Chapter 8. Storage and Flow of Powders - Hopper Design

☞http://www.ecgf.uakron.edu/~chem/fclty/chase/SolidsNotes9.PDF

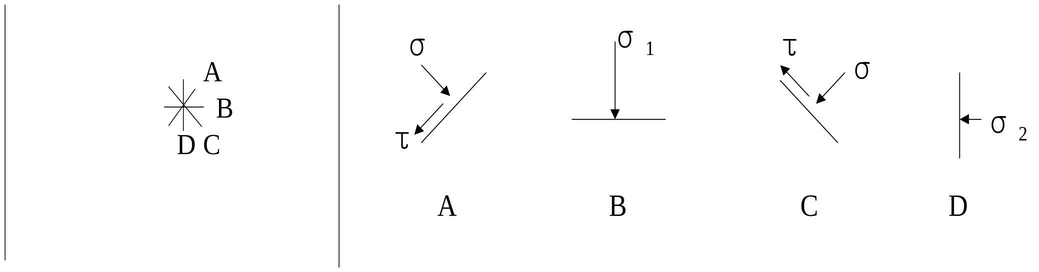
http://www.dietmar-schulze.de/storage.html

http://www.anl.luth.se/abb/siloprojekt/index.html

8.0 Stresses in Bulk Solids

1) Mohr Stress Circle

Two dimensional stresses in the static powder bed



- Normal stress, σ
- Shear(tangential) Stress, τ

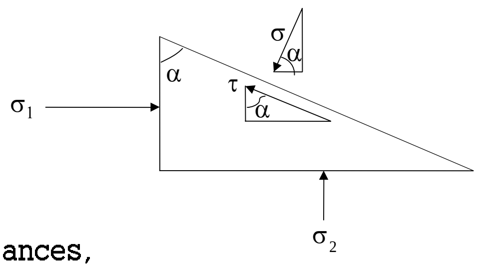
분체층은 정지되어 있어도 유체와는 달리

- 수직응력(유체에서 보통 압력으로 부름)이외에 전단력이 존재하고,
- 수직응력과 전단응력은 면의 배향에 따라서도 달라진다.
- *Principal stresses* (major, minor), σ_1, σ_2

  : normal stresses to the plane in which shear stresses vanish

  where $\sigma_1 \perp \sigma_2$

Correlating σ and τ, in terms of principal stresses



From force balances,

$$\sigma_1 \cos\alpha = \sigma\cos\alpha + \tau\sin\alpha \qquad (1)$$

$$\sigma_2 \sin\alpha = \sigma\sin\alpha - \tau\cos\alpha \qquad (2)$$

$(1)\times\cos\alpha + (2)\times\sin\alpha$

$$\sigma_1\cos^2\alpha + \sigma_2\sin^2\alpha = \sigma$$

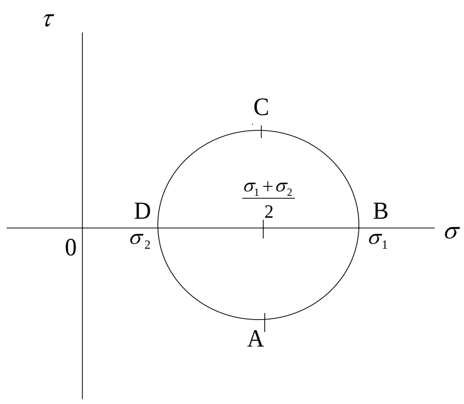
$(1)^2 + (2)^2$

$$\sigma_1^2\cos^2\alpha + \sigma_2^2\sin^2\sigma = \sigma^2 + \tau^2$$

Eliminating $\sin\alpha$ **and** $\cos\alpha$

$$\left(\sigma - \frac{\sigma_1+\sigma_2}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_1-\sigma_2}{2}\right)^2$$

