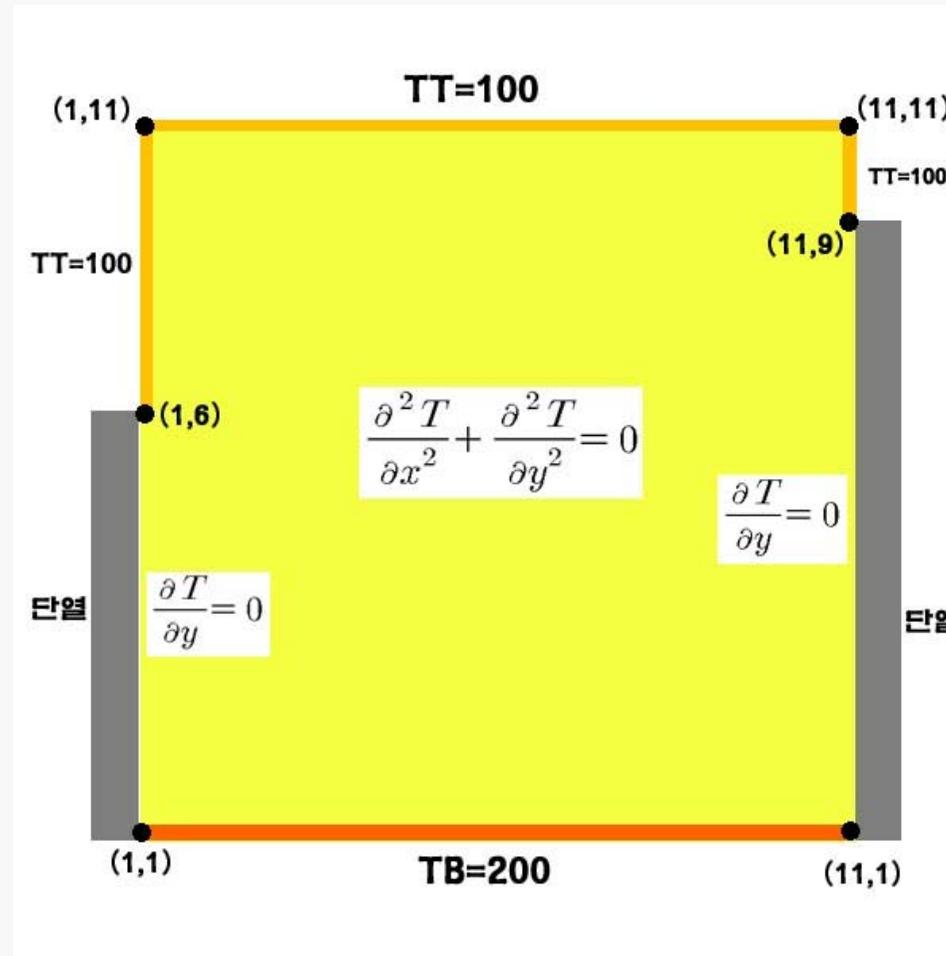




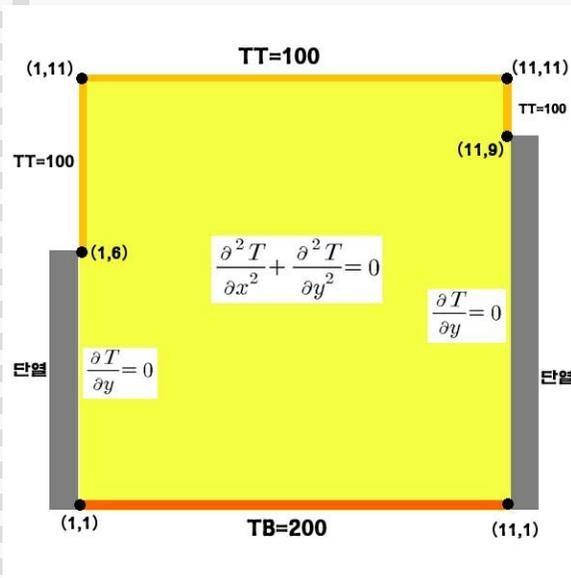
엑셀을 이용한 라플라스 방정식 풀기

Laplace equation



엑셀을 이용하여 위 그림의 각 점들(1,1)~(11,11)의 온도를 구하시오.

