Chapter 8. Storage and Flow of Powders - Hopper Design

입자간의 상호작용과 분체층(분체집단)의 역학에 대해 알아보자.

☞http://www.dietmar-schulze.de/storage.html

8.1 Introduction

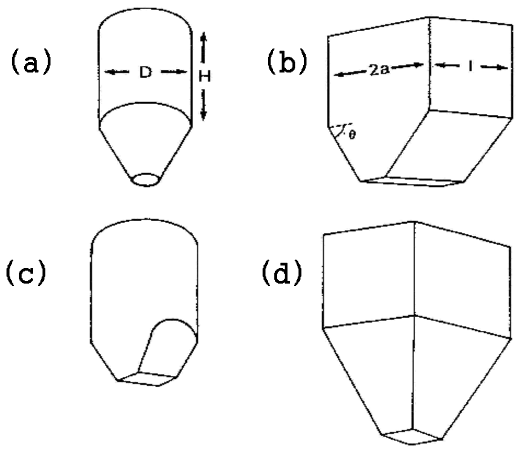
Storage tanks

Silos : section of constant cross sectional area

- Bins : H > 1.5 D
- Bunker : H < 1.5 D

Hopper : section of reducing cross sectional area downwards

Typical bulk solids storage vessels



(a) conical and axisymmetric hopper; (b) plane-flow wedge hopper; (c) plane flow chisel hopper; (d)pyramid hopper

8.2 Mass Flow vs. Core(Funnel) Flow

Mass flow vs. core flow : Figure 8.1

Figure 8.2 Figure 8.3

To see the mass flow in hopper ☞

http://www.cco.caltech.edu/~granflow/movies.html

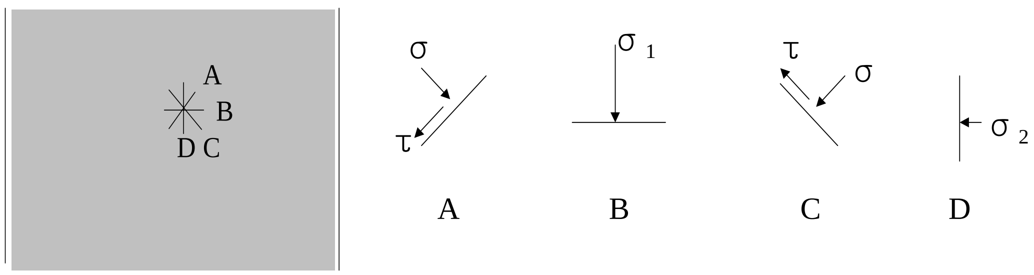
Table Comparison of mass flow and core flow of particulate materials.

Mass flow	Core flow
Characteristics	
No stagnant	Stagnant zone formation
Uses full cross-section of vessel	Flow occurs within a portion of vessel cross-section
First-in, first-out flow	First-in, last-out flow
Advantages	
Minimises segregation, agglomeration of materials during discharge	Small stress on vessel walls during flow due to the 'buffer effect' of stagnant zones Very low particle velocities close to vessel walls; reduced particle attrition and wall wear
Disadvantages	
Large stresses on vessel walls during flow	Promotes segregation and agglomeration during flow
Attrition of particles and erosion and wear of vessel wall surface due to high particle velocities	Discharge rate less predictable as flow region boundary can alter with time.
Small storage volume to vessel height ratio	

8.2S Stresses in Bulk Solids

(1) Mohr Stress Circle

<u>Two dimensional stresses in the powder bed</u>



- Normal stress, σ
- Shear(tangential) Stress, τ

분체층은 정지되어 있어도 유체와는 달리

- 수직응력(유체에서 보통 압력으로 부름)이외에 전단력이 존재하고,

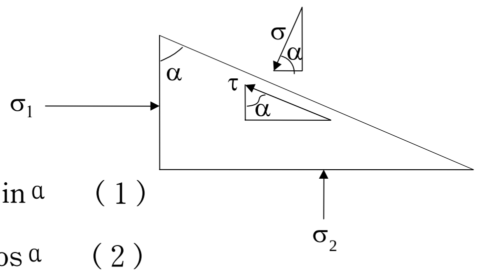
- 수직응력과 전단응력은 면의 배향에 따라서도 달라진다.
- *Principal stresses* (major, minor), σ_1, σ_2

  : normal stresses to the plane in which shear stresses vanish

  where $\sigma_1 \perp \sigma_2$

면의 배향에 따라서는 전단응력이 없어지는 경우가 두 개 생기며 이 두면은 서로 수직하고, 하나는 최대, 다른 하나는 최소 응력을 가진다.

