CHE302 LECTURE XI
CONTROLLER DESIGN AND PID CONTOLLER TUNING

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CONTROLLER DESIGN

- **Performance criteria for closed-loop systems**
  - **Stable**
  - **Minimal effect of disturbance**
  - **Rapid, smooth response to set point change**
  - **No offset**
  - **No excessive control action**
  - **Robust to plant-model mismatch**

$$\min_{K_c, t_I, t_D} \int_0^\infty (w_1 e^2(t) + w_2 \Delta u^2(t))dt$$

- **Trade-offs in control problems**
  - **Set point tracking vs. disturbance rejection**
  - **Robustness vs. performance**

GUIDELINES FOR COMMON CONTROL LOOPS

- **Flow and liquid pressure control**
  - **Fast response with no time delay**
  - **Usually with small high-frequency noise**
  - **PI controller with intermediate controller gain**
- **Liquid level control**
  - **Noisy due to splashing and turbulence**
  - **High gain PI controller for integrating process**
  - **Conservative setting for averaging control when it is used for damping the fluctuation of the inlet stream**
- **Gas pressure control**
  - **Usually fast and self regulating**
  - **PI controller with small integral action (large reset time)**

- **Temperature control**
  - **Wide variety of the process nature**
  - **Usually slow response with time delay**
  - **Use PID controller to speed up the response**

- **Composition control**
  - **Similar to temperature control usually with larger noise and more time delay**
  - **Effectiveness of derivative action is limited**
  - **Temperature and composition controls are the prime candidates for advance control strategies due to its importance and difficulty of control**

TRIAL AND ERROR TUNING

- **Step1: With P-only controller**
  - **Start with low K_c value and increase it until the response has a sustained oscillation (continuous cycling) for a small set point or load change. (K_{cu})**
  - **Set $K_c = K_{cu}$.**
- **Step2: Add I mode**
  - **Decrease the reset time until sustained oscillation occurs. (t_{Iu})**
  - **Set $t_I = 3t_{Iu}$.**
  - **If a further improvement is required, proceed to Step 3.**
- **Step3: Add D mode**
  - **Decrease the reset time until sustained oscillation occurs. (t_{Du})**
  - **Set $t_D = 3t_{Du}$.**

(The sustained oscillation should not be cause by the controller saturation)

CONTINUOUS CYCLING METHOD

- **Also called as loop tuning or ultimate gain method**
  - **Increase controller gain until sustained oscillation**
  - **Find ultimate gain (K_{CU}) and ultimate period (P_{CU})**
- **Ziegler-Nichols controller setting**
  - **¼ decay ratio (too much oscillatory)**

Controller	K_C	t_I	t_D
P	$0.5K_{CU}$	-	-
PI	$0.45K_{CU}$	$P_{CU}/1.2$	-
PID	$0.6K_{CU}$	$P_{CU}/2$	$0.5P_{CU}/8$

  - **Modified Ziegler-Nichols setting**

Controller	K_C	t_I	t_D
Original	$0.6K_{CU}$	$P_{CU}/2$	$P_{CU}/8$
Some overshoot	$0.33K_{CU}$	$P_{CU}/2$	$P_{CU}/3$
No overshoot	$0.2K_{CU}$	$P_{CU}/2$	$P_{CU}/3$

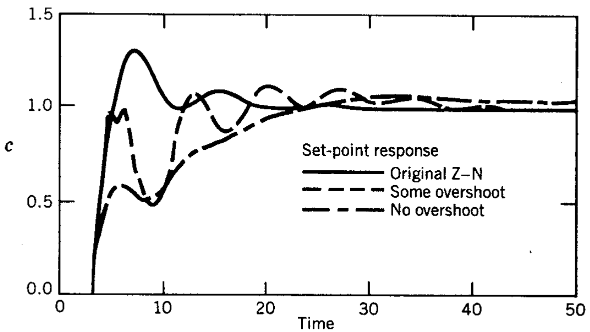
• Examples

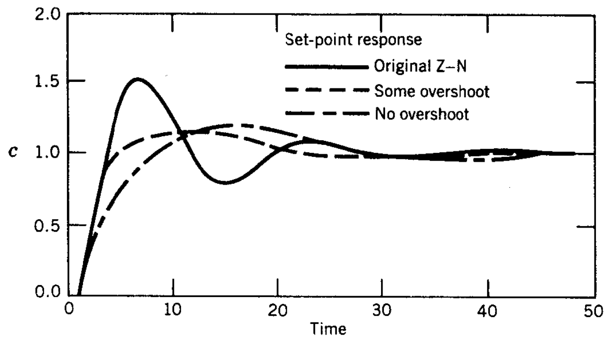
$$G_p(s)=\frac{4e^{-3.5s}}{7s+1} \qquad K_{CU}=0.95 \qquad P_{CU}=12$$