Laplace equation



*내부 ($2 < i < 10, 2 < j < 10$)

$$T_{i,j} = 1/4(T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1})$$

*왼쪽 ($i=1, 2 \leq j \leq 6$)

$$T_{i,j} = 1/4(2T_{i+1,j} + T_{i,j+1} + T_{i,j-1})$$

*오른쪽 ($i=11, 2 \leq j \leq 9$)

$$T_{i,j} = 1/4(2T_{i-1,j} + T_{i,j+1} + T_{i,j-1})$$

*TT=100인 점

$$[1 \leq i \leq 11, j=11], [i=1, 7 \leq j \leq 10], [i=11, j=10]$$

*TB=200인 점

$$1 \leq i \leq 11, j=1$$

데이터 입력

경계조건에 의해

The screenshot shows a Microsoft Excel spreadsheet with a grid of data. The grid has columns labeled A through K and rows numbered 1 through 12. The data values are as follows:

	A	B	C	D	E	F	G	H	I	J	K
1	100	100	100	100	100	100	100	100	100	100	100
2	100	100	100	105	105	105	105	105	103	102	100
3	100	105	110	110	110	115	115	115	115	115	115
4	100	110	120	122	125	125	130	125	125	120	120
5	100	110	130	130	130	130	135	132	130	124	124
6	130	135	140	140	145	145	150	140	130	130	130
7	150	155	150	150	150	150	160	155	155	150	150
8	170	175	170	175	180	180	185	180	175	170	170
9	180	181	180	185	190	190	190	187	185	180	180
10	190	190	190	190	190	190	190	190	190	190	190
11	200	200	200	200	200	200	200	200	200	200	200
12											

The cells in row 1 and column A are highlighted in yellow, representing boundary conditions. The cells in row 11 are highlighted in brown, representing another set of boundary conditions. A red arrow points from the text '경계조건에 의해' to the yellow cells.

각 셀에 위와 같이 대략적인 임의의 값들을 넣어준다.

데이터 입력

The screenshot shows an Excel spreadsheet with columns A through K and rows 1 through 25. The data is as follows:

	A	B	C	D	E	F	G	H	I	J	K
1	100	100	100	100	100	100	100	100	100	100	100
2	100	100	100	105	105	105	105	105	103	102	100
3	100	105	110	110	110	115	115	115	115	115	115
4	100	110	120	122	125	125	130	125	125	120	120
5	100	110	130	130	130	130	135	132	130	124	124
6	130	135	140	140	145	145	150	140	130	130	130
7	150	155	150	150	150	150	160	155	155	150	150
8	170	175	170	175	180	180	185	180	175	170	170
9	180	181	180	185	190	190	190	187	185	180	180
10				190	190	190	190	190	190	190	190
11											
12											
13	1										
14	100	100	100	100	100	100	100	100	100	100	100
15	100	101.25									100
16	100										112.5
17	100										
18	100										
19	130										
20											
21											
22											
23											
24	200	200	200	200	200	200	200	200	200	200	200
25											

$=0.25*(B1+A2+B3+C2)$

$=0.25*(2*J5+K4+K6)$

$=0.25*(2*B6+A5+A7)$

조건에 의해 각 셀에 수식을 입력해줌

데이터 입력

	A	B	C	D	E	F	G	H	I	J	K
1	100	100	100	100	100	100	100	100	100	100	100
2	100	100	100	105	105	105	105	105	103	102	100
3	100	105	110	110	110	115	115	115	115	115	115
4	100	110	120	122	125	125	130	125	125	120	120
5	100	110	130	130	130	130	135	132	130	124	124
6	130	135	140	140	145	145	150	140	130	130	130
7	150	155	150	150	150	150	160	155	155	150	150
8	170	175	170	175	180	180	185	180	175	170	170
9	180	181	180	185	190	190	190	187	185	180	180
10	190	190	190	190	190	190	190	190	190	190	190
11	200	200	200	200	200	200	200	200	200	200	200
12											
13	1										
14	100	100	100	100	100	100	100	100	100	100	100
15	100	101.25	103.75	103.75	105	106.25	106.25	105.75	105.5	104.5	100
16	100	105	108.75	111.75	113.75	113.75	116.25	115	114.5	113	112.5
17	100	108.75	118	121.25	121.75	125	125	125.5	122.5	121	119.75
18	100	118.75	125	130.5	132.5	133.75	135.5	132.5	127.75	126	124.5
19	130	133.75	138.75	141.25	141.25	143.75	145	141.75	138.75	133.5	133.5
20	152.5	152.5	153.75	153.75	156.25	158.75	160	158.75	152.5	151.25	150
21	170	169	170	171.25	173.75	176.25	177.5	175.5	172.5	168.75	167.5
22	180.5	181.25	181.5	183.75	186.25	187.5	188	186.25	183	181.25	180
23	190	190.25	190	191.25	192.5	192.5	192.5	191.75	191.25	190	190
24	200	200	200	200	200	200	200	200	200	200	200

