Chapter 17. Continuum Mechanics

- Three -Dimensional stress and strain
- · Mechanical properties : deformation under loading behavoir
- · Deformation : geometrical shape, the way of load application .

History (sample preparation)

Rate of application load(frequency, time)

 stress existing at any point in a material-three arbitrary coordinate directions



· First subscript : direction of normal to the plane on which the stress acts

- · Second subscript : the direction of force itself (stress)
- \cdot Three normal stresses $\tau_{\scriptscriptstyle 11},~\tau_{\scriptscriptstyle 22},~\tau_{\scriptscriptstyle 33}$
- \cdot Six shear stresses $\tau_{\rm 12}$, $\tau_{\rm 13}$, $\tau_{\rm 21}$, $\tau_{\rm 23}$, $\tau_{\rm 31}$, $\tau_{\rm 32}$
- stress tensor These nine quantities, necessary for specifying the state of stress at a point completely, are the components of the stress

tensor

$$\mathcal{I} = \begin{vmatrix}
 Z_{11} & \mathcal{I}_{12} & \mathcal{I}_{13} \\
 Z_{21} & \mathcal{I}_{22} & \mathcal{I}_{23} \\
 Z_{31} & \mathcal{I}_{32} & \mathcal{I}_{33}
 \end{aligned}$$

• The goal of continuum mechanics - to develop general constitutive relations (between the stress and rate of strain) and use them for predicting material response in the widest variety of situations.

· For equilibrium viscometric flows of incompressible fluids (assumption)

- There are two independent differences of the normal stresses in shear flows.

 $\sigma_1 = \tau_{11} - \tau_{22}$ (first normal stress differences)

 $\sigma_{\scriptscriptstyle 2}$ = $\tau_{\scriptscriptstyle 22}$ - $\tau_{\scriptscriptstyle 33}$ (second normal stress differences)

 Fluid moves along one coordinate direction only and its velocity varies only in one other coordinate direction :

1 direction - the direction of fluid velocity

2 direction - the direction of velocity variation

3 direction - the remaining neutral direction

· For polymeric fluids :

 $|-\sigma_1| > |+\sigma_2|$

 the first normal stress difference is practically always negative and numerically much larger than the second normal stress difference polymeric fluids exhibit an <u>extra tension</u> aling the streamlines, (that is, in the 1 direction), in addition to the shear stresses.



The extra tension arises from the stretching(random
 -> elasticity effect) and alignment of the polymer
 molecules along the streamlines - polymer molecules act as small
 'rubber bonds' wanting to snap back (Weissonberg's effect)

• The second normal stress difference is quite small - in a shear flow the fluid exhibits a small extra tension in the 3 direction.

(Ref. R.B.Bird et al,. Dynamics of Polymeric Liquid, Chapter 2, Vol 1)













ETA	N1	TORQUE	NORMAL	STRESS	RATE
POISE	DYNE/SQ CM	GM CM	GRAMS	DYNE/SQ CM	
4.143E+03	4.422E+02	1.728È-01	1.107E+00	4.143E+01	1.000E-02



FET Only, strain rate= 20%, Feb. 9, 1991

RATE SWEEP

G'	G"	ETA*	STRAIN	TORQUE	RATE
DYNE/SQ CM	DYNE/SQ CM	POISE	7.		RAD/SEC
1.533E+02	8.956E+02	8.988E+03	2.004E+01	7.511E-01	1.000E-01
1.569E+01	1.342E+03	8.466E+03	2.003E+01	1.121E+00	1.585E-01
0.000E+00	1.664E+03	6.625E+03	2.003E+01	1.394E+00	2.512E-01
0.000E+00	2.477E+03	6.223E+03	2.003E+01	2.070E+00	3.981E-01
5.592E+01	3.738E+03	5.925E+03	2.003E+01	3.124E+00	6.310E-01
3.309E+01	5.753E+03	5.753E+03/	2.011E+01	4.825E+00	1.000E+00
3.203E+02	8.789E+03	5.549E+03	2.004E+01	7.350E+00	1.585E+00
4.654E+02	1.348E+04	5.369E+03	2.003E+01	1.127E+01	2.512E+00
9.874E+02	2.034E+04	5.115E+03	2.003E+01	1.701E+01	3.981E+00
1.743E+03	3.079E+04	4.887E+03-	2.001E+01	2.574E+01	6.310E+00
3.241E+03	4.636E+04	4.647E+03	1.999E+01	3.874E+01	1.000E+01
5.988E+03	6.985E+04	4.424E+03	1.992E+01	5.826E+01	1.585E+01
1.103E+04	1.050E+05	4.201E+03	1.975E+01	8.692E+01	2.512E+01
2.045E+04	1.573E+05	3.985E+03	1.933E+01	1.279E+02	3.981E+01
3.624E+04	2.329E+05	3.735E+03	1.841E+01	1.810E+02	6.310E+01
6.163E+04	3.404E+05	3.460E+03	1.670E+01	2.410E+02	1.000E+02

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RATE SWEEP LAST RATE [1.000E+02] POINTS FER DECADE[5] 3 READINGS FER RATE[2] 1 AUTO SWEEP [Y]

TIME BEFORE MEASURE [5.000E-01] MEASURE TIME [5.000E-01]

G = Storage modulus G'' = Loss modulus $\eta^* = complex viscosity$