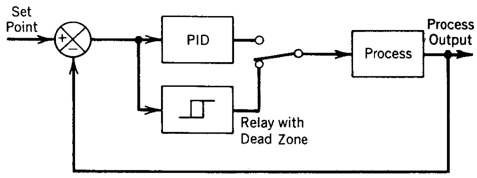
- **Disadvantages of continuous cycling method**
  - Time consuming
  - Loss of product quality and productivity during the tests
  - Continuous cycling may cause the violation of process limitation and safety hazards
  - Not applicable to open-loop unstable process
  - First-order and second-order process without time delay will not oscillate even with very large controller gain

=> Motivates Relay feedback method. (Astrom and Wittenmark)

RELAY FEEDBACK METHOD

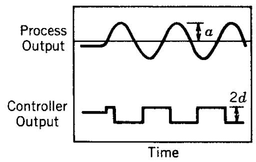
- **Relay feedback controller**
  - Forces the system to oscillate by a relay controller
  - Require a single closed-loop experiment to find the ultimate frequency information
  - No *a priori* information on process is required
  - Switch relay feedback controller for tuning
  - Find P_{CU} and calculate K_{CU}

$$K_{CU} = \frac{4d}{pa}$$



  - User specified parameter: d

    Decide d in order not to perturb the system too much.



  - Use Ziegler-Nichols Tuning rules for PID tuning parameters

DESIGN RELATIONS FOR PID CONTROLLERS

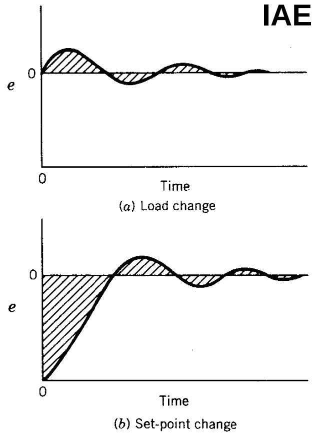
- **Cohen-Coon controller design relations**
  - Empirical relation for ¼ decay ratio for FOPDT model

Table 12.2 Cohen and Coon Controller Design Relations

Controller	*Settings*	*Cohen–Coon*
P	K_c	$\frac{1}{K}\frac{\tau}{\theta}[1 + \theta/3\tau]$
PI	K_c	$\frac{1}{K}\frac{\tau}{\theta}[0.9 + \theta/12\tau]$
	τ_I	$\frac{\theta[30 + 3(\theta/\tau)]}{9 + 20(\theta/\tau)}$
PID	K_c	$\frac{1}{K}\frac{\tau}{\theta}\left[\frac{16\tau + 3\theta}{12\tau}\right]$
	τ_I	$\frac{\theta[32 + 6(\theta/\tau)]}{13 + 8(\theta/\tau)}$
	τ_D	$\frac{4\theta}{11 + 2(\theta/\tau)}$

- **Design relations based on integral error criteria**
  - ¼ decay ratio is too oscillatory
  - Decay ratio concerns only two peak points of the response
  - IAE: Integral of the Absolute Error

$$\text{IAE} = \int_0^\infty |e(t)|\,dt$$



  - ISE: Integral of the Square Error

$$\text{ISE} = \int_0^\infty [e(t)]^2\,dt$$

    - Large error contributes more
    - Small error contributes less
    - Large penalty for large overshoot
    - Small penalty for small persisting oscillation
  - ITAE: Integral of the Time-weighted Absolute Error

$$\text{ITAE} = \int_0^\infty t\,|e(t)|\,dt$$

    - Large penalty for persisting oscillation
    - Small penalty for initial transient response

- **Controller design relation based on ITAE for FOPDT model**

Table 12.3 Controller Design Relations Based on the ITAE Performance Index and a First-Order plus Time-Delay Model [6–8][a]

Type of Input	*Type of Controller*	*Mode*	*A*	*B*
Load	PI	P	0.859	−0.977
		I	0.674	−0.680
Load	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a]Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.
[b]For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$. [8]

- **Similar design relations based on IAE and ISE for other types of models can be found in literatures.**