세 구역의 조건이 각각 다르기 때문에 수식도 각각 다르게 적용

채우기 핸들을 이용하여 나머지 셀에도 수식을 동일하게 적용함

데이터 입력

	A	B	C	D	E	F	G	H	I	J	K
1	100	100	100	100	100	100	100	100	100	100	100
2	100	100	100	105	105	105	105	105	103	102	100
3	100	105	110	110	110	115	115	115	115	115	115
4	100	110	120	122	125	125	130	125	125	120	120
5	100	110	130	130	130	130	135	132	130	124	124
6	130	135	140	140	145	145	150	140	130	130	130
7	150	155	150	150	150	150	160	155	155	150	150
8	170	175	170	175	180	180	185	180	175	170	170
9	180	181	180	185	190	190	190	187	185	180	180
10	190	190	190	190	190	190	190	190	190	190	190
11	200	200	200	200	200	200	200	200	200	200	200
12											
13	1										
14	100	100	100	100	100	100	100	100	100	100	100
15	100	101.25	103.75	103.75	105	106.25	106.25	105.75	105.5	104.5	100
16	100	105	108.75	111.75	113.75	113.75	116.25	115	114.5	113	112.5
17	100	108.75	118	121.25	121.75	125	125	125.5	122.5	121	119.75
18	100	118.75	125	130.5	132.5	133.75	135.5	132.5	127.75	126	124.5
19	130	133.75	138.75	141.25	141.25	143.75	145	141.75	138.75	133.5	133.5
20	152.5	152.5	153.75	153.75	156.25	158.75	160	158.75	152.5	151.25	150
21	170	169	170	171.25	173.75	176.25	177.5	175.5	172.5	168.75	167.5
22	180.5	181.25	181.5	183.75	186.25	187.5	188	186.25	183	181.25	180
23	190	190.25	190	191.25	192.5	192.5	192.5	191.75	191.25	190	190
24	200	200	200	200	200	200	200	200	200	200	200
25											
26	2										
27	100	100	100	100	100	100	100	100	100	100	100
28	100	102.19	103.44	105.13	105.94	106.25	107.06	106.69	106.19	104.63	100
29	100	104.69	109.63	111.88	113.06	115.31	115	115.5	114	113.13	111.44
30	100	110.44	115.94	120.5	123.13	123.56	125.56	123.75	122.19	120.31	119.75
31	100	116.88	126.5	130	131.81	134.19	134.06	132.63	129.94	126.69	126.31
32	130	135	138.44	141.06	143.44	144.69	145.25	143.75	138.88	137.38	135.38
33	151.25	152.25	153.75	155.63	156.88	159.06	160	157.44	155.31	151.19	150.88
34	167.75	168.44	168.88	170.31	172.5	174.38	174.94	173.75	169.94	168.13	166.88
35	180.63	180.31	181.25	182.56	184.38	185.75	185.94	184.56	182.81	180.44	180
36	190.25	190.31	190.75	191.56	192.5	193.13	193.06	192.5	191.19	190.63	190
37	200	200	200	200	200	200	200	200	200	200	200
38											

같은 방법으로 채우기 핸들을 이용하여 다른 셀에도 서식 적용

결과값(n번 반복 실행)

10											
100	100	100	100	100	100	100	100	100	100	100	100
100	102.52	104.54	106.01	107	107.4	107.67	107.33	106.72	104.96	100	100
100	105.49	109.8	112.69	114.41	115.55	115.61	115.49	114.6	113.4	112.08	112.08
100	109.89	116.45	120.6	123.15	124.22	124.77	124.29	123.52	122.69	122.01	122.01
100	117.4	125.83	130.42	132.99	134.44	134.65	134.24	133.52	132.45	132.39	132.39
129.22	134.11	138.71	142.19	144.41	145.48	145.7	145.32	144.15	143.73	142.92	142.92
148.75	150.14	152.7	154.83	156.43	157.28	157.46	156.7	156.19	154.92	155.14	155.14
164.05	164.82	165.97	167.39	168.43	169.08	168.97	168.72	167.58	167.35	166.58	166.58
177.42	177.59	178.34	179.07	179.78	180.11	180.21	179.64	179.35	178.5	178.67	178.67
189.05	189.22	189.47	189.87	190.18	190.4	190.32	190.23	189.78	189.69	189.39	189.39
200	200	200	200	200	200	200	200	200	200	200	200