Correlating σ and τ, in terms of principal stresses



From force balances,

$$\sigma_1 \cos\alpha = \sigma\cos\alpha + \tau\sin\alpha \qquad (1)$$

$$\sigma_2 \sin\alpha = \sigma\sin\alpha - \tau\cos\alpha \qquad (2)$$

$(1)\times\cos\alpha + (2)\times\sin\alpha$

$$\sigma_1\cos^2\alpha + \sigma_2\sin^2\alpha = \sigma$$

$(1)^2 + (2)^2$

$$\sigma_1^2\cos^2\alpha + \sigma_2^2\sin^2\sigma = \sigma^2 + \tau^2$$

Eliminating $\sin\alpha$ and $\cos\alpha$

$$\left(\sigma - \frac{\sigma_1 + \sigma_2}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_1 - \sigma_2}{2}\right)^2$$

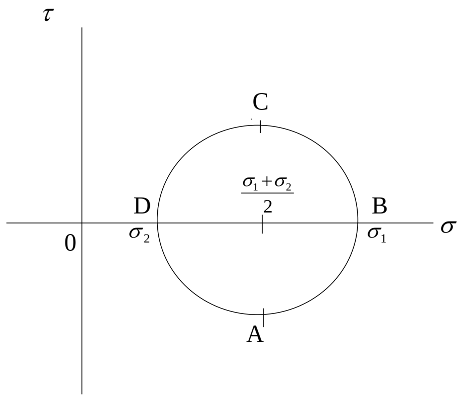
Mohr Stress Circle

정지한 분체층의 한면에 작용하는 수직응력과 전단응력은 *Mohr 원*의 원 둘레에 위치한다.

8.4 Shear Cell Tests - Yield Behavior of Bulk Powder

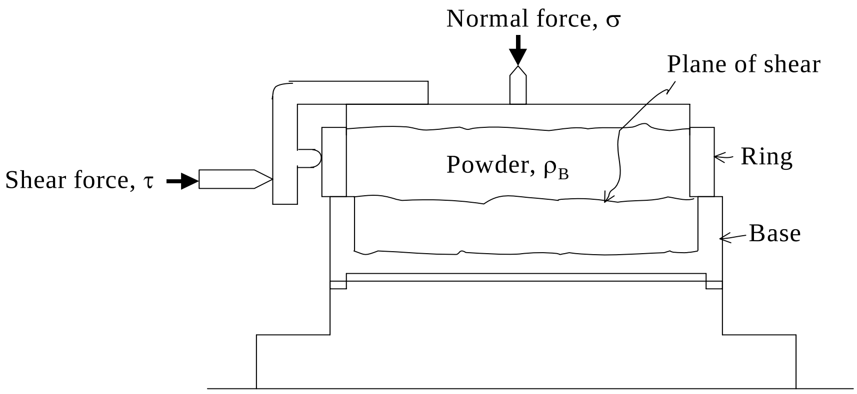
Powder Bed :

- Fixed : Adsorption beds, catalyst beds, packed beds for absorber
- Moving : Feeding in storage tank



