Chapter 7. Separation of Particles from a Gas : Cyclones and Impactors

For either gas cleaning (removal of dusts) or recovery of particulate products

7.S Inertial Motion and Impact of Particles

입자는 질량이 그것을 담고 다니는 유체의 그것보다 크기 때문에 (액체보다는 특히 기체에서 두드러짐) 유체의 흐름(속도와 방향)에 급격한 변화가 일어날 때 적응하지 못하고 관성을 가지고 독자적인 운동을 하게 된다. 이를 입자의 관성운동이라 부르며, 고체표면 가까이서 이 현상이 일어나면 입자는 표면에 부딪히게 된다. 이를 충돌 (impaction)이라 한다.

1) *Stop Distance*

For *Stokesian* particles

Momentum(force) balance for a single sphere

$$m_p \frac{dU}{dt} = -\frac{3\pi\mu d_p U}{C_c}$$

Integrating once

$$U = U_0 e^{-t/\tau}$$

where $\tau = \dfrac{m_p C_c}{3\pi\mu d_p} = \dfrac{\rho_p d_p^2 C_c}{18\mu}$

relaxation time

Integrating twice

$$x = U_0\tau(1 - e^{t/\tau})$$

As $\dfrac{t}{\tau} \to \infty$,

$$x \sim U_0\tau = \frac{\rho_p d_p^2 U_0 C_c}{18\mu} \equiv s$$

stop distance

* out of Stokes' range

2) *Simiulitude Law* for Impaction : Stokesian Particles

For Re < 1

Force balance around a particle (equation of particle motion)

$$m_p \frac{d\vec{U}}{dt} = -3\pi\mu d_p(\vec{U} - \vec{U}_p)$$

Defining dimensionless variables

$$\vec{U}_1 \equiv \frac{\vec{U}}{L}, \quad \vec{U}_{f1} \equiv \frac{\vec{U}_f}{L} \quad \text{and} \quad \theta \equiv \frac{tU}{L}$$

where U, L: *characteristic velocity and length* of the system

$$\therefore St\frac{d\vec{U}_1}{d\theta} = -(\vec{U}_1 - \vec{U}_{f1})$$

or

In terms of displacement,

$$St\frac{d^2\vec{r}_1}{d\theta^2} + \frac{d\vec{r}_1}{d\theta} = \vec{U}_{f1}$$

where $\vec{r}$: dispacement vector

$$\vec{r}_1 \equiv \frac{\vec{r}}{L}$$

$$St \equiv \frac{\rho_p d_p^2 U}{18\mu L} = \frac{\tau U}{L} \equiv \frac{\textit{particle persistence}}{\textit{size of obstacle}}$$

Stokes number

$$\therefore \underset{\substack{\uparrow \\ \text{particle} \\ \text{trajectory}}}{\vec{r}_1} = f\left(St, \underset{\substack{\uparrow \\ \vec{U}_{f1}}}{Re}, \underset{\substack{\uparrow \\ \text{B.C.}}}{R}\right) \sim \eta_R$$

* *For the two particle systems*

If Re, St *and B.C. are the same, particle trajectories are the same.*

* 이와 같은 관성현상은 운동방정식을 풀어 입자의 시간에 따른 변위(궤

Net displacement in 1s due to Brownian motion and gravity for satandard-density spheres at standard conditions

Particle diameter, μm	Re_0	S at U_0=10m/s	time to travel 95% of S
0.01	0.0066	7.0×10^{-5}	2.0×10^{-8}
0.1	0.066	9.0×10^{-4}	2.7×10^{-7}
1.0	0.66	0.035	1.1×10^{-5}
10	6.6	2.3*	8.5×10^{-4}*
100	66	127*	0.065*

적)을 추적하여 입자의 거동을 해석한다.