10번 반복 실행해준 값

20											
100	100	100	100	100	100	100	100	100	100	100	100
100	102.56	104.66	106.17	107.17	107.72	107.95	107.82	107.21	105.45	100	100
100	105.57	109.9	112.9	114.77	115.87	116.31	116.27	115.77	114.66	113.33	113.33
100	109.8	116.47	120.69	123.27	124.7	125.35	125.46	125.08	124.63	124.1	124.1
100	117.1	125.42	130.1	132.86	134.4	135.15	135.24	135.17	134.72	134.73	134.73
128.36	133.08	137.77	141.27	143.53	144.92	145.5	145.77	145.49	145.5	145.17	145.17
147.12	148.61	151.09	153.24	154.91	155.87	156.45	156.43	156.51	156.13	156.32	156.32
162.21	162.88	164.06	165.41	166.39	167.14	167.37	167.56	167.28	167.38	167.08	167.08
175.67	175.89	176.55	177.21	177.85	178.19	178.46	178.37	178.44	178.17	178.32	178.32
188.05	188.18	188.41	188.74	188.98	189.2	189.24	189.3	189.17	189.22	189.1	189.1
200	200	200	200	200	200	200	200	200	200	200	200

20번 반복 실행해준 값

조건부 서식

Microsoft Excel - laplace

파일(F) 편집(E) 보기(V) 삽입(I) 서식(O) 도구(T) 데이터(D) 창(W) 도움말(H)

