Chapter 11. Principles of Heat Flow in Fluids

Heat-Exchange Equipment

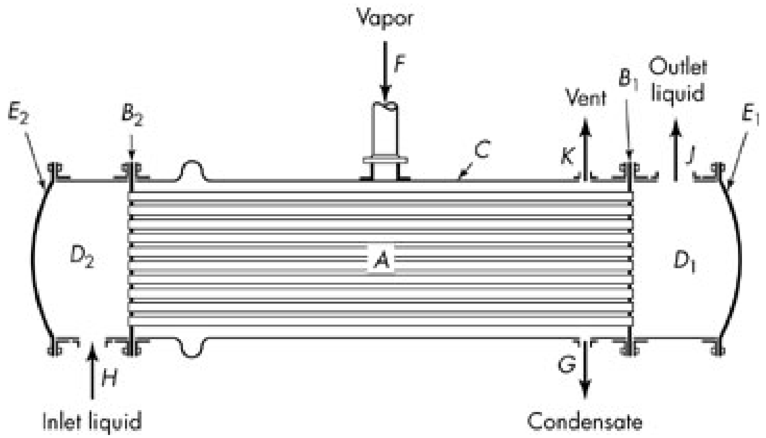


Fig. 11.1. Single pass tubular condenser:

A: tubes; *B_1*, *B_2*: tube sheets; *C*: shell; *D_1*, *D_2*: channels; *E_1*, *E_2*: channel covers; *F*: vapor inlet; *G*: condensate outlet; *H*: cold-liquid inlet; *J*: warm-liquid outlet; *K*: noncondensed gas vent.

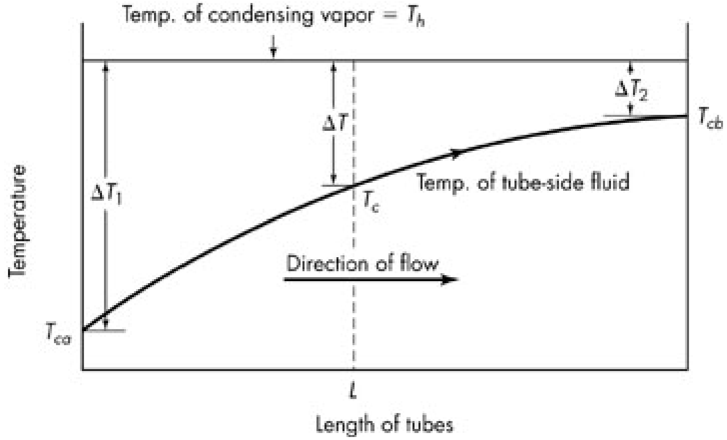


Fig. 11.2. Temperature-length curves for condenser.

Approach: terminal *T* difference, ΔT_1, ΔT_2

Range: *T* change of a fluid, T_{cb}-T_{ca}, T_{ha}-T_{hb}

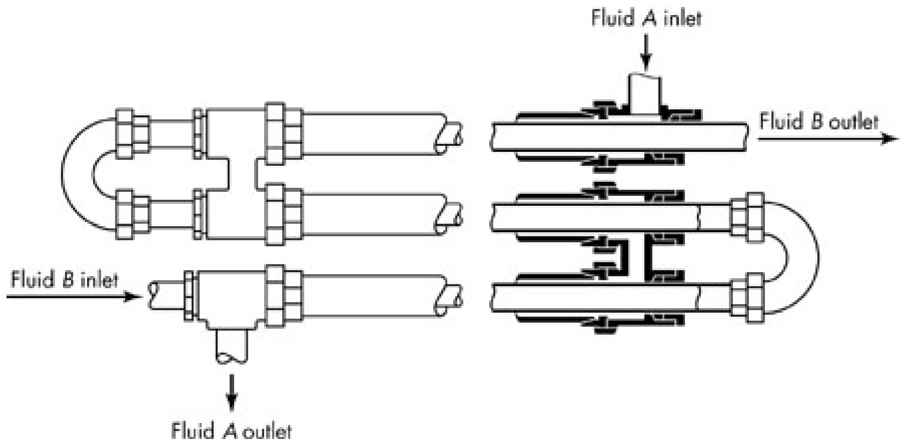
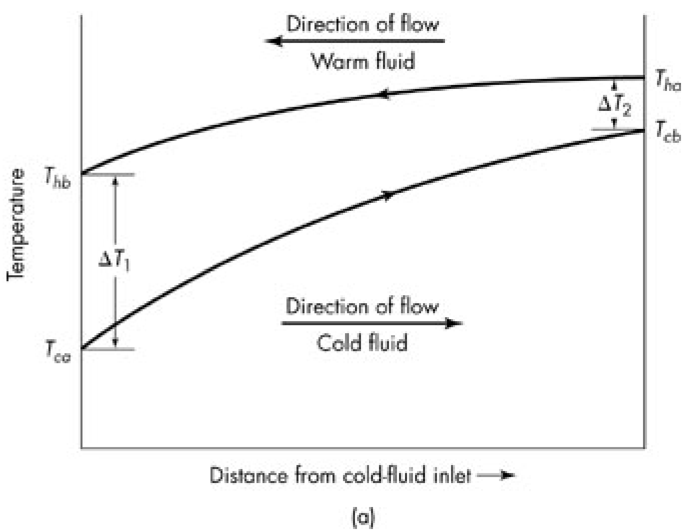