Mohr Stress Circle

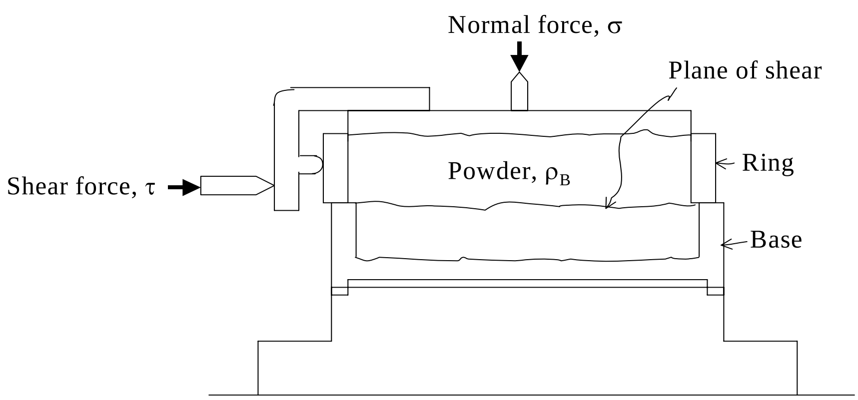


Mohr 원

2) Shear Cell Tests: Yield Behavior of Bulk Powder

Powder Bed: move or not?

Jenike shear cell

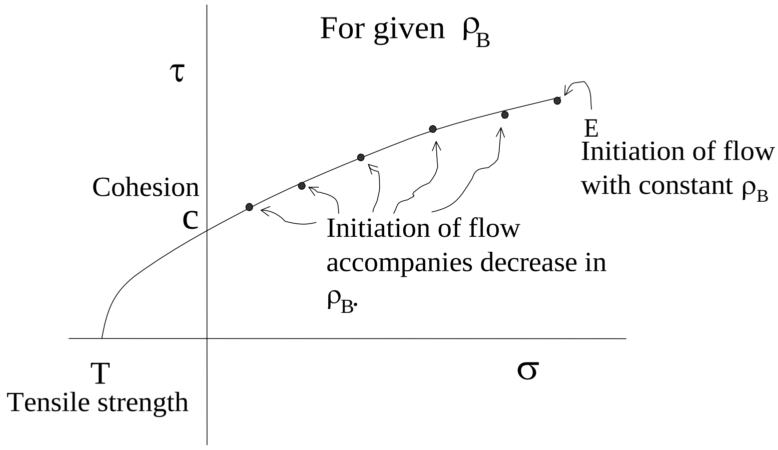


※ Construction of yield locus

- Put the powder sample of ρ_B in the cell.

- Measure the horizontal stress(τ) to initiate flow for the given normal stress(σ). (σ must be low enough for ρ_B to decrease during application of τ)

- Repeat this procedure for each identical powder sample(ρ_B) with greater σ until ρ_B does not decrease. Five or six pairs of (σ, τ) should be generated.



- For other values of ρ_B, Figure 8.10

※ Expanded flow(at the points up to E on the curve)

Free flow (at point E): critical flow

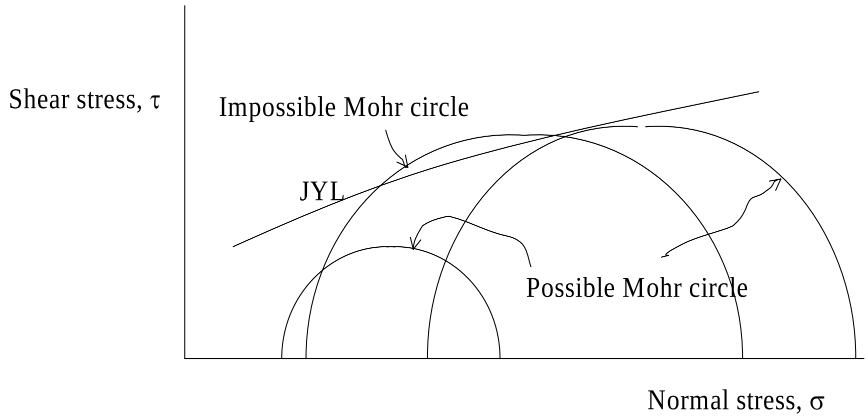
※ Cohesion

Tensile strength

3) Application of Mohr's Circle to Analysis of the Yield Locus:

따라서 JYL은 Mohr 원(정지한 분체층의 상태)이 위치할 수 있는 극한 상황이라 할 수 있다.