11 | 가 가 가 | W % , % % | 100%

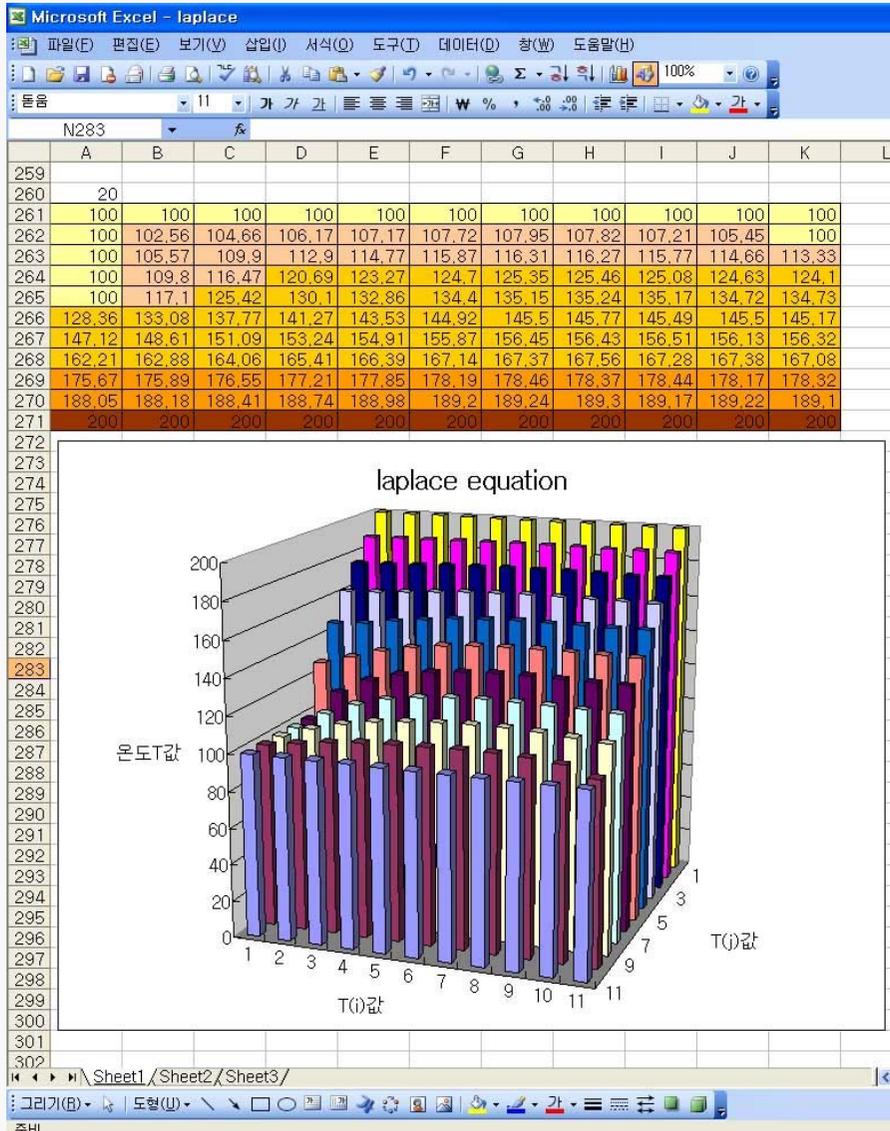
	A	B	C	D	E	F	G	H	I	J	K
1	100	100	100	100	100	100	100	100	100	100	100
2	100	100	100	105	105	105	105	105	103	102	100
3	100	105	110	110	110	115	115	115	115	115	115
4	100	110	120	122	125	125	130	125	125	120	120
5	100	110	130	130	130	130	135	132	130	124	124
6	130	135	140	140	145	145	150	140	130	130	130
7	150	155	150	150	150	150	160	155	155	150	150
8	170	175	170	175	180	180	185	180	175	170	170
9	180	181	180	185	190	190	190	187	185	180	180
10	190	190	190	190	190	190	190	190	190	190	190
11	200	200	200	200	200	200	200	200	200	200	200
12											
13	1										
14	100	100	100	100	100	100	100	100	100	100	100
15	100	101.25	103.75	103.75	105	106.25	106.25	105.75	105.5	104.5	100
16	100	105	108.75	111.75	113.75	113.75	116.25	115	114.5	113	112.5
17	100	108.75	118	121.25	121.75	125	125	125.5	122.5	121	119.75
18	100	118.75	125	130.5	132.5	133.75	135.5	132.5	127.75	126	124.5
19	130	133.75	138.75	141.25	141.25	143.75	145	141.75	138.75	133.5	133.5
20	152.5	152.5	153.75	153.75	156.25	158.75	160	158.75	152.5	151.25	150
21	170	169	170	171.25	173.75	176.25	177.5	175.5	172.5	168.75	167.5
22	180.5	181.25	181.5	183.75	186.25	187.5	188	186.25	183	181.25	180
23	190	190.25	190	191.25	192.5	192.5	192.5	191.75	191.25	190	190
24	200	200	200	200	200	200	200	200	200	200	200
25											
26	2										
27	100	100	100	100	100	100	100	100	100	100	100
28	100	102.19	103.44	105.13	105.94	106.25	107.06	106.69	106.19	104.63	100
29	100	104.69	109.63	111.88	113.06	115.31	115	115.5	114	113.13	111.44
30	100	110.44	115.94	120.5	123.13	123.56	125.56	123.75	122.19	120.31	119.75
31	100	116.88	126.5	130	131.81	134.19	134.06	132.63	129.94	126.69	126.31
32	130	135	138.44	141.06	143.44	144.69	145.25	143.75	138.88	137.38	135.38
33	151.25	152.25	153.75	155.63	156.88	159.06	160	157.44	155.31	151.19	150.88
34	167.75	168.44	168.88	170.31	172.5	174.38	174.94	173.75	169.94	168.13	166.88
35	180.63	180.31	181.25	182.56	184.38	185.75	185.94	184.56	182.81	180.44	180
36	190.25	190.31	190.75	191.56	192.5	193.13	193.06	192.5	191.19	190.63	190
37	200	200	200	200	200	200	200	200	200	200	200
38											