- **Example1**

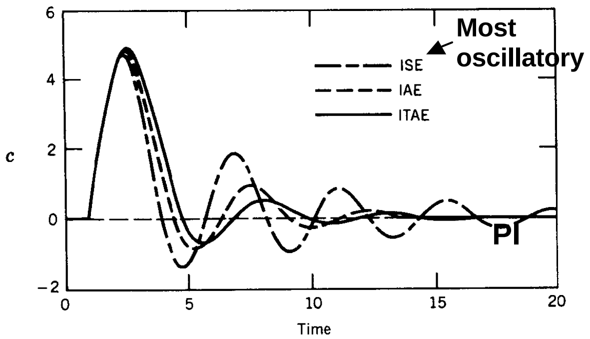
$$G(s)=\frac{10e^{-s}}{2s+1}$$

$$KK_c=(0.859)(1/2)^{-0.977}=1.69$$

$$\Rightarrow K_c=0.169$$

$$t/t_I=(0.674)(1/2)^{-0.680}=1.08$$

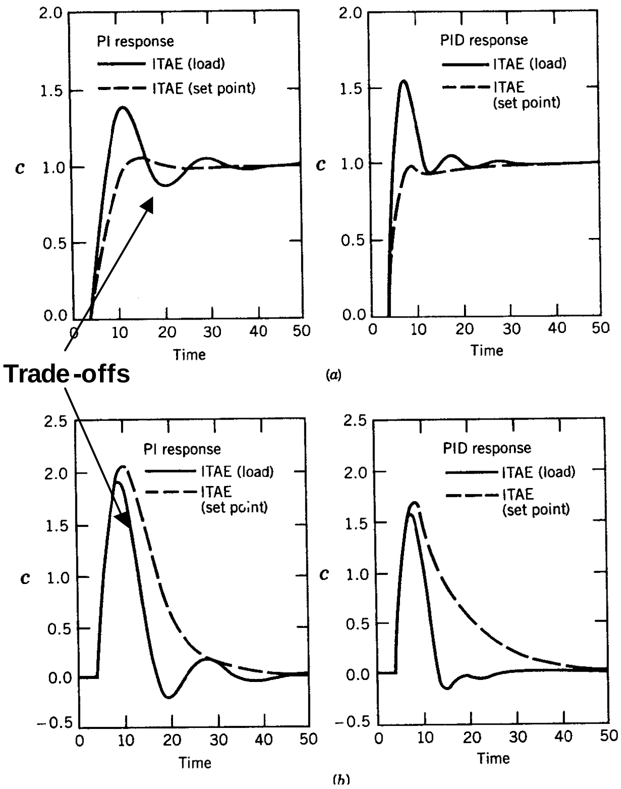
$$\Rightarrow t_I=1.85$$



Method	K_c	t_I
IAE	0.195	2.02
ISE	0.245	2.44
ITAE	0.169	1.85

Example2

$$G(s)=\frac{4e^{-3.5s}}{7s+1}$$



- **Design relations based on process reaction curve**
  - **For the processes who have sigmoidal shape step responses**

    (Not for underdamped processes)
  - **Fit the curve with FOPDT model**

$$G(s)=\frac{Ke^{-qs}}{(ts+1)} \qquad S=K\Delta u/t \qquad S^*=S/\Delta u=K/t$$

Table 13.3 Ziegler–Nichols Tuning Relations (Process Reaction Curve Method)

Controller Type	K_c	τ_I	τ_D
P	$\frac{1}{\theta S^*}$	—	—
PI	$\frac{0.9}{\theta S^*}$	3.33θ	—
PID	$\frac{1.2}{\theta S^*}$	2θ	0.5θ

- **Very simple**
- **Inherits all the problems of FOPDT model fitting**

DIRECT SYNTHESIS METHOD

- **Analysis: Given** $G_c(s)$**, what is** $y(t)$**?**
- **Design: Given** $y_d(t)$**, what should** $G_c(s)$ **be?**
- **Derivation**

Let $G_{OL} = K_m G_c G_v G_p \triangleq G_c G$

$$\frac{Y(s)}{R(s)} = \frac{G_{OL}}{1+G_{OL}} = \frac{G_c G}{1+G_c G} \quad \Rightarrow \quad G_c = \frac{1}{G}\left(\frac{Y/R}{1-Y/R}\right)$$

Specify $(Y/R)_d \quad \Rightarrow \quad G_c = \frac{1}{G}\left(\frac{(Y/R)_d}{1-(Y/R)_d}\right)$

- **If** $(Y/R)_d = 1$**, then it implies perfect control. (infinite gain)**
- **The resulting controller may not be physically realizable**
- **Or, not in PID form and too complicated.**
- **Design with finite settling time:** $(Y/R)_d = \frac{1}{t_c s+1}$

- **Examples**

1. Perfect control (K_c **becomes infinite)**

$$G(s) = \frac{K}{(t_1 s+1)(t_2 s+1)} \quad \text{and} \quad (Y/R)_d = 1$$

$$G_c(s) = \frac{1}{G(s)}\left(\frac{1}{1\text{-}1}\right) = \frac{\infty}{G(s)} \text{ (infinite gain, unrealizable)}$$