Mohr 원

<u>Jenike shear cell</u>

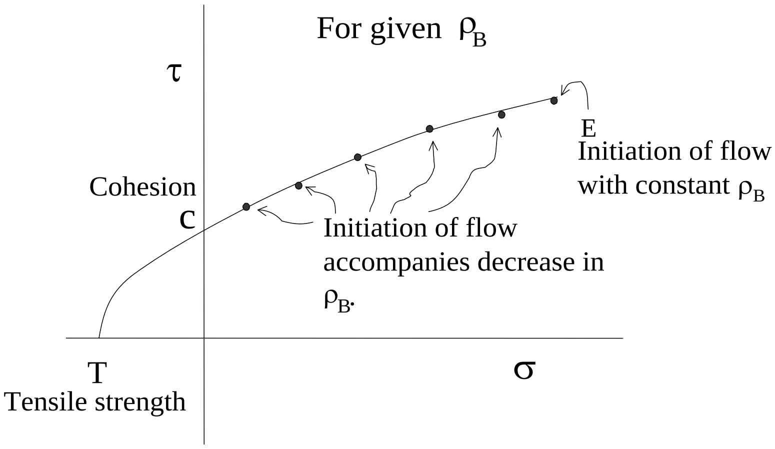


<u>Jenike yield locus</u>

- Put the powder sample of ρ_B in the cell.
- Note the horizontal stress(τ) to initiate flow for the given normal stress(σ). (σ must be low enough for ρ_B to decrease during application of τ)
- Repeat this procedure for each identical powder sample(ρ_B) with greater σ until ρ_B does not decrease. Five or six pairs of (σ, τ) should be generated.

이를 τ *vs.* σ로 그림을 그리면 다음과 같은 *Janike yield locus*가 얻어진다. 이 선 상에 있는 점들은 주어진 ρ_B에서 분체층이 이제 막

움직이기(흐르기) 시작하는 바로 그 상황을 의미한다.

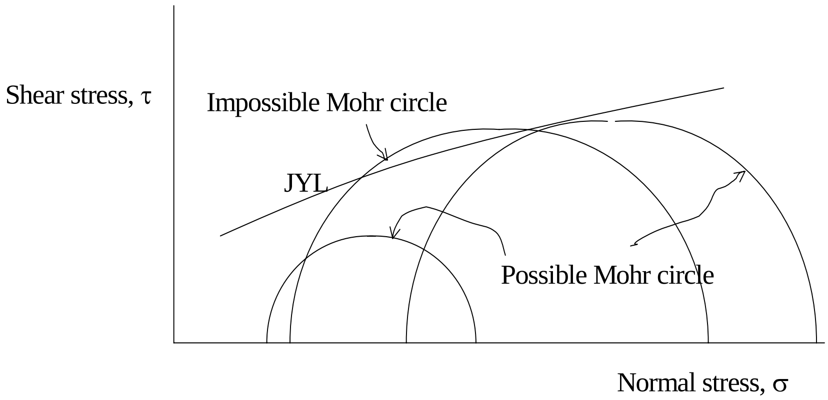


- Expanded flow(at the points up to E on the curve)
- Free flow (at point E): critical flow

- Cohesion
- Tensile strength

8.5 Analysis of Shear Cell Test Results

(1) JYL vs. Mohr stress circle



따라서 JYL은 Mohr 원(정지한 분체층의 상태)이 위치할 수 있는 극한 상황이라 할 수 있다.

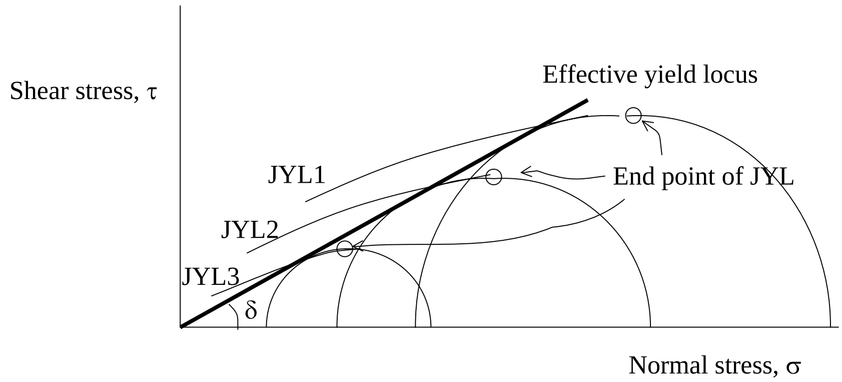
(2) Determination of δ from Shear Cell Tests

* *Effective Yield Locus*

서로 다른 ρ_B에서 JYL의 끝점(E) 들을 접하는 원들을 그리고, 이 원들을 접하는 선을 그리면 원점을 지나는 직선이 얻어진다. 여기서 마찰각 σ를 얻는다.