4) Effective Yield Locus: Determination of σ

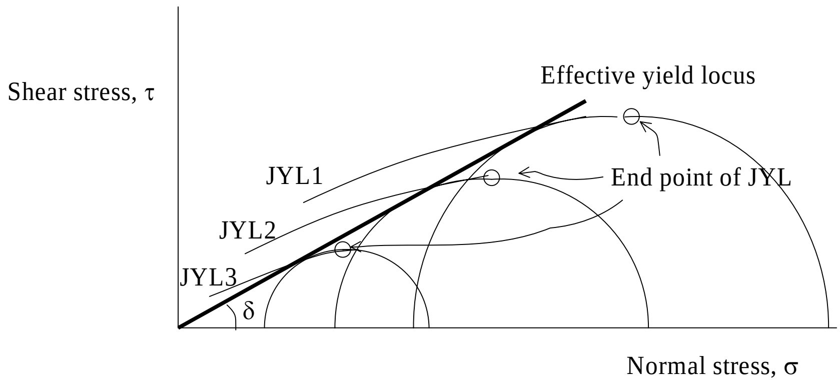


Experiments have demonstrated that

$$\frac{\delta_1}{\delta_2} = constant$$

for Mohr's circles at end points of yield loci

→ 서로 다른 ρ_B에서 JYL의 끝점(E) 들을 접하는 원들을 그리고, 이 원들을 접하는 선을 그리면 원점을 지나는 직선이 얻어진다. 여기서 내부마찰각 σ를 얻는다.



$$\therefore \tau = \sigma \tan\delta$$

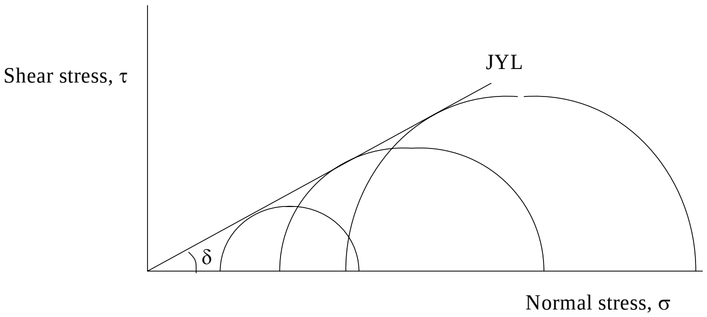
where δ : effective (internal) angle of friction

Worked Example 8.1(a)

Ex.8.4, 8.5

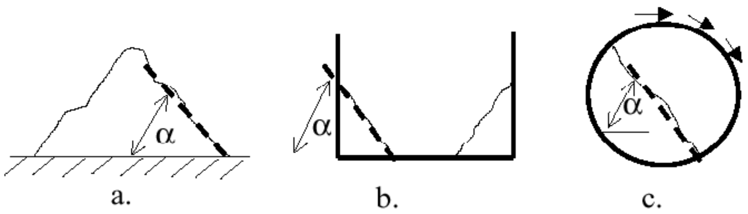
* For free flowing(noncohesive) powder bed, Figure 8.5

Yield locus: only one → coincides with EYL



Yield locus for noncohesive (free-flow) powder

* 안식각 *Angle of Repose*, α



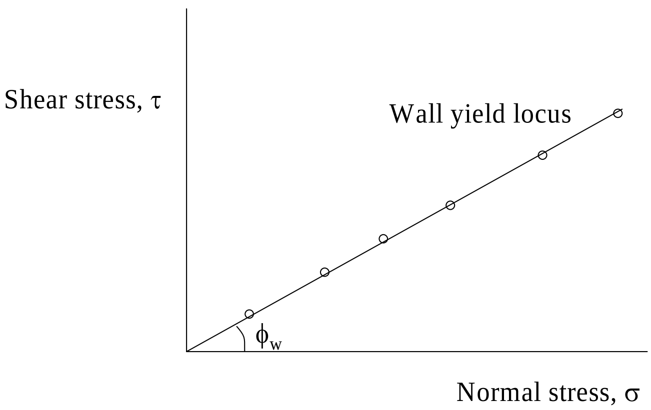
Angle of repose, α, of (a) a pile of powder, (b) powder in a container, and (c) powder in a rolling drum.