7.0 Introduction

1) Separation Mechanisms

Sedimentation :

Settling chamber, centrifuge

Migration of charged particle in an electric field :

Electrostatic precipitator

Inertial deposition :

Cyclone, scrubber, filters, inertial impactor

Brownian diffusion :

Diffusion batteries

※ Filters

2) Collection efficiency

Fractional (grade) efficiency Figure 7.1

$$G_N(d_p) \equiv \frac{N_{feed}(d_p) - N_{product}(d_p)}{N_{feed}(d_p)}$$

based on number of particles

$$G_M(d_p) \equiv \frac{M_{feed}(d_p) - M_{product}(d_p)}{M_{feed}(d_p)}$$

based on mass of particles

Total efficiency

$$E_T = \int_0^\infty G(d_p) \underbrace{dF(d_p)}_{\downarrow}$$

Fraction of feed particles $d_p \sim d_p + dd_p$

3) Inertial Separators

입자의 관성을 이용하여 유체에서 분리한다...

Dimensional analysis for $G(d_p)$

$$G(d_p)=f(d_p,\rho_p,\rho_f,L,v) \rightarrow G(d_p)=f(St,Re,d_p/L)$$

where L : characteristic length of the separator

U : characteristic velocity of the particle in the separator

$$St\equiv\frac{\rho_p d_p^2 U}{18\mu L} \quad \text{and} \quad Re\equiv\frac{\rho_f UL}{\mu}$$

$$\boxed{Eu=f(Re)}$$

Define *cut size*, $d_{p,50}\equiv\ d_p$ at $G(d_p)=0.5$

Economy of the collectors

Based on $/(1000 m^3 cleaned gas /h)

annualized capital cost + *operating cost** :

* Power requirement $\equiv Q\,\Delta p$, [W]

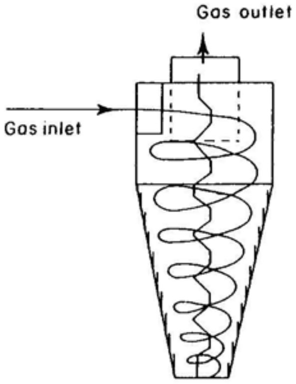
where $\Delta p=f(L,v,\rho_f,\mu)$ $\rightarrow$

By dimensional analysis

where $Eu\equiv\dfrac{\Delta p}{\rho_f v^2/2}$

7.1 Gas Cyclones - Description

Figure 7.2 reverse flow cyclone



7.2 Flow Characteristics

Rotational flow in the forced vortex

Radial pressure gradient

Characteristic velocity

$$v = \frac{4q}{\pi D^2}$$

where q: Gas flow rate

D:cyclone inside diameter

7.3 Efficiency of Separation

1) Total and Grade Efficiencies

M: solids mass rate to cyclone

M_f:fine solids mass flow rate leaving cyclone with gas

M_c: Coarse solids mass flow rate leaving from oriface

Total: $M = M_f + M_c$

Component: $M\frac{dF}{dd_p} = M_f\frac{dF_f}{dd_p} + M_c\frac{dF_c}{dd_p}$

Total efficiency: $E_T = \frac{M_c}{M}$

Grade efficiency: $G(d_p) = \frac{M_c\frac{dF_c}{dd_p}}{M\frac{dF}{dd_p}} = E_T\frac{\frac{dF_c}{dd_p}}{\frac{dF}{dd_p}}$

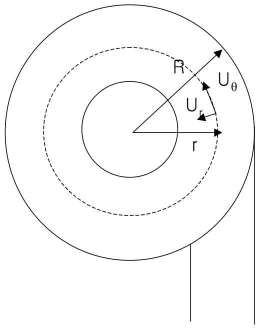
2) Simple Analysis for Particle Collection

Figure 7.3

At equilibrium orbit, r

$$3\pi d_p \mu U_r = \frac{\pi d_p^3}{6}(\rho_p - \rho_f)\frac{U_\Theta^2}{r}$$

$$F_D \qquad\qquad F_C - F_B$$



where $U_\Theta r^{1/2} = constant$

for confined vortex

$= U_{\Theta R} R^{1/2}$

$U_r r = constant$

for radially inward flow

$= U_R R$

$$\therefore d_p^2 = \frac{18\mu}{\rho_p - \rho_f} \frac{U_R}{U_{\Theta R}^2} r$$

where r : *the radius of the equilibrium orbit*
(displacement) for a particle of diameter d_p