$$\therefore \tau = \sigma \ \tan\delta$$

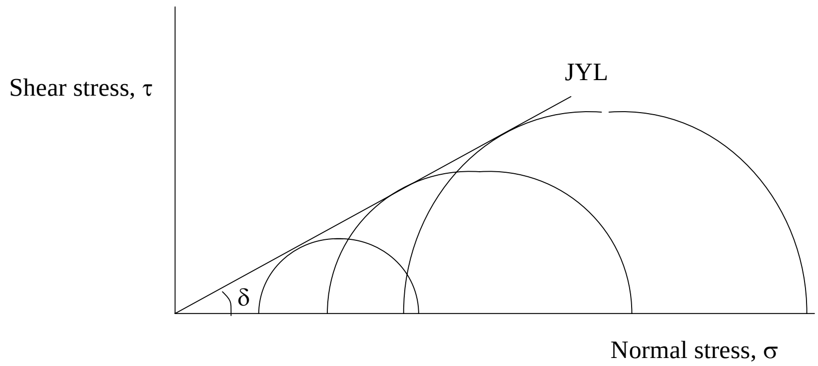
where δ : effective (internal) angle of friction



Worked Example 8.1(a)

Ex.8.4, 8.5

* For free flowing powder

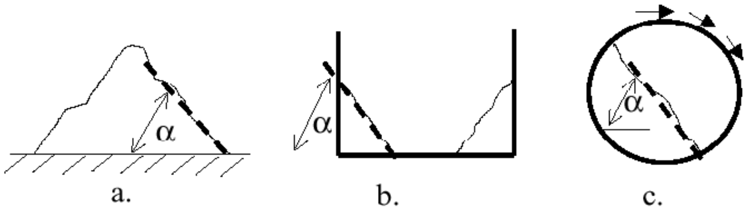


Yield locus for noncohesive (free-flow) powder

* 안식각 *Angle of Repose*, α

For noncohesive(free-flowing) particles

$$\phi \sim \alpha$$

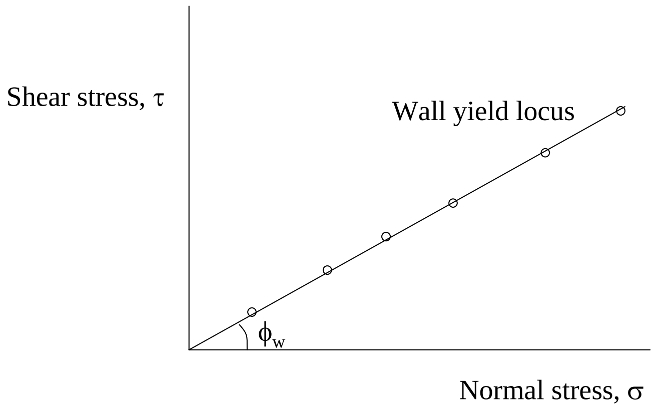


Angle of repose, α, of (a) a pile of powder, (b) powder in a container, and (c) powder in a rolling drum.

(5) The kinematic Angle of Friction between Powder and Hopper Wall Φ_W

Wall Yield Locus

비슷한 방법으로 벽과 분체층과의 yield locus도 구할 수 있다.

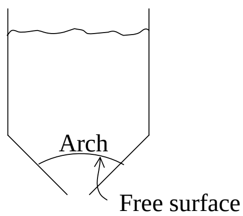


From wall shear test ☞ Figure 8.16

$$\tau_w = \sigma_w \tan\Phi_w$$

8.3 Design Philosophy

Hopper에서 항상 문제가 되는 것은 막힘현상이다. 이는 다음 그림에서 보는 것처럼 arch를 만드는 현상이며 흐름을 유지하기 위해서는 arching 없이 mass flow가 일어나도록 설계되어야 한다.



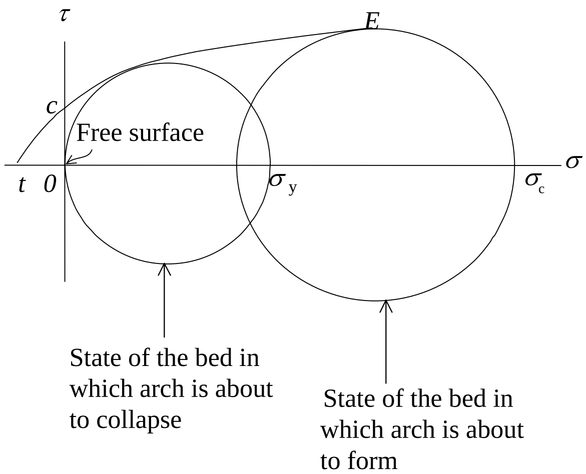
Arching

Arch - free surface, no flow

e.g. a salt shaker (a salt pourer?)