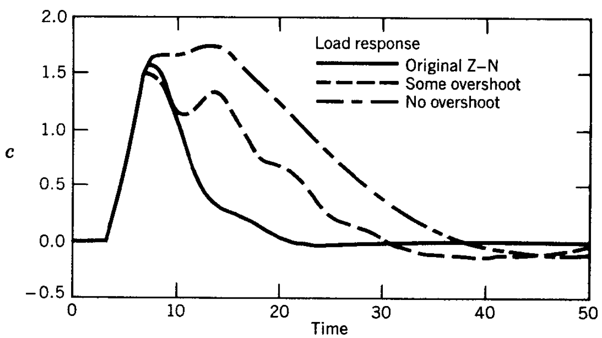
$$G_p(s)=\frac{2e^{-s}}{(10s+1)(5s+1)} \qquad K_{CU}=7.88 \qquad P_{CU}=11.6$$

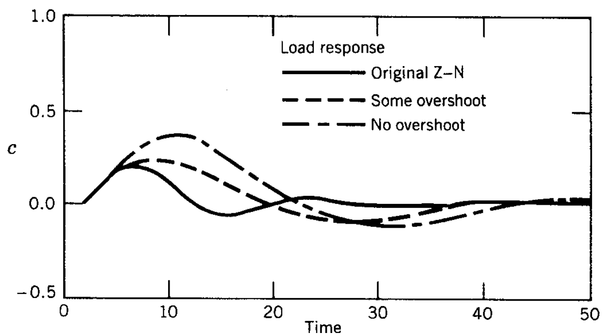
Controller	K_C	t_I	t_D
Original	0.57	6.0	*1.5*
Some overshoot	0.31	6.0	*4.0*
No overshoot	0.19	6.0	*4.0*

Controller	K_C	t_I	t_D
Original	4.73	5.8	*1.45*
Some overshoot	2.60	5.8	*3.87*
No overshoot	1.58	5.8	*3.87*









- **Advantages of continuous cycling method**
  - No a priori information on process required
  - Applicable to all stable processes

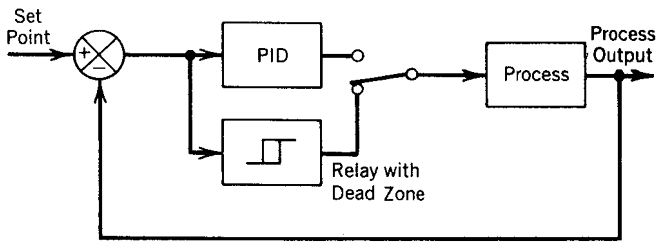
- **Disadvantages of continuous cycling method**
  - Time consuming
  - Loss of product quality and productivity during the tests
  - Continuous cycling may cause the violation of process limitation and safety hazards
  - Not applicable to open-loop unstable process
  - First-order and second-order process without time delay will not oscillate even with very large controller gain

=> Motivates Relay feedback method. (Astrom and Wittenmark)

RELAY FEEDBACK METHOD

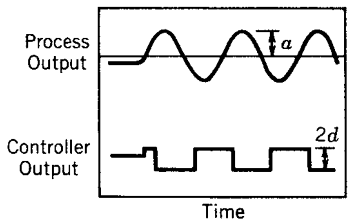
- **Relay feedback controller**
  - **Forces the system to oscillate by a relay controller**
  - **Require a single closed-loop experiment to find the ultimate frequency information**
  - **No *a priori* information on process is required**
  - **Switch relay feedback controller for tuning**
  - **Find P_{CU} and calculate K_{CU}**

$$K_{CU} = \frac{4d}{p\,a}$$



  - **User specified parameter: d**

Decide d in order not to perturb the system too much.