For noncohesive(free-flowing) particles

$$\delta \sim \alpha$$

4) Wall Yield Locus

비슷한 방법으로 벽과 분체층과의 yield locus도 구할 수 있다.



From wall shear test ☞ Figure 8.16

$$\tau_w = \sigma_w \tan\phi_w$$

8.1 Introduction

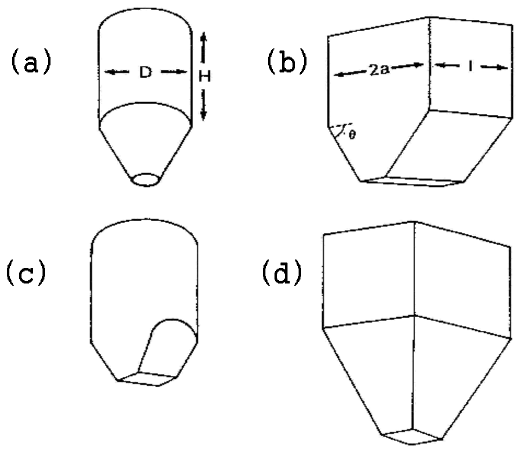
Storage tanks

Silos : section of constant cross sectional area

- Bins : $H > 1.5\ D$
- Bunker : $H < 1.5\ D$

Hopper : section of reducing cross sectional area downwards

Typical bulk solids storage vessels



(a) conical and axisymmetric hopper; (b) plane-flow wedge hopper;
(c) plane flow chisel hopper; (d)pyramid hopper

8.2 Mass Flow and Core(Funnel) Flow

Mass flow vs. core flow : Figure 8.1

Figure 8.2   Figure 8.3

To see the mass flow in hopper ☞

http://www.cco.caltech.edu/~granflow/movies.html

8.3 Design Philosophy - *Arching* and Hopper Design

Hopper에서 항상 문제가 되는 것은 막힘현상이다. 이는 다음 그림에서 보는 것처럼 arch를 만드는 현상이며 흐름을 유지하기 위해서는 arching 없이

mass flow가 일어나도록 설계되어야 한다.

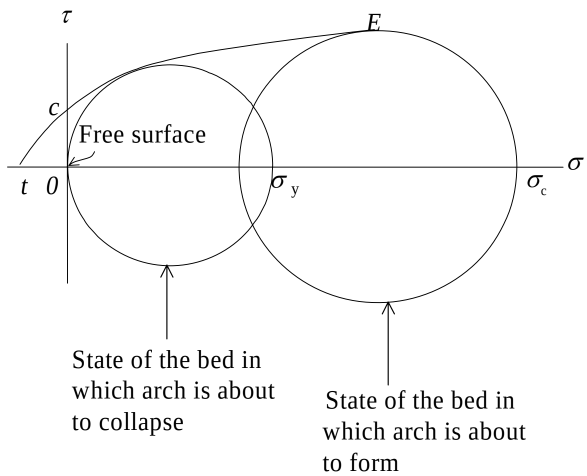
1) Arching Figure 8.4

Arch - free surface, no flow

e.g. a salt shaker (a salt pourer?)

2) Two Circles: Determination of σ_y and σ_C

그림에서 YL는 powder bed에서 flow가 일어나는 경계선이다. 따라서 이에 접하는 Mohr circle은 바로 이 상황의 bed 상황이며, 다음 그림의 두 개의 원은 각각 arch가 깨져 흐름이 막 시작되거나(왼쪽 원), free flow의 상태에서 arch가 이제 막 형성(오른 쪽 원)되는 상황을 나타낸다.