10											
100	100	100	100	100	100	100	100	100	100	100	100
100	102.52	104.54	106.01	107	107.4	107.67	107.33	106.72	104.96	100	
100	105.49	109.8	112.69	114.41	115.55	115.61	115.49	114.6	113.4	112.08	
100	109.89	116.45	120.6	123.15	124.22	124.77	124.29	123.52	122.69	122.01	
100	117.4	125.83	130.42	132.99	134.44	134.65	134.24	133.52	132.45	132.39	
129.22	134.11	138.71	142.19	144.41	145.48	145.7	145.32	144.15	143.73	142.92	
148.75	150.14	152.7	154.83	156.43	157.28	157.46	156.7	156.19	154.92	155.14	
164.05	164.82	165.97	167.39	168.43	169.08	168.97	168.72	167.58	167.35	166.58	
177.42	177.59	178.34	179.07	179.78	180.11	180.21	179.64	179.35	178.5	178.67	
189.05	189.22	189.47	189.87	190.18	190.4	190.32	190.23	189.78	189.69	189.39	
200	200	200	200	200	200	200	200	200	200	200	

20											
100	100	100	100	100	100	100	100	100	100	100	100
100	102.56	104.66	106.17	107.17	107.72	107.95	107.82	107.21	105.45	100	
100	105.57	109.9	112.9	114.77	115.87	116.31	116.27	115.77	114.66	113.33	
100	109.8	116.47	120.69	123.27	124.7	125.35	125.46	125.08	124.63	124.1	
100	117.1	125.42	130.1	132.86	134.4	135.15	135.24	135.17	134.72	134.73	
128.36	133.08	137.77	141.27	143.53	144.92	145.5	145.77	145.49	145.5	145.17	
147.12	148.61	151.09	153.24	154.91	155.87	156.45	156.43	156.51	156.13	156.32	
162.21	162.88	164.06	165.41	166.39	167.14	167.37	167.56	167.28	167.38	167.08	
175.67	175.89	176.55	177.21	177.85	178.19	178.46	178.37	178.44	178.17	178.32	
188.05	188.18	188.41	188.74	188.98	189.2	189.24	189.3	189.17	189.22	189.1	
200	200	200	200	200	200	200	200	200	200	200	

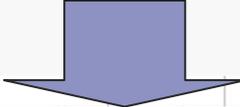
조건부 서식을 통해 각 셀의 온도 경향을 쉽게 알 수 있음

결과값(Graph)