  - **Use Ziegler-Nichols Tuning rules for PID tuning parameters**

DESIGN RELATIONS FOR PID CONTROLLERS

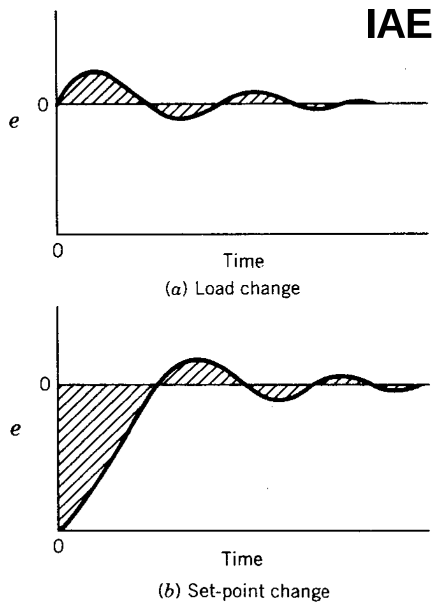
- **Cohen-Coon controller design relations**
  - **Empirical relation for ¼ decay ratio for FOPDT model**

Table 12.2 Cohen and Coon Controller Design Relations

Controller	*Settings*	*Cohen–Coon*
P	K_c	$\frac{1}{K}\frac{\tau}{\theta}[1 + \theta/3\tau]$
PI	K_c	$\frac{1}{K}\frac{\tau}{\theta}[0.9 + \theta/12\tau]$
	τ_I	$\frac{\theta[30 + 3(\theta/\tau)]}{9 + 20(\theta/\tau)}$
PID	K_c	$\frac{1}{K}\frac{\tau}{\theta}\left[\frac{16\tau + 3\theta}{12\tau}\right]$
	τ_I	$\frac{\theta[32 + 6(\theta/\tau)]}{13 + 8(\theta/\tau)}$
	τ_D	$\frac{4\theta}{11 + 2(\theta/\tau)}$

- **Design relations based on integral error criteria**
  - ¼ decay ratio is too oscillatory
  - Decay ratio concerns only two peak points of the response
  - IAE: Integral of the Absolute Error

$$\text{IAE} = \int_0^\infty |e(t)|\, dt$$



  - ISE: Integral of the Square Error

$$\text{ISE} = \int_0^\infty [e(t)]^2\, dt$$

    - Large error contributes more
    - Small error contributes less
    - Large penalty for large overshoot
    - Small penalty for small persisting oscillation
  - ITAE: Integral of the Time-weighted Absolute Error

$$\text{ITAE} = \int_0^\infty t\,|e(t)|\, dt$$

    - Large penalty for persisting oscillation
    - Small penalty for initial transient response

- **Controller design relation based on ITAE for FOPDT model**

Table 12.3 Controller Design Relations Based on the ITAE Performance Index and a First-Order plus Time-Delay Model [6–8][a]

Type of Input	*Type of Controller*	*Mode*	*A*	*B*
Load	PI	P	0.859	−0.977
		I	0.674	−0.680
Load	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a]Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.
[b]For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$. [8]

- **Similar design relations based on IAE and ISE for other types of models can be found in literatures.**

