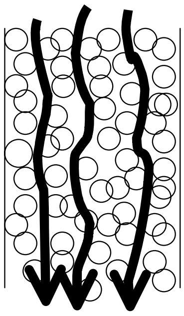
Chapter 4. Fluid Flow through a Packed Bed of Particles

4.1 Pressure Drop - Flow Relationship

이제 입자층은 움직이지 않고 유체가 이 층을 지나간다고 생각하자. 여기서 서로가 미치는 항력은 유체흐름의 압력 강하로 나타나는데....

1) Laminar Flow

Fluid flow through a packed bed: simulated by fluid flow through a hypothetical tubes



$$\therefore \frac{(-\Delta p)}{H} = \frac{32\mu U}{D^2}$$

$$\Rightarrow \frac{(-\Delta p)}{H_e} = \frac{K_1 \mu U_i}{D_e^2}$$

Hagen-Poiseille equation

Substituting suitable relations

$$\therefore \frac{(-\Delta p)}{H} = 180 \frac{\mu U}{d_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

Carman-Kozeny equation

3) General Equation for Turbulent and Laminar Flow

Ergun equation

$$\frac{(-\Delta p)}{H} = \underbrace{150 \frac{\mu U}{d_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3}}_{\text{Laminar}} + \underbrace{1.75 \frac{\rho_f U^2}{d_p} \frac{(1-\varepsilon)}{\varepsilon^3}}_{\text{Turbulent}}$$

Laminar flow for $Re* = \frac{d_p U \rho_f}{\mu(1-\varepsilon)} < 10$

Turbulent flow for $Re* = \frac{d_p U \rho_f}{\mu(1-\varepsilon)} > 2000$

or

$$f* = \frac{150}{Re*} + 1.75$$

$$\text{where} \quad f* \equiv \frac{(-\Delta p)}{H} \frac{d_p}{\rho_f U^2} \frac{\varepsilon^3}{(1-\varepsilon)}$$

Friction factor

Figure 4.1

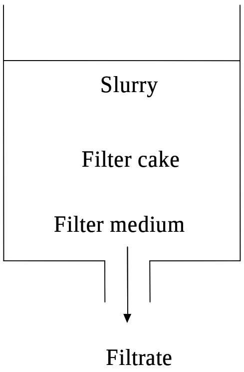
4) Nonspherical Particles

$d_{p,sv}$ (surface-volume diameter) instead of d_p

Worked Example 4.1

4.2 (Liquid) Filtration

For filtration theory and practice, see http://coel.ecgf.uakron.edu/~chem/fclty/chase/FILTRATION%20FUNDAMENTALS_files/frame.htm



1) Introduction: Cake filtration

Cake on filter medium: its depth increasing with time

Incompressible: constant voidage of cake

Compressible:

Filter media : Canvas cloth, woolen cloth, metal cloth, glass cloth, paper, synthetic fabrics

Filter aids : *To avoid cake plugging*

- Diatomaceous silica, perlite, purified woolen cellulose, other inert porous solids

- By either adding slurry (increasing cake permeability)

  or precoating the filter media surface

  (preventing gelatinous solids from plugging the media)

Types of liquid filters

	Batch	Continuous
Pressure Filtration	Filter Press Shell-and-Leaf Filters	Automatic Belt Filters
Vacuum Filtration	Discontinuous Vacuum Filters	Rotary Drum Filters Horizontal Belt Filters
Centrifugal Filtration	Batch Types	Continuous Types

2) Compressible Cake

For *cake filter*

From laminar part of Ergun equation

$$\frac{(-\Delta p)}{H} = \frac{150\mu U(1-\varepsilon)^2}{d_p^2\varepsilon^3}$$

where L : cake thickness

d_p : surface-volume diameter of particle

By defining cake resistance r_c

$$r_c = \frac{150(1-\varepsilon)^2}{d_p^2\varepsilon^3},$$

$$\frac{(-\Delta p)}{H} = r_c\mu U$$

where $U = \frac{1}{A}\frac{dV}{dt}$

V: volume of slurry fed to filter

여기서는 비압축성 cake를 가정하여

Also defining ϕ(volume formed by passage of unit volume filtrate)

$$\phi = \frac{HA}{V},$$

$$\frac{dV}{dt} = \frac{A^2(-\Delta p)}{r_c\mu\phi V}$$

Including the resistance of filter medium,

since *the resistances of the cake and the filter medium* are in series,

$$(-\Delta p)=(-\Delta p_m)+(-\Delta p_c)$$

$$\downarrow$$

$$\frac{1}{A}\frac{dV}{dt}r_c\mu H_c$$

By ***analogy*** for the filter medium

$$(-\Delta p_m)=\frac{1}{A}\frac{dV}{dt}r_m\mu H_m$$

$$\therefore\ (-\Delta p)=\frac{1}{A}\frac{dV}{dt}(r_m\mu H_m+r_c\mu H_c)$$

Defining *equivalent height of filter cake and volume of filtrate*

$$r_mH_m=r_cH_{eq}\ \text{and}\ H_{eq}=\frac{\Phi V_{eq}}{A}$$

where V_{eq}: volume of filtrate passing to create a cake of thickness H_{eq}

$$\therefore\ \frac{1}{A}\frac{dV}{dt}=\frac{(-\Delta p)A}{r_c\mu(V+V_{eq})\Phi}$$