Fig. 11.3. Double-pipe heat exchanger.

*** Countercurrent flow (or counter flow)** 향류



Two fluids enter at different ends of HX.

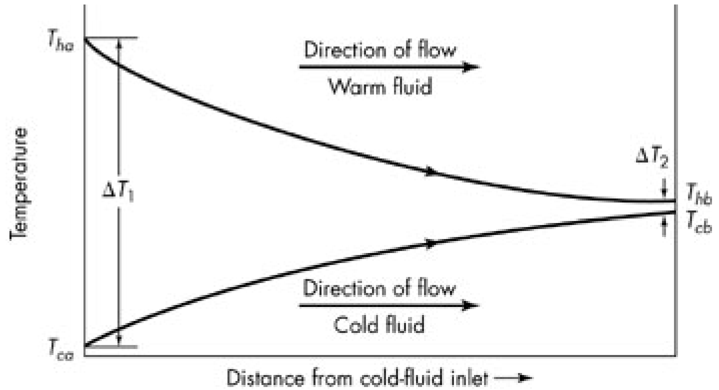
" pass in opposite directions.

approaches: ΔT_1, ΔT_2

warm fluid range: $T_{ha} - T_{hb}$

cold fluid range: $T_{cb} - T_{ca}$

*** Parallel flows (or cocurrent flow) 병류**



(b)

Two fluids flow in the same direction.

*cf.) **cross flow** 교차류*

Energy Balances

In heat exchangers, W_s, E_p & $E_k \approx 0$.

$$\dot{m}(H_b - H_a) = q$$

mass flow rate → $\dot{m}$

enthalpies/mass at exit and entrance → H_b, H_a

rate of heat transfer → q

For the warm fluid, $\dot{m}_h (H_{hb} - H_{ha}) = q_h < 0$

For the cold fluid, $\dot{m}_c (H_{cb} - H_{ca}) = q_c > 0$

q_c = -q_h (← The heat lost by the warm fluid is gained by the cold fluid)

$$\therefore\ \dot{m}_h (H_{ha} - H_{hb}) = \dot{m}_c (H_{cb} - H_{ca}) = q$$

: overall enthalpy balance

For a condenser,

$$H_{ha} = \lambda + c_{ph} T_{ha}, \quad H_{hb} = c_{ph} T_{hb}, \quad H_{ca} = c_{pc} T_{ca}, \quad H_{cb} = c_{pc} T_{cb}$$

$$\therefore\ \dot{m}_h \left[\lambda + c_{ph}(T_{ha} - T_{hb})\right] = \dot{m}_c\, c_{pc} (T_{cb} - T_{ca})$$

latent heat (λ)

specific heat of the condensate (c_{ph})

specific heat of cold fluid (c_{pc})

Heat Flux and Heat-Transfer Coefficients

. Heat flux: the rate of heat transfer per unit area

. Average stream temperature (or mixing-cup temperature):

average temperature of fluid stream

*** Overall heat-transfer coefficient (총괄 열전달계수) *U***

Driving force: $T_h - T_c$ (overall local temperature ΔT)

$$\frac{dq}{dA} (\text{local flux}) \propto \Delta T$$

$$\therefore \frac{dq}{dA} = U\,\Delta T = U(T_h - T_c) \qquad \text{--- Eq. (11.9)}$$

