Chapter 2 Multiple Particle Systems

2.1 Settling of a Suspension of Particles

Hindered Settling

입자가 모여 있으면 서로의 영향이 침강에 미친다. 이를 간섭침강이라 한다.

Effective viscosity of suspension

$$\mu_e = \frac{\mu}{f(\varepsilon)}$$

Effective density of suspension

$$\rho_{ave} = \varepsilon\rho_f + (1-\varepsilon)\rho_p$$

U_p , U_f: actual(interstitial) velocity of particles and fluid, respectively

U_{ps} , U_{fs}:superficial velocity

$$U_{ps} = U_p(1-\varepsilon)$$

$$U_{fs} = U_f\varepsilon$$

where ε : voidage or void fraction

$\therefore 1-\varepsilon = \frac{particle\ volume}{suspension\ volume} = volume\ concentration\ of\ particles = c_v$ = volume

$\therefore$ U_{ps} and U_{fs}: volume flux of particles and fluid

($m/s \cdot \frac{m^3\ fluid\ or\ particles}{m^3\ suspension}$)

☞ U_{ps}와 U_{fs}는 superficial velocity임과 동시에 부피 flux 임!

Corrected terminal settling velocity

$$U_{rel_T} \equiv U_p - U_f$$

$$\neq U_T$$

$$\equiv U_T\varepsilon f(\varepsilon)$$

Because of no net flow

$$U_{ps} + U_{fs} = 0$$

2.2 Batch Settling

(1) Settling Flux as a Function of Suspension Concnetration

$$U_p(1-\varepsilon)+U_f\varepsilon=0$$

$$\therefore\ U_p(1-\varepsilon)+[U_p-U_T\varepsilon f(\varepsilon)]\varepsilon=0$$

$$\therefore\ U_p=U_T\varepsilon^2 f(\varepsilon)$$

e.g. Richardson and Zaki(1954)

$$U_p=U_T\varepsilon^n$$

where

$$\frac{4.8-n}{n-2.4}=0.043Ar^{0.57}\left[1-2.4\left(\frac{d_p}{D}\right)^{0.27}\right]$$

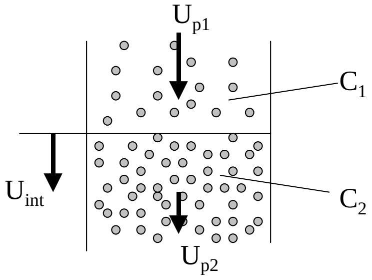
Superficial solid velocity or volumetric solid flux(m/s)

$$U_{ps}=U_p(1-\varepsilon)=U_T\varepsilon^2 f(\varepsilon)(1-\varepsilon)$$

$$=U_T(1-\varepsilon)\varepsilon^n$$

Settling flux curve (U_{ps} vs. C_v): Figure 2.1

(2) Sharp Interfaces in Sedimentation



Material Balance over the interface

$$(U_{p1}-U_{int})C_1=(U_{p2}-U_{int})C_2$$

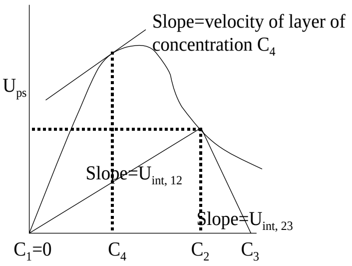
where $C=1-\varepsilon$, solids fraction

$$\therefore\ U_{int}=\frac{U_{ps_1}-U_{ps2}}{C_1-C_2}$$

As $\Delta C\rightarrow 0$,

$$U = \frac{dU_{ps}}{dC}$$

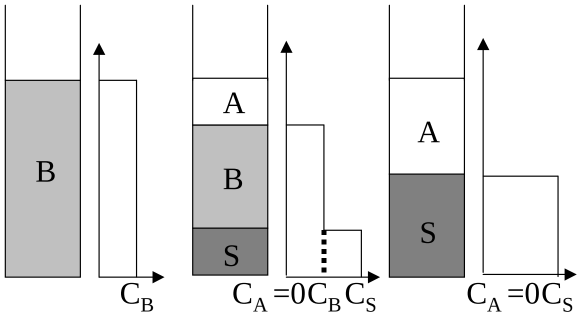
The velocity of layer of concentration C

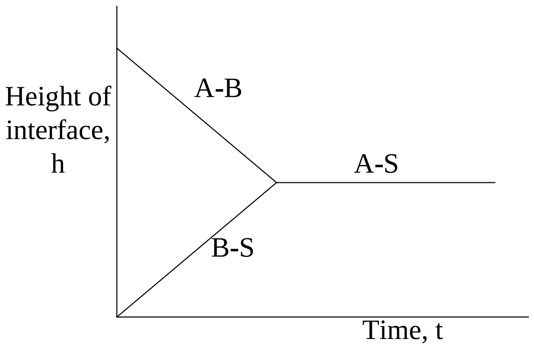


(3) **Batch Settling**

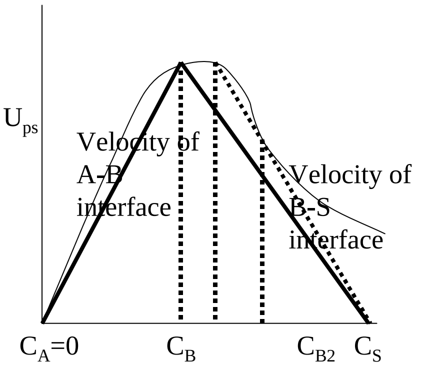
Supplying information for the design of a thickener