Constant rate filtration

$$\frac{1}{A}\frac{dV}{dt}=\frac{(-\Delta p)A}{r_c\mu(V+V_{eq})\Phi}=constant$$

Constant pressure filtration

Integrating

$$\frac{t}{V}=\frac{r_c\Phi\mu}{A^2(-\Delta p)}\left(\frac{V}{2}+V_{eq}\right)$$

4) Washing the Cake

Displacement the filtrate in the voids with clean solvent

Diffusion into the wash water

5) Compressible Cake

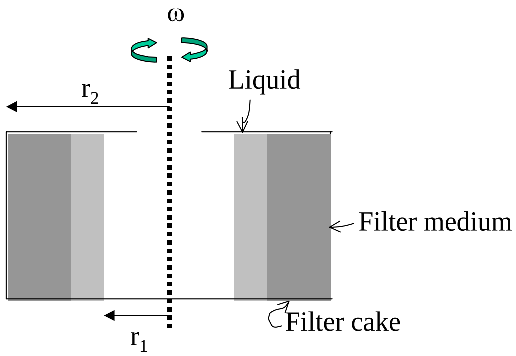
For *compressible* filter cake,

$$\frac{dp}{dL} = r_c \mu U$$

where r_c: a function of pressure difference

Worked Example 4.2

6) Centrifugal Filtration



From (*) and substituting the expression for R_c

$$-\Delta p = Q\mu\left(\frac{m_c \alpha}{A^2} + \frac{R_m}{A}\right)$$

where $m_c = c_s V$

In a centrifugal field, $dp = \rho r \omega^2 dr$

Integration

$$-\Delta p = \frac{\rho\omega^2 (r_2^2 - r_1^2)}{2}$$

$$\therefore \quad Q = \frac{\rho\omega^2 (r_2^2 - r_1^2)}{2\mu\left(\frac{m_c \alpha}{A^2} + \frac{R_m}{A}\right)}$$

4.3 Commercial Liquid Filter

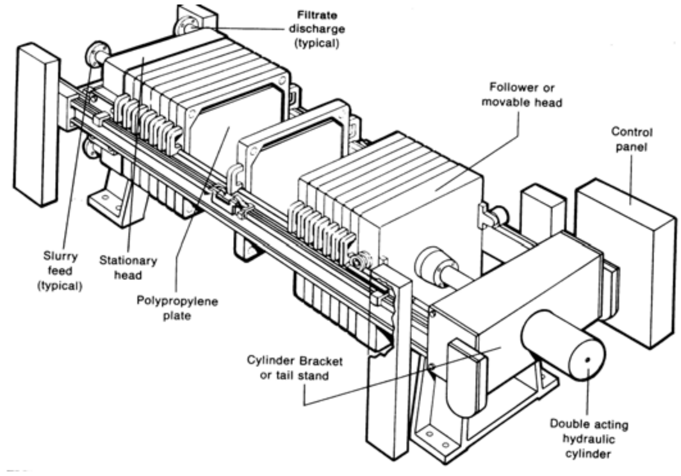
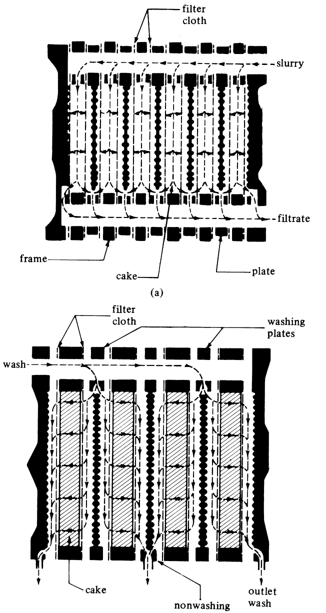
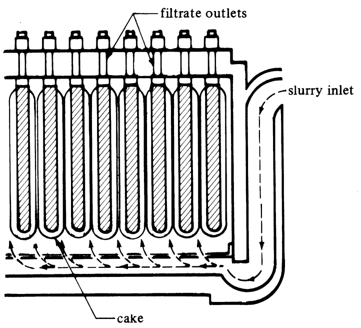