U: local overall heat-transfer coefficient

$$\frac{U_o}{U_i} = \frac{dA_i}{dA_o} = \frac{D_i}{D_o}$$

U_o: overall heat-transfer coefficient based on outside surface area

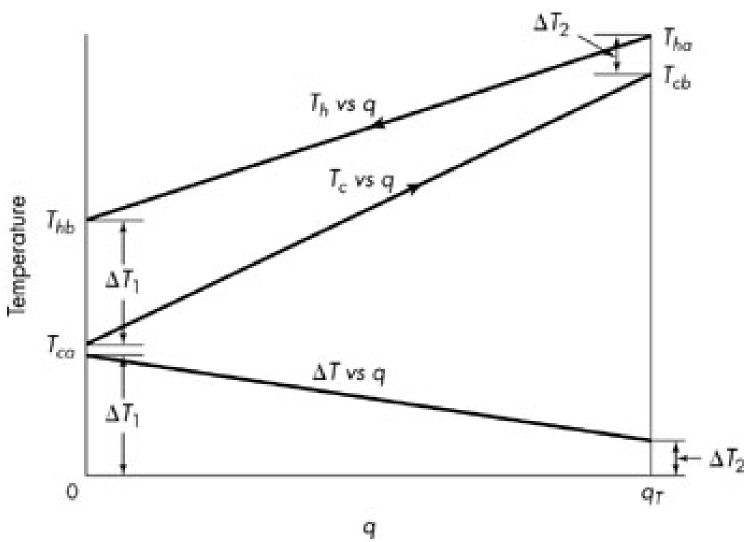
U_i: " " inside surface area

* Integration over total surface

Integration of Eq. (11.9) to the entire area of a heat exchanger

Assumptions:

1) U --- constant
2) c_{pc}, c_{ph} --- constant
3) heat exchange with ambient --- negligible
4) flow --- steady, either parallel or countercurrent



T vs. q in countercurrent flow
(가정 2와 4 하에서의 그래프)

T_c & T_h vary linearly with q. (가정 2와 4 적용)

→ ΔT “ .

$$\frac{d(\Delta T)}{dq} = \frac{\Delta T_2 - \Delta T_1}{q_T} \longrightarrow \text{기울기 constant}$$

rate of heat transfer
in entire heat exchanger

$dq = U\Delta T dA$ 에 대입하면

$$\frac{d(\Delta T)}{U\Delta T dA} = \frac{\Delta T_2 - \Delta T_1}{q_T}$$

적분: $0 \rightarrow A_T$ for A

$\Delta T_1 \rightarrow \Delta T_2$ for ΔT

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = \frac{U(\Delta T_2 - \Delta T_1)}{q_T} \int_0^{A_T} dA$$

$$\therefore q_T = UA_T \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} = UA_T \overline{\Delta T_L}$$

logarithmic mean temperature difference (**LMTD**):

$$\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}$$

* Individual heat-transfer coefficient (개별 열전달계수) *h*

U depends on many variables.