• Example1

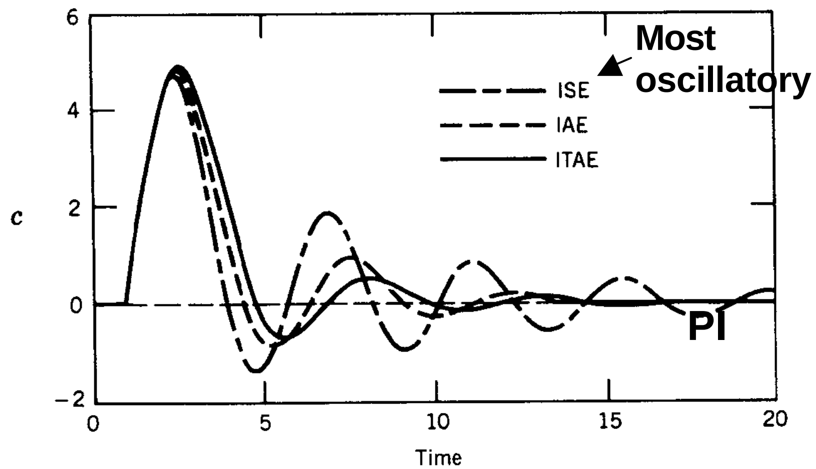
$$G(s) = \frac{10e^{-s}}{2s+1}$$

$$KK_c = (0.859)(1/2)^{-0.977} = 1.69$$

$$\Rightarrow K_c = 0.169$$

$$t / t_I = (0.674)(1/2)^{-0.680} = 1.08$$

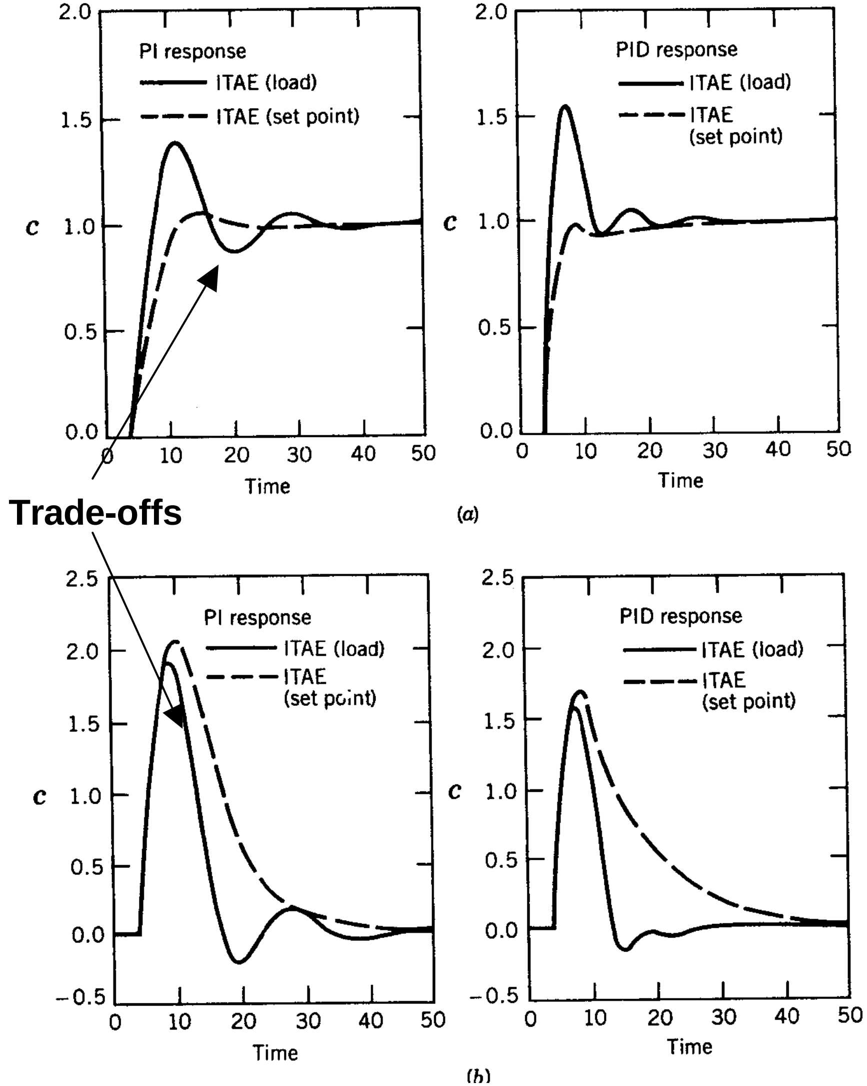
$$\Rightarrow t_I = 1.85$$



Method	K_c	t_I
IAE	**0.195**	**2.02**
ISE	**0.245**	**2.44**
ITAE	**0.169**	**1.85**

Example2

$$G(s) = \frac{4e^{-3.5s}}{7s+1}$$



- **Design relations based on process reaction curve**
  - **For the processes who have sigmoidal shape step responses**

    (Not for underdamped processes)
  - **Fit the curve with FOPDT model**

$$G(s)=\frac{Ke^{-qs}}{(ts+1)} \qquad S = K\Delta u/t \qquad S^{*}=S/\Delta u = K/t$$

Table 13.3 Ziegler–Nichols Tuning Relations (Process Reaction Curve Method)

Controller Type	K_c	τ_I	τ_D
P	$\frac{1}{\theta S^*}$	—	—
PI	$\frac{0.9}{\theta S^*}$	3.33θ	—
PID	$\frac{1.2}{\theta S^*}$	2θ	0.5θ

  - **Very simple**
  - **Inherits all the problems of FOPDT model fitting**

DIRECT SYNTHESIS METHOD

- **Analysis: Given** $G_c(s)$**, what is** $y(t)$**?**
- **Design: Given** $y_d(t)$**, what should** $G_c(s)$ **be?**
- **Derivation**

$$\text{Let } G_{OL} = K_m G_c G_v G_p \triangleq G_c G$$

$$\frac{Y(s)}{R(s)} = \frac{G_{OL}}{1+G_{OL}} = \frac{G_c G}{1+G_c G} \quad \Rightarrow \quad G_c = \frac{1}{G}\left(\frac{Y/R}{1-Y/R}\right)$$

$$\text{Specify } (Y/R)_d \quad \Rightarrow \quad G_c = \frac{1}{G}\left(\frac{(Y/R)_d}{1-(Y/R)_d}\right)$$

  - **If** $(Y/R)_d$ **= 1, then it implies perfect control. (infinite gain)**
  - **The resulting controller may not be physically realizable**
  - **Or, not in PID form and too complicated.**
  - **Design with finite settling time:** $(Y/R)_d = \dfrac{1}{t_c s + 1}$