(1) Determination of σ_y and σ_c

τ vs. σ 그림에서 YL는 powder bed에서 flow가 일어나는 경계선이다. 따라서 이에 접하는 Mohr circle은 바로 이 상황의 bed 상황이며, 다음 그림의 두 개의 원이 각각 arch가 깨져 흐름이 막 시작되거나(왼쪽 원), free flow의 상태에서 arch가 이제 막 형성(오른 쪽 원)되는 상황을 나타낸다.



σ_y:: Unconfined yield strength (UYS),

σ_C: Compacting(consolidating) stress

(2) Powder flow function

$$\sigma_y = fn(\sigma_C)$$

Figure 8.6

(3) Flow-nonflow condition(Arch가 깨지는 조건)

For flow

$$\sigma_D > \sigma_y$$

where σ_D: actual stress of the powder developed under hopper condition

(4) Hopper flow factor

$$ff \equiv \frac{compacting\ stress\ of\ the\ powder\ under\ hopper\ condition}{actual\ stress\ of\ the\ powder\ developed\ under\ hopper\ condition}$$

$$= \frac{\sigma_C}{\sigma_D(\delta, \Phi_w, \Theta)}$$

where $ff = f(\delta, \Phi_w, \Theta)$

* Flow factor chart : Figure 8.18, Figure 8.19

(5) Critical Conditions for Flow

For flow

$$\frac{\sigma_C}{ff} > \sigma_y$$

Hopper Design

Find YL's and δ.

↓

Find the relation between σ_y and σ_C.

↓

Find WYL and Φ_W .

↓

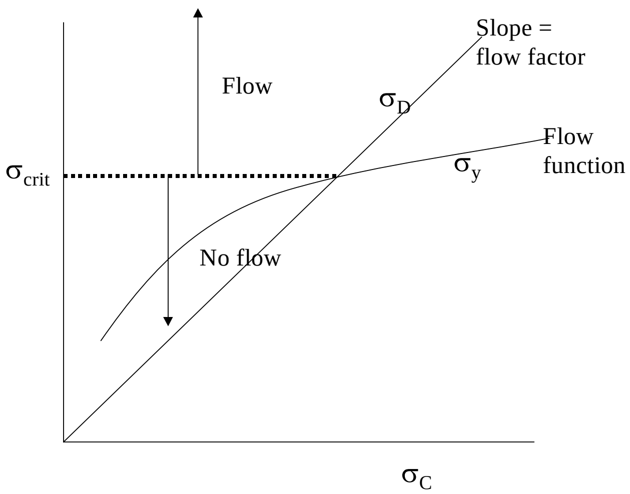
With Φ_W and δ, determine ff and Θ from Figures 8.18 and 8.19.

↓

Draw σ_D vs. σ_C(FF) and σ_y vs. σ_C.

↓

Determine $\overline{\sigma_{crit}}$



(6) Critical Outlet Dimension

$$D = H(\theta)\frac{\sigma_{crit}}{\rho_B g}$$

where

for conical hopper $H(\theta) = 2 + \frac{\theta}{60}$

for slot-type hopper $H(\theta) = 1 + \frac{\theta}{180}$

Example : σ=30° and Φ_w=19℃ → θ=(30.5°→27.5°) → ff=1.8

Worked Example 8.1

Worked Example 8.2

8.8 Pressure on the Base of a Tall Cylindrical Bin - Stresses in the Storage Tank

Vertical stress, σ_v

- In the cylindrical bins

Force balance on a slice of thickness ΔH in the powder bed,

$$D\Delta\sigma_v + 4\tan\Phi_w\sigma_h\Delta H = D\rho_B g\Delta H$$

Assuming $\sigma_H = k\sigma_v$ and $\Delta H \to 0$

$$\frac{d\sigma_v}{dH} + \left(\frac{4\tan\Phi_w k}{D}\right)\sigma_v = \rho_B g$$

Integrating

$$\sigma_v = \frac{D\rho_B g}{4\tan\Phi_w k}[1 - e^{-4\tan\Phi_w kH/D}] + \sigma_{v0}e^{-4\tan\Phi_w kH/D}$$