Consider a specific point in double-pipe heat exchanger

& Assume { turbulent flow
surface of tube – clear of dirt or scale

Individual heat-transfer coefficient, *h*

$$h = \frac{dq/dA}{T - T_w}$$

- dq/dA → heat flux
- T → average T
- T_w → wall T

1/*h*: thermal resistance *cf.*) x_w/k for conduction

$$h_i = \frac{dq/dA_i}{T_h - T_{wh}} \quad \text{for the inside tube}$$

$$h_o = \frac{dq/dA_o}{T_{wc} - T_c} \quad \text{for the outside tube}$$

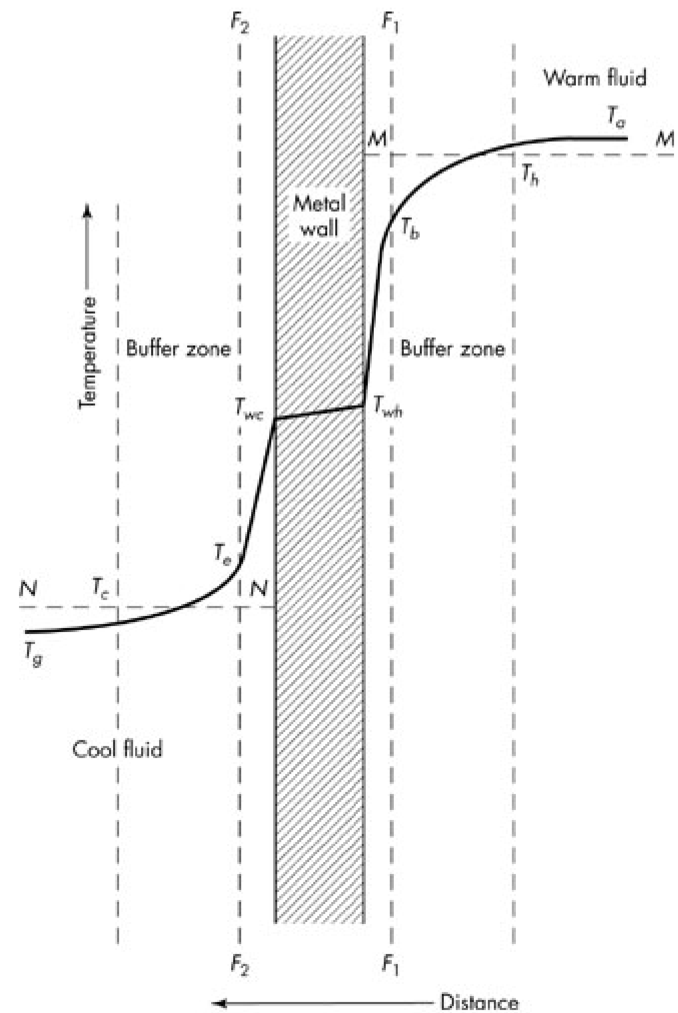


Fig. 11.8. *T* gradients in forced convection

. Heat transfer very near the wall occurs only by conduction.

$$\frac{dq}{dA} = -k\left(\frac{dT}{dy}\right)_w \quad \Rightarrow \quad h = -k\frac{(dT/dy)_w}{T - T_w}$$

$$\boxed{\frac{hD}{k} = -D\frac{(dT/dy)_w}{T - T_w}} \quad \sim \frac{(dT/dy)_w}{(T - T_w)/D}$$

$(dT/dy)_w$ → T gradient at the wall

$(T - T_w)/D$ → average T gradient across the entire pipe

Nu (**Nusselt number**)

: *the ratio of the total heat transferred to the heat by conduction*

. Another interpretation of the Nusselt number

If all the resistance to heat transfer is in a laminar layer of thickness x

in which heat transfer is only by conduction.

$$\frac{dq}{dA} = \frac{k(T - T_w)}{x} \qquad h = \frac{k}{x}$$

$$\text{Nu} = \frac{hD}{k} = \frac{k}{x}\frac{D}{k} = \frac{D}{x}$$

$\therefore$ *the ratio of the tube diameter to the equivalent thickness of the laminar layer*

* Calculation of overall coefficients from individual coefficients

From Fig. 11.8,

$$(T_h - T_{wh}) + (T_{wh} - T_{wc}) + (T_{wc} - T_c) = T_h - T_c = \Delta T$$

$$= dq\left(\frac{1}{dA_i h_i} + \frac{x_w}{d\overline{A}_L k_m} + \frac{1}{dA_o h_o}\right)$$

x_w → tube wall thickness　　k_m → thermal conductivity of wall

. Heat flux based on the outside area

$$\frac{dq}{dA_o} = \frac{T_h - T_c}{\frac{1}{h_i}\left(\frac{dA_o}{dA_i}\right) + \frac{x_w}{k_m}\left(\frac{dA_o}{d\overline{A}_L}\right) + \frac{1}{h_o}}$$

$$= \frac{T_h - T_c}{\frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{x_w}{k_m}\left(\frac{D_o}{\overline{D}_L}\right) + \frac{1}{h_o}}$$

$$\therefore \frac{1}{U_o} = \frac{D_o}{D_i h_i} + \frac{x_w}{k_m}\frac{D_o}{\overline{D}_L} + \frac{1}{h_o}$$