- **Examples**

1. Perfect control (K_c becomes infinite)

$$G(s) = \frac{K}{(t_1 s + 1)(t_2 s + 1)} \quad \text{and} \quad (Y/R)_d = 1$$

$$G_c(s) = \frac{1}{G(s)} \left(\frac{1}{1\text{-}1} \right) = \frac{\infty}{G(s)} \text{ (infinite gain, unrealizable)}$$

2. Finite settling time for 1st-order process

$$G(s) = \frac{K}{(t s + 1)} \quad \text{and} \quad (Y/R)_d = \frac{1}{t_c s + 1}$$

$$G_c(s) = \frac{1}{G(s)} \left(\frac{1/(t_c s + 1)}{1\text{-}1/(t_c s + 1)} \right) = \frac{t s + 1}{K t_c s} = \frac{t}{t_c K} \left(1 + \frac{1}{t s} \right) \text{ (PI)}$$

3. Finite settling time for 2nd-order process

$$G(s) = \frac{K}{(t_1 s + 1)(t_2 s + 1)} \quad \text{and} \quad (Y/R)_d = \frac{1}{t_c s + 1}$$

$$G_c(s) = \frac{(t_1 + t_2)}{t_c K} \left(1 + \frac{1}{(t_1 + t_2) s} + \frac{t_1 t_2}{(t_1 + t_2)} s \right) \text{ (PID)}$$

- **Process with time delay**
  - **If there is a time delay, any physically realizable controller cannot overcome the time delay. (Need time lead)**
  - **Given circumstance, a reasonable choice will be**

$$\left(Y/R\right)_d = \frac{e^{-q_c s}}{t_c s+1}$$

  - **Examples**

1. $G(s) = \dfrac{Ke^{-qs}}{(t s+1)}$ and $(Y/R)_d = \dfrac{e^{-qs}}{t_c s+1}$ $(q_c = q)$

Physically realizable

$$G_c(s) = \frac{1}{G(s)}\left(\frac{e^{-qs}/(t_c s+1)}{1-e^{-qs}/(t_c s+1)}\right) = \frac{t s+1}{K}\boxed{\frac{1}{t_c s+1-e^{-qs}}} \text{ (not a PID)}$$

2. With 1st-order Taylor series approx. ($e^{-qs} \approx 1-qs$ **)**

$$G_c(s) = \frac{t s+1}{K}\frac{1}{(t_c+q)s} = \frac{t}{K(t_c+q)}\left(1+\frac{1}{t s}\right) \text{ (PI)}$$

3. $G(s) = \dfrac{Ke^{-qs}}{(t_1 s+1)(t_2 s+1)}$ and $(Y/R)_d = \dfrac{e^{-qs}}{t_c s+1}$ $(q_c = q)$

$$G_c(s) = \frac{(t_1 s+1)(t_2 s+1)}{K}\frac{1}{(t_c+q)s} = \frac{(t_1+t_2)}{K(t_c+q)}\left(1+\frac{1}{(t_1+t_2)s}+\frac{t_1 t_2}{(t_1+t_2)}s\right) \text{ (PID)}$$

- **Observations on Direct Synthesis Method**
  - **Resulting controllers could be quite complex and may not even be physically realizable.**
  - **PID parameters will be decided by a user-specified parameter: The desired closed-loop time constant (t_c)**
  - **The shorter t_c makes the action more aggressive. (larger K_c)**
  - **The longer t_c makes the action more conservative. (smaller K_c)**
  - **For a limited cases, it results PID form.**
    - **1st-order model without time delay: PI**
    - **FOPDT with 1st-order Taylor series approx.: PI**
    - **2nd-order model without time delay: PID**
    - **SOPDT with 1st-order Taylor series approx.: PID**
    - **Delay modifies the K_c.**

$$\frac{t}{Kt_c} \rightarrow \frac{t}{K(t_c+q)} \text{ (1st order)} \qquad \frac{(t_1+t_2)}{Kt_c} \rightarrow \frac{(t_1+t_2)}{K(t_c+q)} \text{ (2nd order)}$$

    - **With time delay, the K_c will not become infinite even for the perfect control (Y/R=1).**

INTERNAL MODEL CONTROL (IMC)