σ_y:: Unconfined yield strength (UYS),

σ_C: Compacting(consolidating) stress

Powder flow function

$$\sigma_y = fn(\sigma_C)$$

Figure 8.6

3) Hopper flow factor

$$ff \equiv \frac{\text{compacting stress of the powder under hopper condition}}{\text{actual stress of the powder developed under hopper condition}}$$

$$= \frac{\sigma_C}{\sigma_D}$$

where $ff = f(\delta, \Phi_w, \Theta)$

이 비에서 분모는 주어진 hopper 조건하에서 분체층에 형성되는 실제 응력으로 Jenike가 이를 구하여 ff의 값을 나타내는 다음 그림을 만들었다..

↓

Flow factor chart : Figure 8.18, Figure 8.19

4) Flow-nonflow condition(Arch가 깨지는 조건)

For flow

$$\sigma_D > \sigma_y$$

$$\therefore \frac{\sigma_C}{ff} > \sigma_y$$

Critical condition for flow

$$\sigma_{D,\,crit} = \sigma_y$$

5) Determination of critical stress

Find YL's and δ.

↓

Find the relation between σ_y and σ_C.

↓

Find WYL and Φ_W .

↓

With Φ_W and δ, determine ff and Θ from Figures 8.18 and 8.19.

where Θ: semi-included angle of the conical section

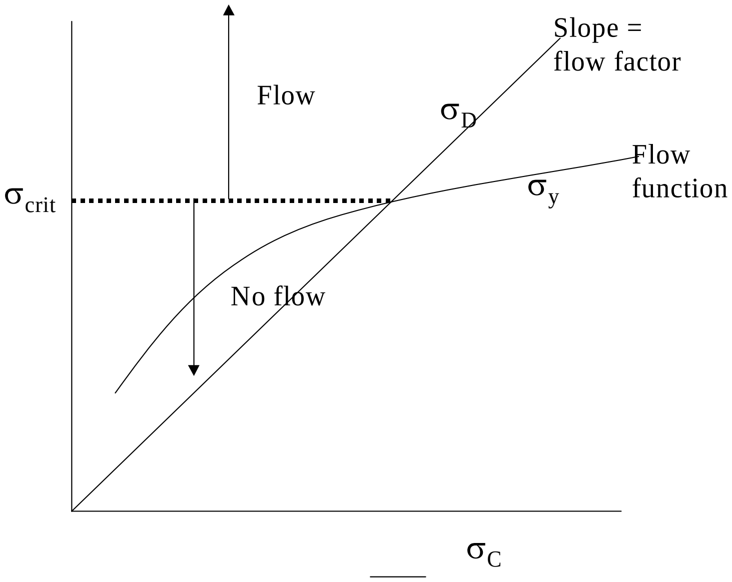
↓

Draw σ_D vs. σ_C(FF) and σ_y vs. σ_C.

↓

Determine $\overline{\sigma_{crit}}$

6) Critical outlet dimension: Determination of hopper opening



$$B = H(\Theta)\frac{\overline{\sigma_{crit}}}{\rho_B g}$$

where

for conical hopper $H(\Theta) = 2 + \frac{\Theta}{60}$

for slot-type hopper $H(\Theta) = 1 + \frac{\Theta}{180}$

Example : σ=30° and Φ_w=19°C → Θ=(30.5°→27.5°) → ff=1.8

Worked Example 8.1

Worked Example 8.2

8.8 Pressure on the Base of a Tall Cylindrical Bin: Stress in Powder Bed

Pressure: vertical stress, σ_v

Force balance on a slice of thickness ΔH in the powder bed,

$$D\Delta\sigma_v + 4\tan\Phi_w \sigma_h \Delta H = D\rho_B g \Delta H$$

Assuming $\sigma_H = k\sigma_v$ and $\Delta H \to 0$

$$\frac{d\sigma_v}{dH} + \left(\frac{4\tan\Phi_w k}{D}\right)\sigma_v = \rho_B g$$

Integrating

$$\sigma_v = \frac{D\rho_B g}{4\tan\Phi_w k}[1 - e^{-4\tan\Phi_w kH/D}] + \sigma_{v0} e^{-4\tan\Phi_w kH/D}$$