2. Finite settling time for 1st**-order process**

$$G(s) = \frac{K}{(t s+1)} \quad \text{and} \quad (Y/R)_d = \frac{1}{t_c s+1}$$

$$G_c(s) = \frac{1}{G(s)}\left(\frac{1/(t_c s+1)}{1\text{-}1/(t_c s+1)}\right) = \frac{t s+1}{K t_c s} = \frac{t}{t_c K}\left(1+\frac{1}{t s}\right) \text{ (PI)}$$

3. Finite settling time for 2nd**-order process**

$$G(s) = \frac{K}{(t_1 s+1)(t_2 s+1)} \quad \text{and} \quad (Y/R)_d = \frac{1}{t_c s+1}$$

$$G_c(s) = \frac{(t_1+t_2)}{t_c K}\left(1+\frac{1}{(t_1+t_2)s}+\frac{t_1 t_2}{(t_1+t_2)}s\right) \text{ (PID)}$$

- **Process with time delay**
  - **If there is a time delay, any physically realizable controller cannot overcome the time delay. (Need time lead)**
  - **Given circumstance, a reasonable choice will be**

$$\left(Y/R\right)_d = \frac{e^{-q_c s}}{t_c s+1}$$

  - **Examples**

1. $G(s) = \dfrac{Ke^{-qs}}{(t\,s+1)}$ and $(Y/R)_d = \dfrac{e^{-qs}}{t_c s+1}$ $(q_c = q)$

$$G_c(s) = \frac{1}{G(s)}\left(\frac{e^{-qs}/(t_c s+1)}{1 - e^{-qs}/(t_c s+1)}\right) = \frac{t\,s+1}{K}\boxed{\frac{1}{t_c s+1-e^{-qs}}} \text{ (not a PID)}$$

Physically realizable

2. With 1st-order Taylor series approx. ($e^{-qs} \approx 1 - qs$ **)**

$$G_c(s) = \frac{t\,s+1}{K}\frac{1}{(t_c+q)s} = \frac{t}{K(t_c+q)}\left(1+\frac{1}{t\,s}\right) \text{ (PI)}$$

3. $G(s) = \dfrac{Ke^{-qs}}{(t_1 s+1)(t_2 s+1)}$ and $(Y/R)_d = \dfrac{e^{-qs}}{t_c s+1}$ $(q_c = q)$

$$G_c(s) = \frac{(t_1 s+1)(t_2 s+1)}{K}\frac{1}{(t_c+q)s} = \frac{(t_1+t_2)}{K(t_c+q)}\left(1+\frac{1}{(t_1+t_2)s}+\frac{t_1 t_2}{(t_1+t_2)}s\right) \text{ (PID)}$$

- **Observations on Direct Synthesis Method**
  - **Resulting controllers could be quite complex and may not even be physically realizable.**
  - **PID parameters will be decided by a user-specified parameter: The desired closed-loop time constant (** t_c **)**
  - **The shorter** t_c **makes the action more aggressive. (larger** K_c**)**
  - **The longer** t_c **makes the action more conservative. (smaller** K_c**)**
  - **For a limited cases, it results PID form.**
    - **1st-order model without time delay: PI**
    - **FOPDT with 1st-order Taylor series approx.: PI**
    - **2nd-order model without time delay: PID**
    - **SOPDT with 1st-order Taylor series approx.: PID**
    - **Delay modifies the** K_c**.**

$$\frac{t}{Kt_c} \rightarrow \frac{t}{K(t_c+q)} \text{ (1st order)} \qquad \frac{(t_1+t_2)}{Kt_c} \rightarrow \frac{(t_1+t_2)}{K(t_c+q)} \text{ (2nd order)}$$

    - **With time delay, the** K_c **will not become infinite even for the perfect control (*Y/R*=1).**

INTERNAL MODEL CONTROL (IMC)