그림 1. Filter Press





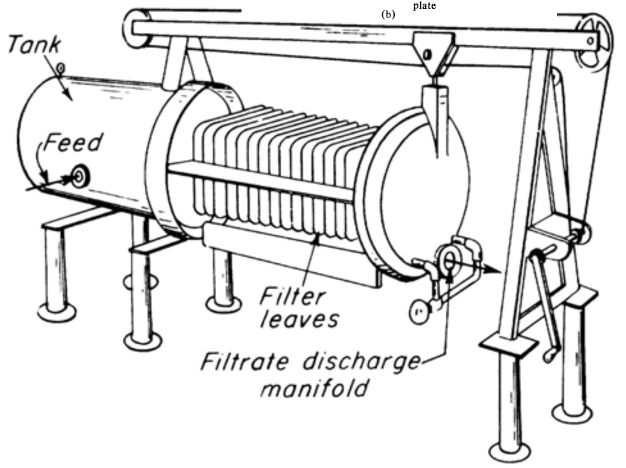
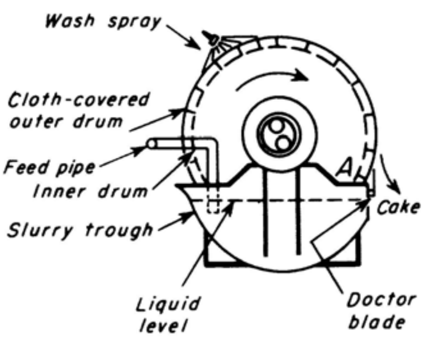
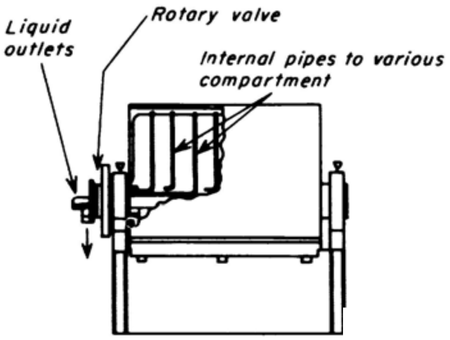


그림 2. Leaf Filter





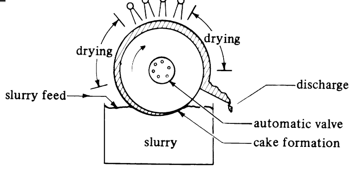


그림 3 Rotary Vacuum Filter

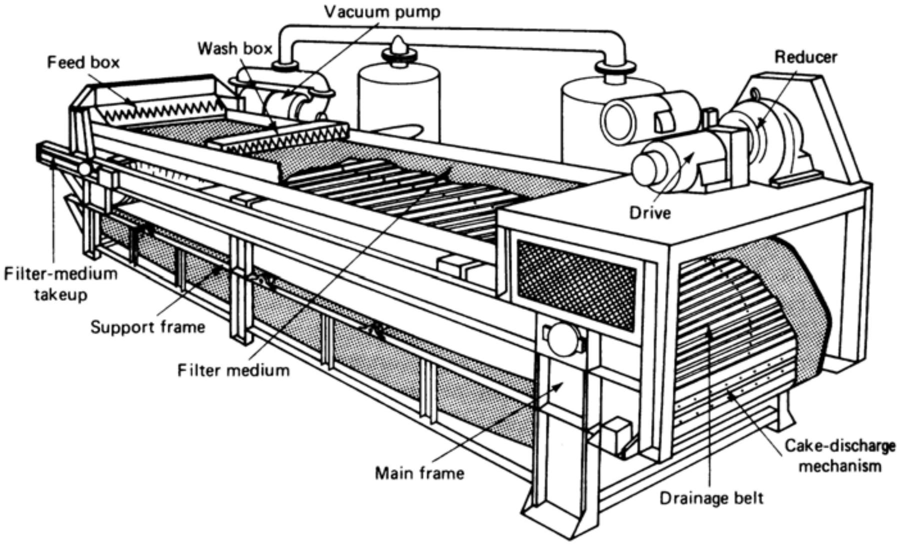


그림 4. Belt Filter

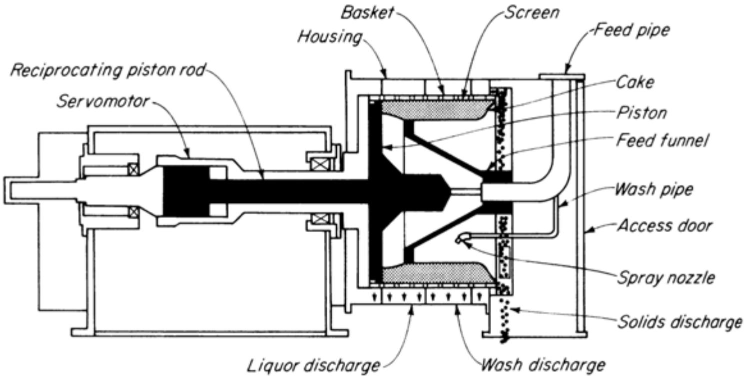


그림 5. Centrifuge Filter