Type I Settling



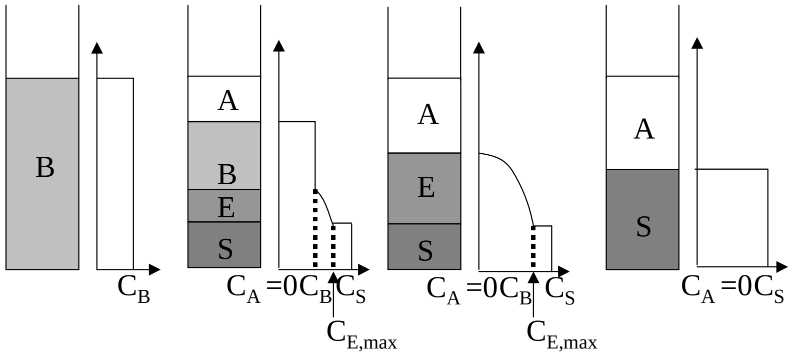


Variation of heights of interfaces with respect to time



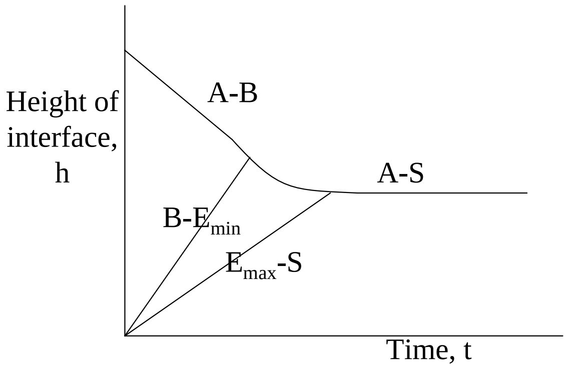
Flux vs. concentration

Type II Settling



Recommended web site :

http://www.aem.umn.edu/Solid-Liquid_Flows/video.html



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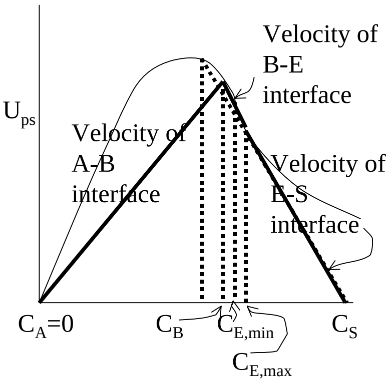


Flux vs. concentration for particle slurry

즉 두 type의 침강은 초기농도에 의존하는 것으로서 후자의 경우 하나의 층이 더 나타나는 것은 C_B의 위치에서 C_S가 변곡점 때문에 가려서 생기는 문제이다. 따라서 농도가 진하여 변곡점 아주 가까이에 있을 때 type II의 침강이 생긴다.

2.3 Continuous Settling

(1) Settling of a Suspension in a Flowing Fluid

Thickener vs. Clarifier Figure 2.11

Downward flow

$$U_{ps} = \frac{Q(1-\varepsilon)}{A} + U_T \varepsilon^2 f(\varepsilon)$$

Total solid flux	*Flux due to bulk flow*	*Flux due to settling*

Define $C_v \equiv 1-\varepsilon$, *particle volume concentration*

Figure 2.12:

Feed concentration, $C_F \rightarrow$ bottom section concentration, C_B

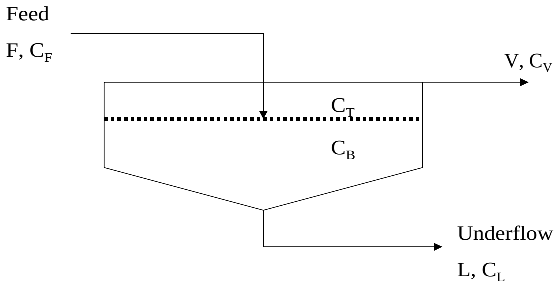
Upward flow

$$U_{ps} = \frac{-Q(1-\varepsilon)}{A} + U_T \varepsilon^2 f(\varepsilon)$$

Figure 2.13:

Feed concentration, $C_F \rightarrow$ Top section concentration, C_T

(2) Real Thickener



Feed/ Under(down)flow/ up(over)flow:

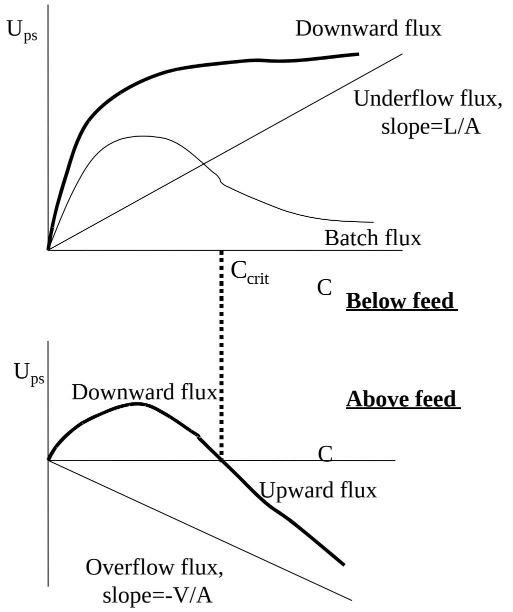
F(C_F) L(C_L) V(C_V)

Below feed

$$U_{ps} = \frac{L(1-\varepsilon)}{A} + U_T \varepsilon^2 f(\varepsilon)$$

Above feed

$$U_{ps} = \frac{-V(1-\varepsilon)}{A} + U_T \varepsilon^2 f(\varepsilon)$$



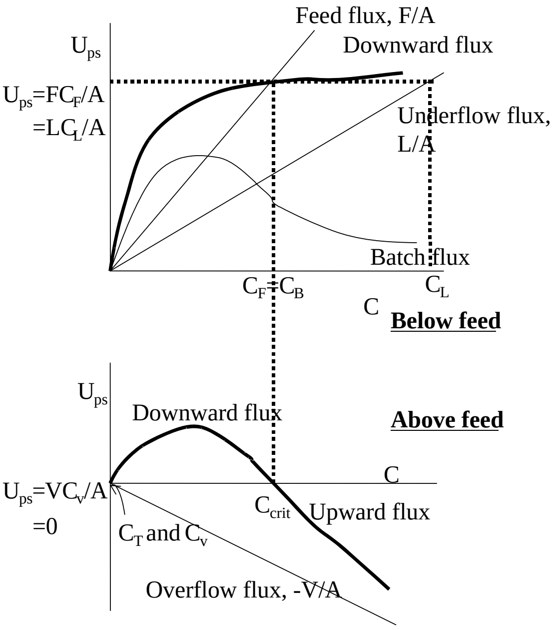
* Critical concentration: $C_{crit} \equiv C$ at $U_{p,\,upward} = 0$

(3) Critically Loaded Thickener

$$C_F = C_{crit}$$

∴ $U_{p,\,upward} = 0$, $C_B = C_F$, $C_T = C_V = 0$ and

$$U_{ps,\,downflow} = \frac{FC_F}{A} = \frac{LC_L}{A}$$



(4) Underloaded Thickener

$$C_F < C_{crit}$$

$$\therefore \quad U_{p,\,upward} = 0, \quad C_B < C_F \text{ and } C_T = C_V = 0$$

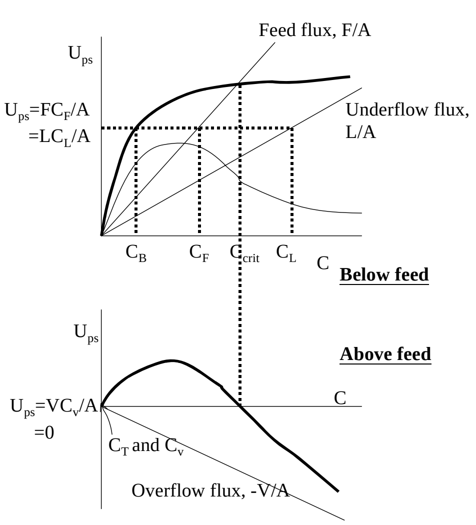
$$U_{ps,\,downflow} = \frac{FC_F}{A} = \frac{LC_L}{A}$$

(5) Overloaded Thickener

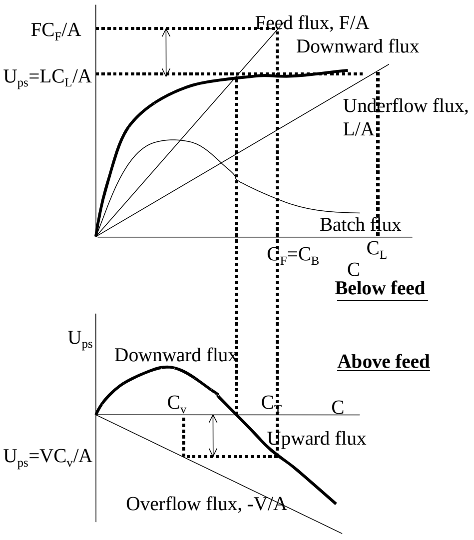
$$C_F > C_{crit}$$

$$\therefore \quad U_{p,\,upward} = \frac{FC_F}{A} - U_{ps,\,downflow}, \quad C_B = C_F \text{ and } C_T > C_V \neq 0$$

$$U_{ps,\,downflow} = \frac{LC_L}{A}$$



Underloaded Thickener



Overloaded thickener

* Centrifugal Sedimentation $r\omega^2$ instead of g

Worked Example 2.4