- **Motivation**
  - **The resulting controller from direct synthesis method may not be physically unrealizable.**
  - **If there is RHP zero in the process, the resulting controller from direct synthesis method will be unstable.**
  - **Unmeasured disturbance and modeling error are not considered in direct synthesis method.**
- **Source of trouble**
  - **From direct synthesis method**

$$G_c = \boxed{\frac{1}{G}\left(\frac{(Y/R)_d}{1-(Y/R)_d}\right)}$$

Resulting controller may have higher-order numerator than denominator

Direct inversion of process causes many problems

Process is unknown

- **IMC**
  - **Feedback the error between the process output and model output.**
  - **Equivalent conventional controller:** $G_c = \dfrac{G_c^*}{1-G_c^*\tilde{G}}$
  - **Using block diagram algebra**

$$C = GP + L \qquad P = G_c^* E \qquad E = R - (C - \tilde{C}) = R - C + \tilde{G}P$$

$$P = G_c^*(R - C + \tilde{G}P)$$

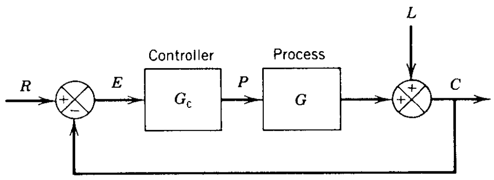
$$\Rightarrow \quad P = G_c^*(R-C)/(1-G_c^*\tilde{G})$$

$$C = GG_c^*(R-C)/(1-G_c^*\tilde{G}) + L$$

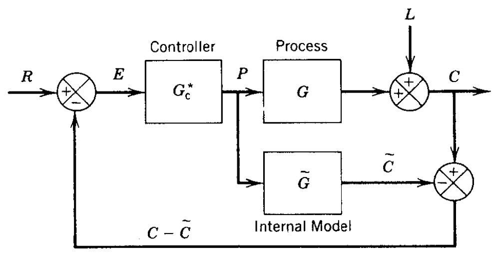
$$(1 + GG_c^* - G_c^*\tilde{G})C = GG_c^* R + (1 - G_c^*\tilde{G})L$$

$$C = \frac{G_c^* G}{1+G_c^*(G-\tilde{G})}R + \frac{(1-G_c^*\tilde{G})}{1+G_c^*(G-\tilde{G})}L$$

If $\tilde{G} = G$, $C = G_c^* G R + (1 - G_c^* G)L$



(a) Classical feedback control



(b) Internal Model Control

- **IMC design strategy**
  - **Factor the process model as**

$$\tilde{G} = \tilde{G}_{+}\tilde{G}_{-}$$

Uninvertibles

- $\tilde{G}_{+}$ **contains any time delays and RHP zeros and is specified so that the steady-state gain is one**
- $\tilde{G}_{-}$ **is the rest of** G**.**

  - **The controller is specified as**

$$G_c^* = \frac{1}{\tilde{G}_{-}} f$$

- **IMC filter** f **is a low-pass filter with steady-state gain of one**
- **Typical IMC filter:** $f = \dfrac{1}{(t_c s + 1)^r}$
- **The** t_c **is the desired closed-loop time constant and parameter** r **is a positive integer that is selected so that the order of numerator of** G_c^* **is same as the order of denominator or exceeds the order of denominator by one.**

- **Example**
  - **FOPDT model with 1/1 Pade approximation**

$$\tilde{G} = \frac{K(1 - q s/2)}{(1 + q s/2)(t s + 1)}$$

$$\tilde{G}_{+} = 1 - q s/2 \qquad \tilde{G}_{-} = \frac{K}{(1 + q s/2)(t s + 1)}$$

$$G_c^* = \frac{1}{\tilde{G}_{-}} f = \frac{(1 + q s/2)(t s + 1)}{K} \frac{1}{(t_c s + 1)}$$

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} = \frac{(1 + q s/2)(t s + 1)}{K(t_c + q/2)s} \quad \text{(PID)}$$

$$K_c = \frac{1}{K} \frac{(t + q/2)}{(t_c + q/2)} \qquad t_I = t + q/2 \qquad t_D = \frac{t q/2}{t + q/2}$$

IMC based PID controller settings

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ [4][a]

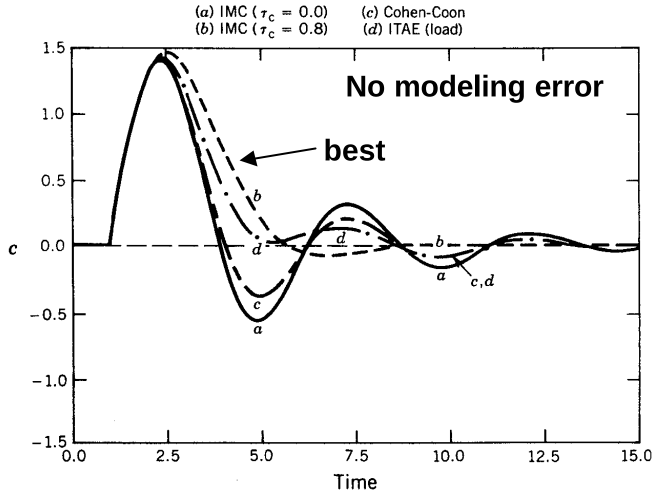
Case	*Model*	K_cK	τ_I	τ_D
A	$\frac{K}{\tau s + 1}$	$\frac{\tau}{\tau_c}$	τ	—
B	$\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\tau_1 + \tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\frac{\tau_1\tau_2}{\tau_1 + \tau_2}$
C	$\frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}, \beta > 0$	$\frac{2\zeta\tau}{\tau_c + \beta}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{1}{\tau_c}$	—	—
F	$\frac{K}{s(\tau s + 1)}$	$\frac{1}{\tau_c}$	—	τ