- **Motivation**
  - The resulting controller from direct synthesis method may not be physically unrealizable.
  - If there is RHP zero in the process, the resulting controller from direct synthesis method will be unstable.
  - Unmeasured disturbance and modeling error are not considered in direct synthesis method.
- **Source of trouble**
  - From direct synthesis method

$$G_c = \boxed{\frac{1}{G}\left(\frac{(Y/R)_d}{1-(Y/R)_d}\right)}$$

Resulting controller may have higher-order numerator than denominator

Direct inversion of process causes many problems

Process is unknown

IMC

- Feedback the error between the process output and model output.
- Equivalent conventional controller: $$G_c = \frac{G_c^*}{1-G_c^*\tilde{G}}$$
- Using block diagram algebra

$$C = GP + L \qquad P = G_c^* E \qquad E = R-(C-\tilde{C}) = R - C + \tilde{G}P$$

$$P = G_c^*(R - C + \tilde{G}P)$$

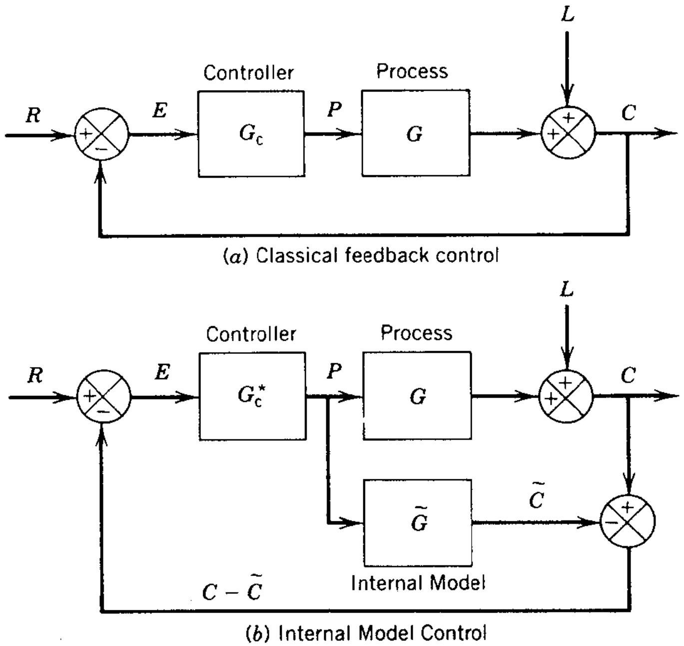
$$\Rightarrow \quad P = G_c^*(R-C)/(1-G_c^*\tilde{G})$$

$$C = GG_c^*(R-C)/(1-G_c^*\tilde{G}) + L$$

$$(1+GG_c^* - G_c^*\tilde{G})C = GG_c^*R + (1-G_c^*\tilde{G})L$$

$$C = \frac{G_c^*G}{1+G_c^*(G-\tilde{G})}R + \frac{(1-G_c^*\tilde{G})}{1+G_c^*(G-\tilde{G})}L$$

If $\tilde{G} = G$, $C = G_c^*GR + (1-G_c^*G)L$



- **IMC design strategy**
  - **Factor the process model as**

$$\tilde{G} = \tilde{G}_{+}\tilde{G}_{-}$$

Uninvertibles

    - $\tilde{G}_{+}$ **contains any time delays and RHP zeros and is specified so that the steady-state gain is one**
    - $\tilde{G}_{-}$ **is the rest of** G**.**
  - **The controller is specified as**

$$G_c^* = \frac{1}{\tilde{G}_{-}} f$$

    - **IMC filter** f **is a low-pass filter with steady-state gain of one**
    - **Typical IMC filter:**

$$f = \frac{1}{(t_c s + 1)^r}$$

    - **The** t_c **is the desired closed-loop time constant and parameter** r **is a positive integer that is selected so that the order of numerator of** G_c^* **is same as the order of denominator or exceeds the order of denominator by one.**

- **Example**
  - **FOPDT model with 1/1 Pade approximation**

$$\tilde{G} = \frac{K(1 - q s/2)}{(1 + q s/2)(t s + 1)}$$

$$\tilde{G}_{+} = 1 - q s/2 \qquad \tilde{G}_{-} = \frac{K}{(1 + q s/2)(t s + 1)}$$

$$G_c^{*} = \frac{1}{\tilde{G}_{-}} f = \frac{(1 + q s/2)(t s + 1)}{K} \frac{1}{(t_c s + 1)}$$

$$G_c = \frac{G_c^{*}}{1 - G_c^{*}\tilde{G}} = \frac{(1 + q s/2)(t s + 1)}{K(t_c + q/2)s} \quad \text{(PID)}$$

$$K_c = \frac{1}{K}\frac{(t + q/2)}{(t_c + q/2)} \qquad t_I = t + q/2 \qquad t_D = \frac{tq/2}{t + q/2}$$