For *all* the particles to be collected, $r \geq R$

$$d_{p,crit}^2 = \frac{18\mu}{\rho_p - \rho_f} \frac{U_R}{U_{\Theta R}^2} R$$

where $d_{p,crit}$: *Critical(minimum) diameter*
of the particles to be collected

↓

or

If $d_p > d_{p,crit}$, $G(d_p) = 1$ and otherwise, $G(d_p) = 0$

Grade efficiency curve ($G(d_p)$ vs. d_p)는 진정 step function인가?

3) Cyclone Grade Efficiency in Practice

More Practical Analysis

Leith and Licht (1980)

Velocity distribution

$$U_\Theta r^m = constant$$

where $m = 1 - (1 - 0.67 D_c^{0.14}) \left(\frac{T}{283} \right)^{0.3}$

Grade efficiency

$$G_N(d_p) = 1 - \exp(-\Psi d_p^M)$$

where $M = \frac{1}{m+1}$

$$\Psi = 2\left[\frac{KQ\rho_p C_c(m+1)}{18\mu D_c^3}\right]^{M/2}$$

K : *geometric configuration parameter*

∴ *Grade efficiency curve* for cyclone - Figure 7.4, Figure 7.6

Why *not a step function*? ☞ distorted due to *velocity fluctuation*

particle-particle interaction

$$\therefore\ \underset{\underset{G_N(0.5)}{\uparrow}}{d_{p,50}} \text{ and } St_{50} \text{ in stead of } \underset{\underset{G_N(1.0)}{\uparrow}}{d_{p,crit}} \text{ and } St_{crit}$$

7.4 Scale-Up of Cyclones

Design of Cyclone

From both *theoretical and actual* analysis for given cyclone,

$$St_{50}\left(\equiv \frac{\rho_p d_{p,50}^2 U}{18\mu D}\right) \sim constant \rightarrow d_{p,50} \propto \sqrt{\mu D^3/\rho_p Q}$$

$$Eu\ \left(\equiv \frac{\Delta p}{\rho_f U^2/2}\right) \underset{\underset{\textit{independent of } Re}{\uparrow}}{\sim} constant \underset{\underset{U = Q/\frac{\pi}{4}D^2}{\uparrow}}{\rightarrow} \Delta p \propto Q^2/D^4$$

Standard Cyclone Designs - dimension

Figure 7.5

For suspension concentration less than ~ 5g/m^3

- *High efficiency* Stairmand cyclone:

$$St_{50} = 1.4\times10^{-4} \text{ and } Eu = 320$$

- *High flow rate* Stairmand cyclone

$$St_{50} = 6\times10^{-3} \text{ and } Eu = 46$$

Approximately

$$Eu = \sqrt{\frac{12}{Stk_{50}}}$$

Practical grade efficiency $= \dfrac{\left(\dfrac{d_p}{d_{p,50}}\right)^2}{1+\left(\dfrac{d_p}{d_{p,50}}\right)^2}$

for typical dimension

Figure 7.6

7.5 Range of Operation

Efficiency and pressure drop Figure 7.7

From theory, E_T↑ as q↑

But! there is maximum E_T due to re-entrainment of separated solids at high q...

Recommended range of Δp: 500 to 1500Pa...

In this range E_T↑ as q↑....