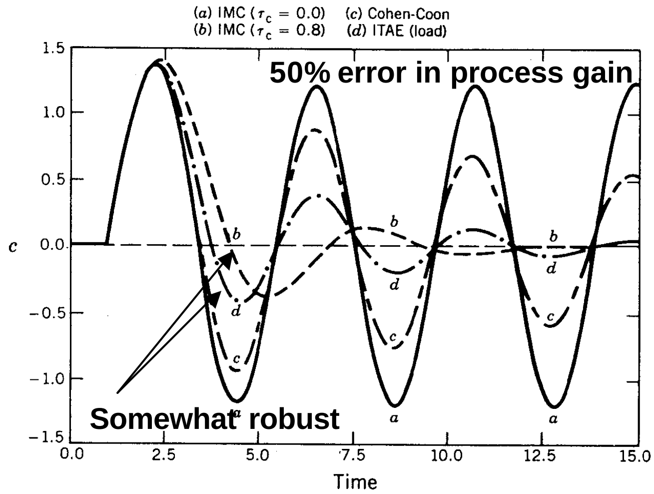
[a]Based on Eq. 12-30 with $r = 1$.

COMPARISON OF CONTROLLER DESIGN RELATIONS

- **PI controller settings for different methods**

$$G(s) = \frac{2e^{-s}}{s+1}$$





EFFECT OF MODELING ERROR

- **Actual plant**

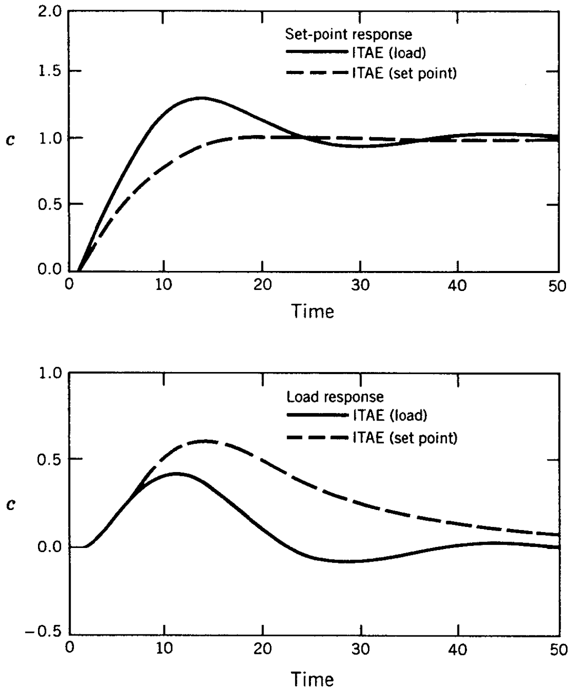
$$G(s) = \frac{2e^{-s}}{(10s+1)(5s+1)}$$

- **Approx. model**

$$\tilde{G}(s) = \frac{2e^{-4.7s}}{12s+1}$$

  - **Satisfactory for this case**
  - **Use with care**

As the estimated time delay gets smaller, the performance degradation will be pronounced.



- **All kinds of tuning method should be used for initial setting and fine tuning should be done!!**

GENERAL CONCLUSION FOR PID TUNING

- **The controller gain should be inversely proportional to the products of the other gains in the feedback loop.**
- **The controller gain should decrease as the ratio of time delay to dominant time constant increases.**
- **The larger the ratio of time delay to dominant time constant is, the harder the system is to control.**
- **The reset time and the derivative time should increase as the ratio of time delay to dominant time constant increases.**
- **The ratio between derivative time and reset time is typically between 0.1 to 0.3.**
- **The ¼ decay ratio is too oscillatory for process control. If less oscillatory response is desired, the controller gain should decrease and reset time should increase.**
- **Among IAE, ISE and ITAE, ITAE is the most conservative and ISE is the least conservative setting.**