IMC based PID controller settings

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ [4][a]

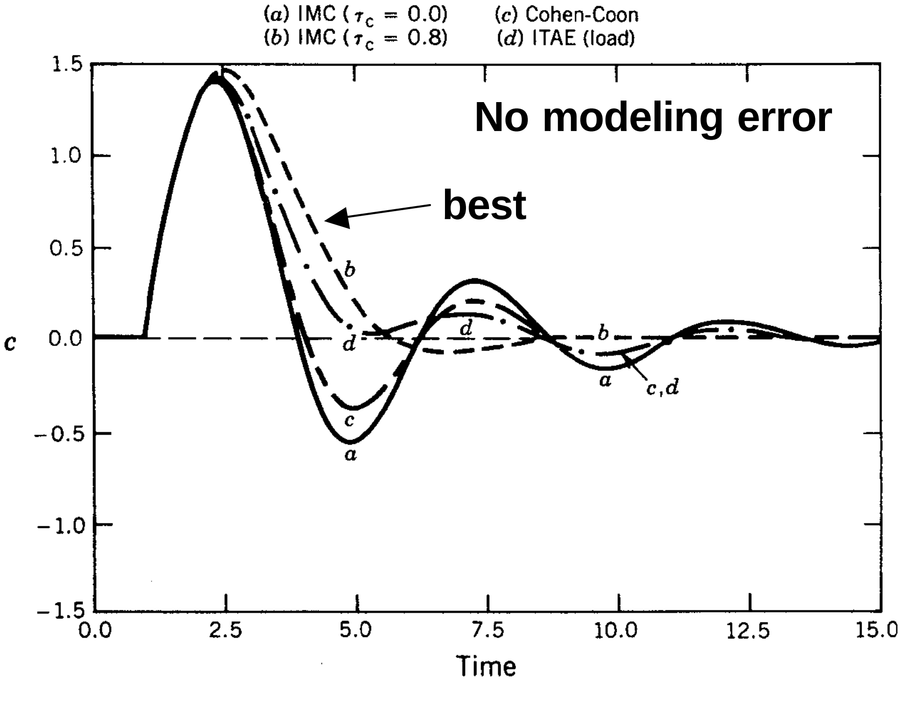
Case	*Model*	$K_c K$	τ_I	τ_D
A	$\dfrac{K}{\tau s + 1}$	$\dfrac{\tau}{\tau_c}$	τ	—
B	$\dfrac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\dfrac{\tau_1 + \tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\dfrac{\tau_1 \tau_2}{\tau_1 + \tau_2}$
C	$\dfrac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\dfrac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
D	$\dfrac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}, \beta > 0$	$\dfrac{2\zeta\tau}{\tau_c + \beta}$	$2\zeta\tau$	$\dfrac{\tau}{2\zeta}$
E	$\dfrac{K}{s}$	$\dfrac{1}{\tau_c}$	—	—
F	$\dfrac{K}{s(\tau s + 1)}$	$\dfrac{1}{\tau_c}$	—	τ

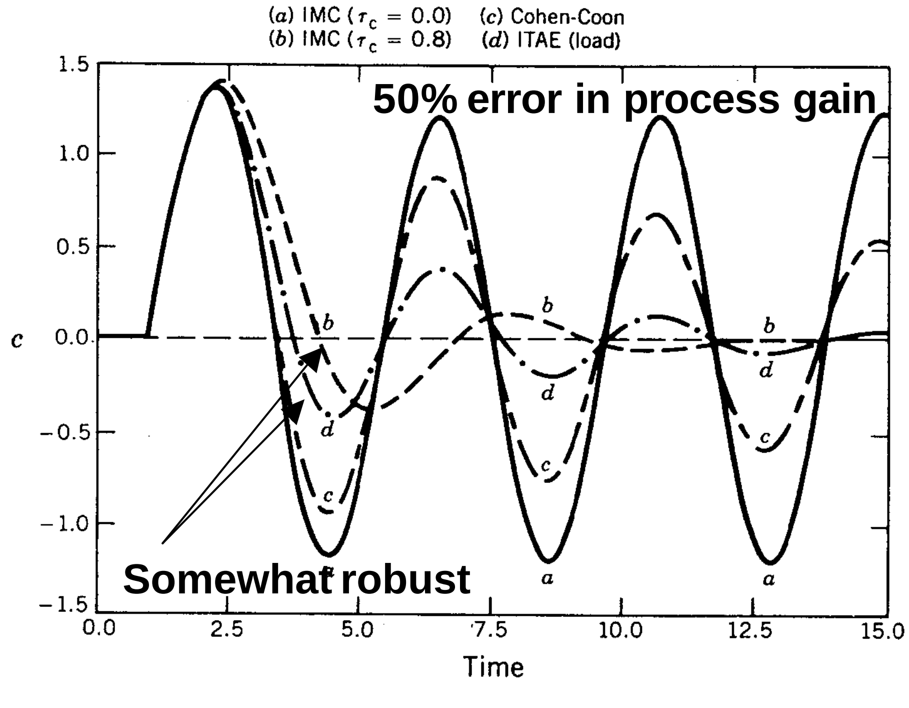
[a]Based on Eq. 12-30 with $r = 1$.