When no force acting on the free surface of the powder $\sigma_{v0} = 0$,

$$\sigma_v = \frac{D\rho_B g}{4\tan\Phi_w k}[1 - e^{-4\tan\Phi_w kH/D}]$$

Janssen equation

For small H,

$$\sigma_v \cong \rho_B H g$$

(liquid-like)

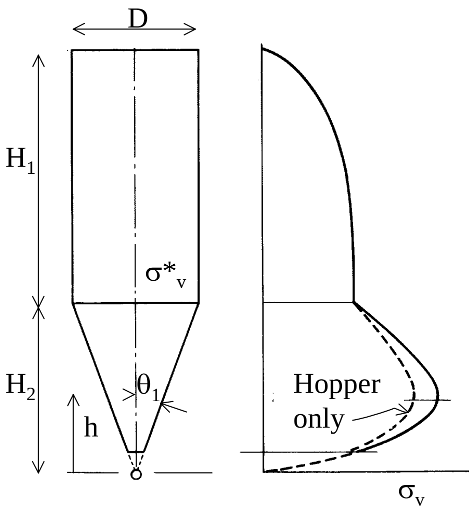
For large H ($> 4D$)

$$\sigma_v \cong \frac{D\rho_B g}{4\tan\Phi_w k}$$

independent of H and σ_{v0}

Figure 8.21

- In hopper



青木(1963), 坂下(1990)

For $C' \neq 1$,

$$\sigma_{v,2} = \sigma_v{*}\left(\frac{h}{H_2}\right)^{C'} + \frac{\gamma H_2}{C'-1}\left(\frac{h}{H_2}\right)\left[1-\left(\frac{h}{H_2}\right)\right]$$

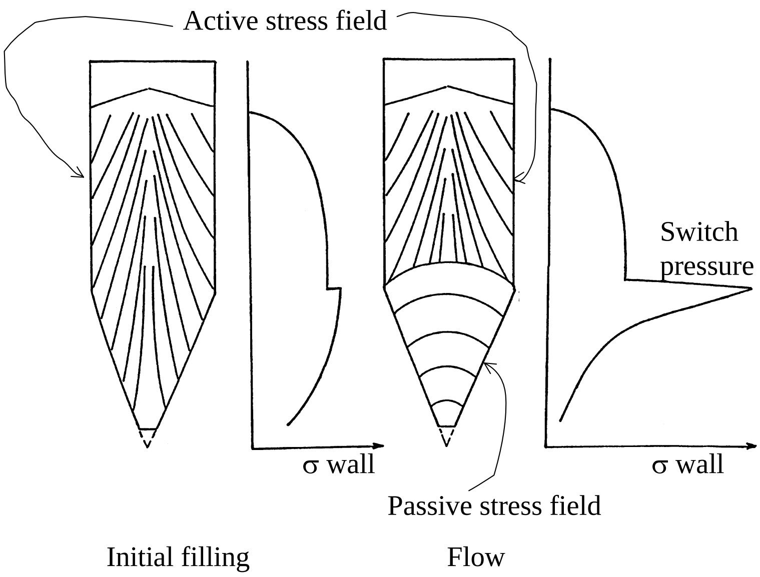
For $C' = 1$

$$\sigma_{v,2} = \sigma_v{*}\left(\frac{h}{H_2}\right)^{C'} + \gamma H_2\left(\frac{h}{H_2}\right)\ln\left(\frac{H_2}{h}\right)$$

where $C' \equiv 2\tan\Phi_W \cot\Theta_1 (K\cos^2\Theta_1 + \sin^2\Theta_1)$

<u>Wall stress distribution in silo-hopper</u>

Storage tank내의 압력은 저장중에는 위의 정압과 일치하나 feeding 과 discharge시에는 달라진다.



8.9 Mass Flow (Discharge) Rate

For cylindrical and conical hoppers

Beverloo(1961)

Dimensional analysis

$$M_p = C\rho_B g^{1/2}(B_0 - kx)^{5/2}$$

where C : 0.55 ~ 0.65

k : 1.5 or somewhat larger depending on particle shape

- Independent of H, D

For cohesionless coarse particles

$$M_p = \frac{\pi}{4}\sqrt{2}\rho_B g^{0.5} h^{0.5} B^2$$