When no force acting on the free surface of the powder $\sigma_{v0} = 0$,

$$\sigma_v = \frac{D\rho_B g}{4\tan\Phi_w k}[1 - e^{-4\tan\Phi_w kH/D}]$$

Janssen equation

For small H,

$\sigma_v \cong \rho_B Hg$ (liquid-like)

For large H ($>4D$)

$$\sigma_v \cong \frac{D\rho_B g}{4\tan\Phi_w k}$$

independent of H and σ_{v0}

Figure 8.21

8.8S1. Hopper

青木(1963), 坂下(1990)

For $C' \neq 1$,

$$\sigma_{v,2} = \sigma_v * \left(\frac{h}{H_2}\right)^{C'} + \frac{\gamma H_2}{C'-1}\left(\frac{h}{H_2}\right)\left[1 - \left(\frac{h}{H_2}\right)\right]$$

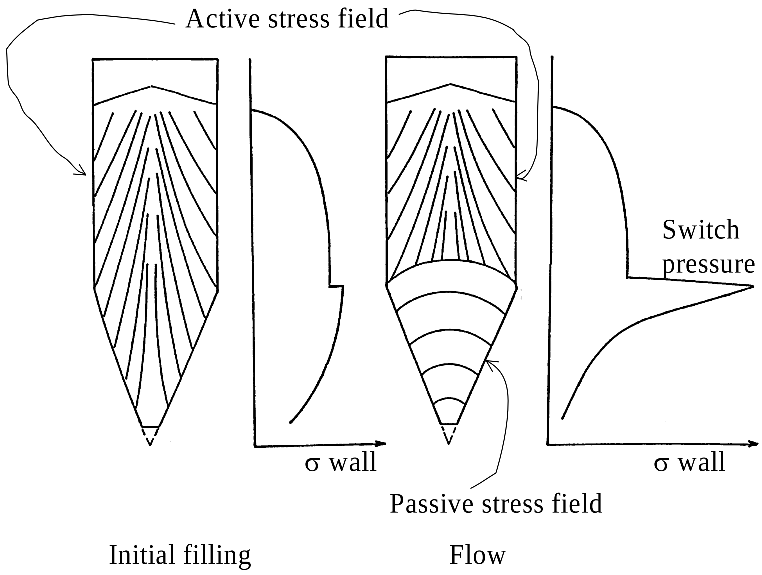
For $C' = 1$

$$\sigma_{v,2} = \sigma_v * \left(\frac{h}{H_2}\right)^{C'} + \gamma H_2 \left(\frac{h}{H_2}\right)\ln\left(\frac{H_2}{h}\right)$$

where $C' \equiv 2\tan\Phi_W \cot\Theta_1 (K\cos^2\Theta_1 + \sin^2\Theta_1)$

8.8S2 Wall stress distribution in silo-hopper

Storage tank내의 압력은 저장중에는 위의 정압과 일치하나 feeding 과 discharge시에는 달라진다.

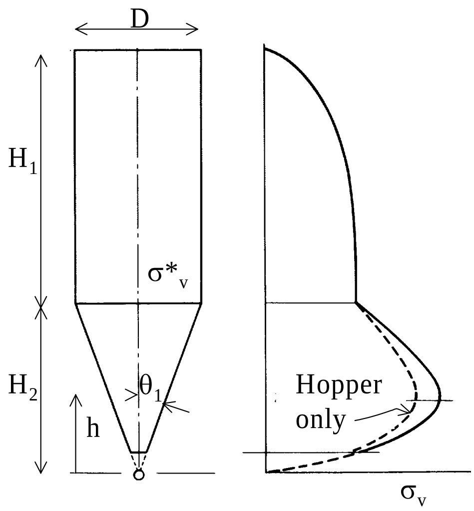


8.9 Mass Flow (Discharge) Rate

For cylindrical and conical hoppers

Beverloo(1961)

Dimensional analysis



$$M_p = C\rho_B g^{1/2}(B_0 - kd_p)^{5/2}$$

where C : 0.55 ~ 0.65

k : 1.5 or somewhat larger depending on particle shape

- Independent of H, D