COMPARISON OF CONTROLLER DESIGN RELATIONS

- **PI controller settings for different methods**

$$G(s) = \frac{2e^{-s}}{s+1}$$





EFFECT OF MODELING ERROR

- **Actual plant**

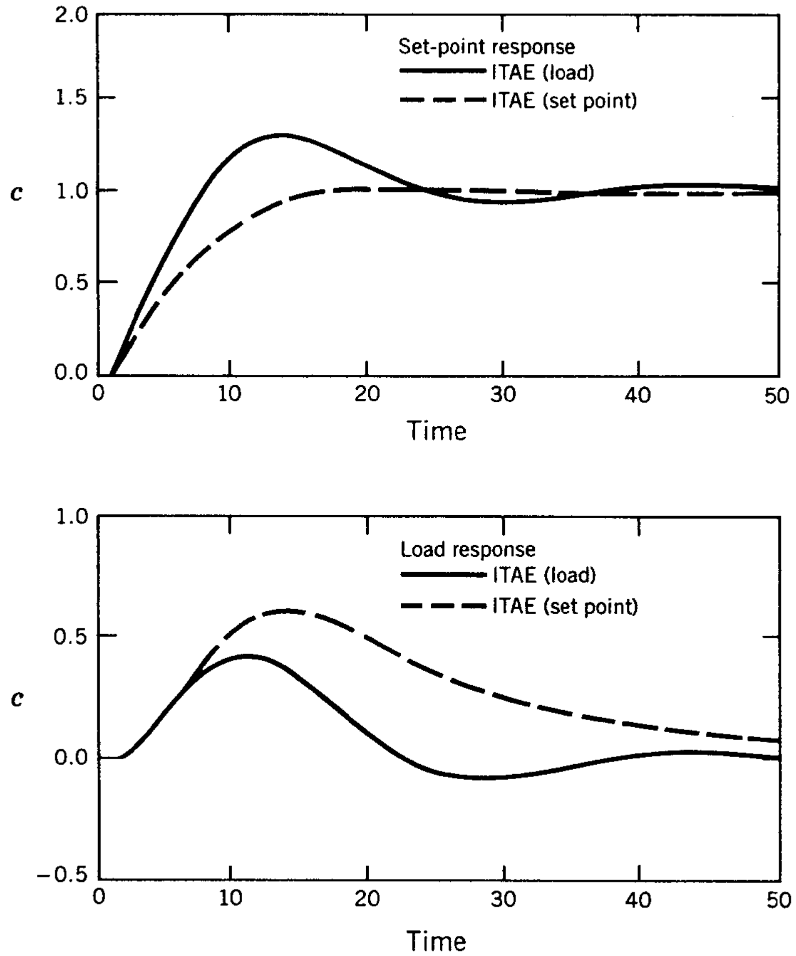
$$G(s)=\frac{2e^{-s}}{(10s+1)(5s+1)}$$

- **Approx. model**

$$\tilde{G}(s)=\frac{2e^{-4.7s}}{12s+1}$$

  - **Satisfactory for this case**
  - **Use with care**

As the estimated time delay gets smaller, the performance degradation will be pronounced.



- **All kinds of tuning method should be used for initial setting and fine tuning should be done!!**

GENERAL CONCLUSION FOR PID TUNING

- **The controller gain should be inversely proportional to the products of the other gains in the feedback loop.**
- **The controller gain should decrease as the ratio of time delay to dominant time constant increases.**
- **The larger the ratio of time delay to dominant time constant is, the harder the system is to control.**
- **The reset time and the derivative time should increase as the ratio of time delay to dominant time constant increases.**
- **The ratio between derivative time and reset time is typically between 0.1 to 0.3.**
- **The ¼ decay ratio is too oscillatory for process control. If less oscillatory response is desired, the controller gain should decrease and reset time should increase.**
- **Among IAE, ISE and ITAE, ITAE is the most conservative and ISE is the least conservative setting.**