N cyclones in parallel

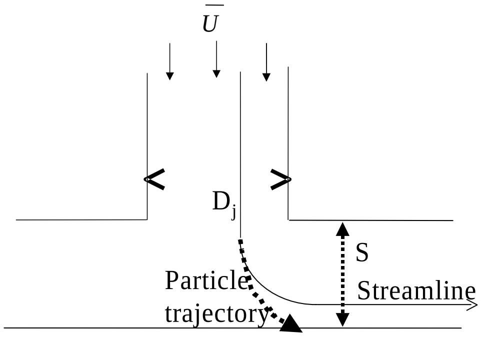
처리양이 많으면 한 개의 cyclone으로는 부족하다.. 그래서 여러 개를 병렬 연결하여 처리한다.

$$Q \rightarrow Q/N$$

Worked Example 7.1

Worked Example 7.2

7.6 Aerosol Impactor



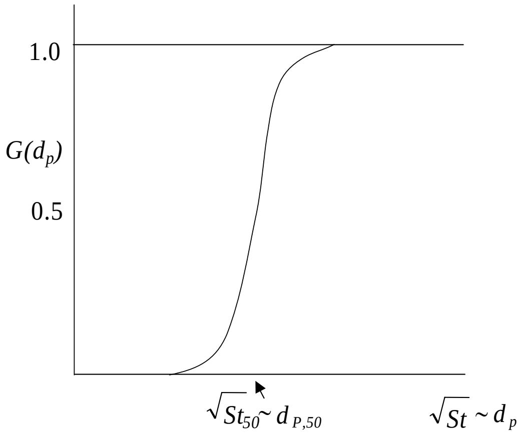
In general, for inertial motion of particles from Chapter 3,

$$G(d_p) = f\left(St(d_p), Re, \frac{S}{D_j}\right)$$

where $St(d_p) = \frac{\tau(d_p)\overline{U}}{D}$

For given geometry (S/D_j)

$0.5 = f(St_{50}, Re) \rightarrow St_{50} = f_1(Re)$



From *numerical and/or experimental* analysis

$St(d_p)$: ***almost independent*** of Re

Or for 500 < Re < 3000 and S/D > 1.5

For *circular* nozzle, St_{50} = 0.22

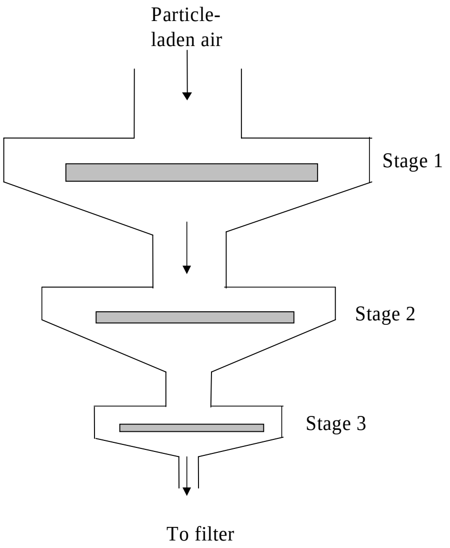
For *rectangular* nozzle, St_{50} = 0.53

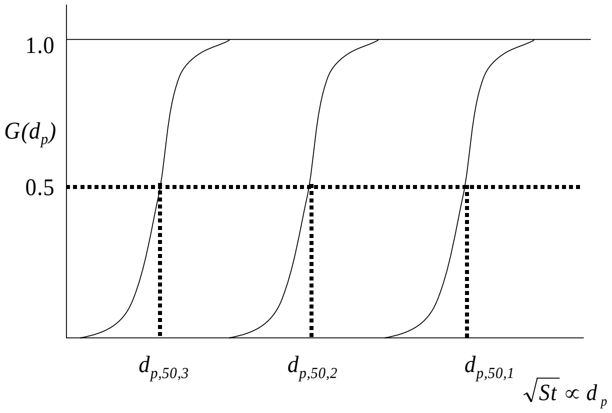
$$\therefore\ d_{p,50} = \left[\frac{9\mu D St_{50}}{\rho_p U C_c}\right]^{1/2}$$

작은 입자를 잡으려면 노즐 입경을 줄이고, 유속을 올리는 방법과 C_c를 올리는 방법이 있다.

C_c를 올리려면 어떻게 해야 하나?

* Cascade impactor





- Measurement of *particle size distribution*
- *Classification* of particles