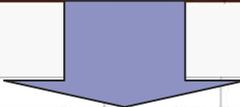


3차원 막대 그래프를 통해
 각 점들의 온도를
 한 눈에 알 수 있음

조건 변화 결과값



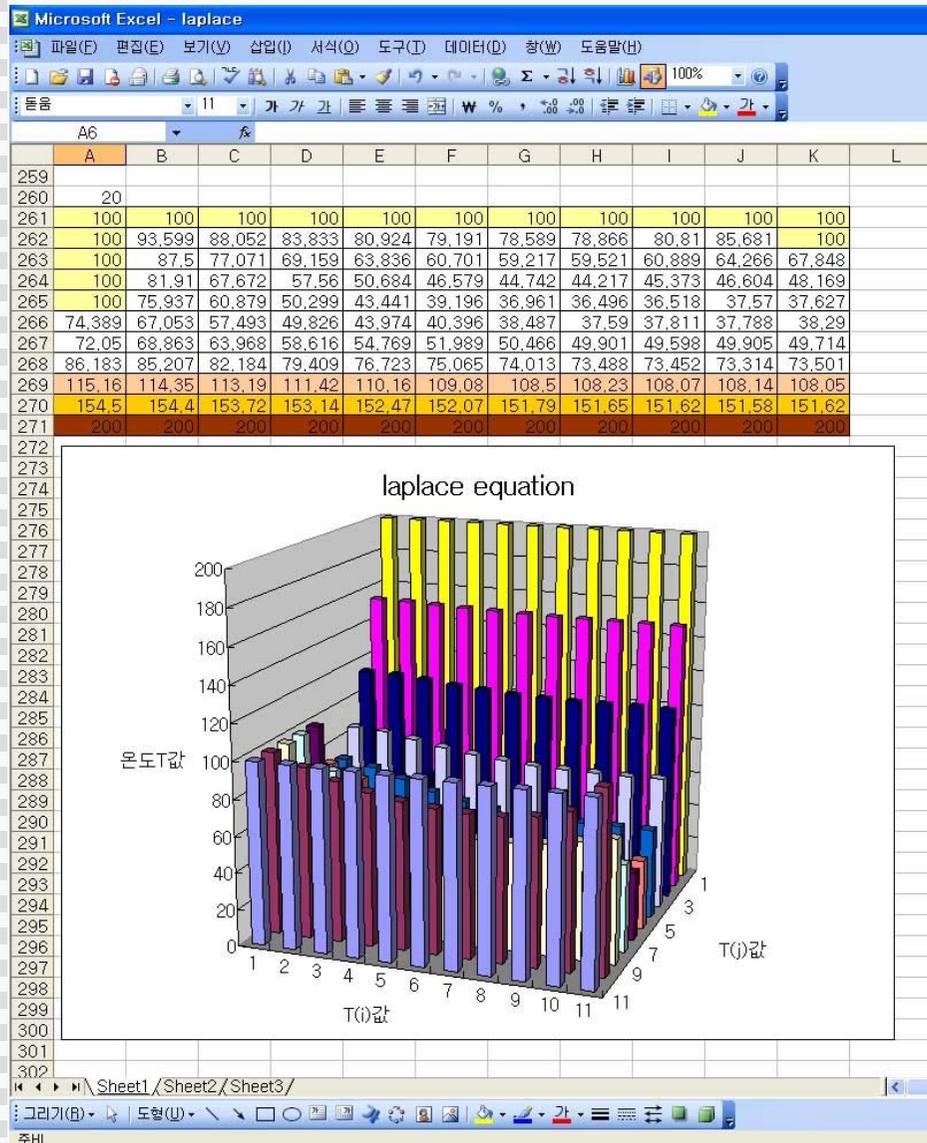
10										
100	100	100	100	100	100	100	100	100	100	100
100	88,539	79,164	72,793	69,216	67,598	67,631	68,522	71,972	79,159	100
100	78,259	61,105	49,586	43,212	40,52	39,831	41,484	43,838	49,566	54,52
100	69,307	46,613	32,601	24,786	21,393	20,788	21,147	23,74	25,5	28,34
100	58,53	35,38	21,321	14,356	11,124	10,131	10,747	11,237	13,18	13,058
50,313	38,888	25,312	16,741	11,346	9,2342	8,53	8,4364	9,1767	9,2562	10,035
36,378	31,676	26,299	20,949	18,437	16,979	16,601	16,638	16,653	17,055	16,888
45,903	45,164	42,094	40,323	38,761	38,295	38,062	38,034	38,127	38,106	38,232
79,916	79,047	78,46	77,382	77,029	76,744	76,704	76,696	76,694	76,731	76,71
133,53	133,58	133,14	132,99	132,78	132,75	132,73	132,73	132,73	132,73	132,74
200	200	200	200	200	200	200	200	200	200	200



20										
100	100	100	100	100	100	100	100	100	100	100
100	93,599	88,052	83,833	80,924	79,191	78,589	78,866	80,81	85,681	100
100	87,5	77,071	69,159	63,836	60,701	59,217	59,521	60,889	64,266	67,848
100	81,91	67,672	57,56	50,684	46,579	44,742	44,217	45,373	46,604	48,169
100	75,937	60,879	50,299	43,441	39,196	36,961	36,496	36,518	37,57	37,627
74,389	67,053	57,493	49,826	43,974	40,396	38,487	37,59	37,811	37,788	38,29
72,05	68,863	63,968	58,616	54,769	51,989	50,466	49,901	49,598	49,905	49,714
86,183	85,207	82,184	79,409	76,723	75,065	74,013	73,488	73,452	73,314	73,501
115,16	114,35	113,19	111,42	110,16	109,08	108,5	108,23	108,07	108,14	108,05
154,5	154,4	153,72	153,14	152,47	152,07	151,79	151,65	151,62	151,58	151,62
200	200	200	200	200	200	200	200	200	200	200

셀 값들이 조건을 변화하지 않았을 때의 결과 값과 가까워 지고 있다.

조건 변화 결과값(Graph)



이 그래프는 20번 반복해준 그래프이다. 만약 반복을 많이 해준다면 조건을 변화하지 않았을 때의 결과 그래프와 비슷해질 것이다.

초기 임의 값의 정확도는 반복 횟수를 좌우 한다.

토론 및 검토

- ◆ 한 점의 온도는 그 점을 둘러싸고 있는 점들의 평균값에 영향을 받음.
- ◆ Laplace equation을 반복 실행하면 할수록 더 정확한 데이터를 얻을 수 있음.
- ◆ 임의의 초기값의 정확도는 반복 실행의 척도가 될 수 있음.(초기값의 정확도가 떨어지면 떨어질수록 반복을 많이 해줘야 결과값이 정확해짐)