$$\overline{D}_L = \frac{D_o - D_i}{\ln(D_o / D_i)}$$

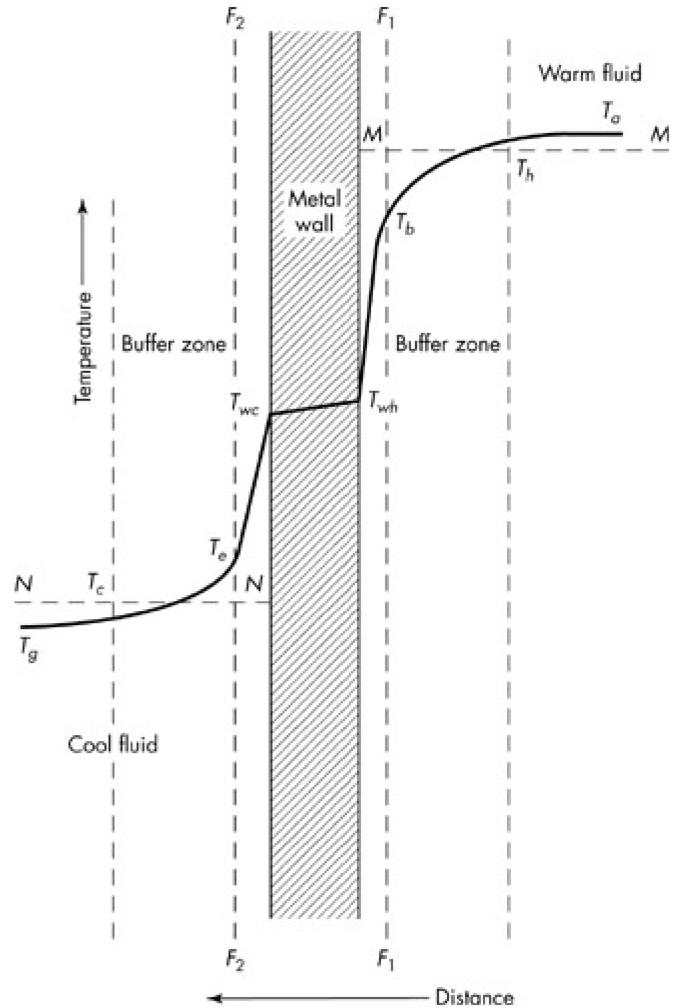


Fig. 11.8. *T* gradients

. Heat flux based on the inside area

앞과 마찬가지로 정리해 보면,

$$\therefore \frac{1}{U_i} = \frac{1}{h_i} + \frac{x_w}{k_m}\frac{D_i}{\overline{D}_L} + \frac{D_i}{D_o h_o}$$

. Overall temperature drop (ΔT) $\propto \frac{1}{U}$

. Temperature drop in two fluids & wall $\propto$ individual resistances

$$\frac{\Delta T}{1/U_o} = \frac{\Delta T_i}{D_o / D_i h_i} = \frac{\Delta T_w}{(x_w / k_m)(D_o / \overline{D}_L)} = \frac{\Delta T_o}{1/h_o}$$

ΔT_i: T drop through inside fluid

ΔT_w: T drop through metal wall

ΔT_o: T drop through outside fluid

← Eq. (10.13)과 같은 resistance 형태:

$$\frac{\Delta T}{R} = \frac{\Delta T_A}{R_A} = \frac{\Delta T_B}{R_B} = \frac{\Delta T_C}{R_C}$$

. Overall resistance, $R_o = \frac{1}{U_o} = \frac{D_o}{D_i h_i} + \frac{x_w}{k_m}\frac{D_o}{\overline{D}_L} + \frac{1}{h_o}$

*** Fouling factors (오염계수)**

Actually, heat-transfer surfaces do not remain clean – Scale, dirt & solid deposits form.

→ provide additional resistances to heat flow

→ reduce the overall coefficient

h_{di}, h_{do}: the fouling factors for the scale deposits on the inside & outside tube surfaces

Then,

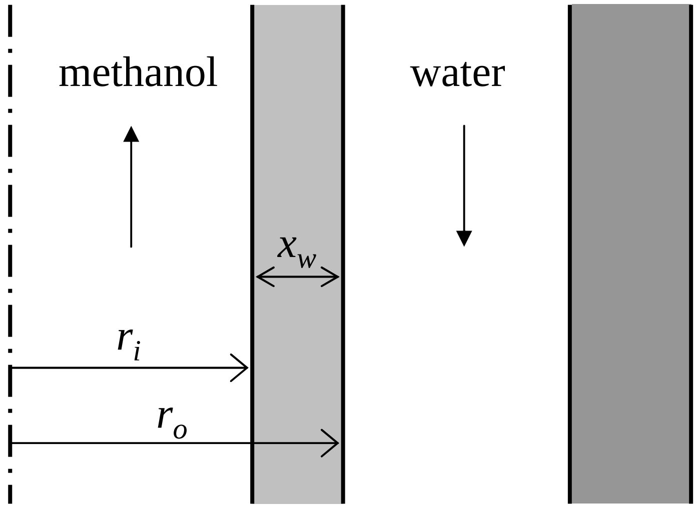
$$U_o = \frac{1}{(D_o / D_i h_{di}) + (D_o / D_i h_i) + (x_w / k_m)(D_o / \overline{D}_L) + (1/h_o) + (1/h_{do})} \quad \text{--- Eq. (11.37)}$$

and

$$U_i = \frac{1}{(1/h_{di}) + (1/h_i) + (x_w / k_m)(D_i / \overline{D}_L) + (D_i / D_o h_o) + (D_i / D_o h_{do})} \quad \text{--- Eq. (11.38)}$$

Fouling factors --- *a safety factor for design*

Ex. 11.1) MeOH flowing in the inner pipe of a double-pipe exchanger is cooled with water.



steel pipe wall
$(k_m = 26 \text{Btu/ft}^2 \cdot {}^oF)$

. 1 inch Schedule 40 steel pipe:
(from Appendix 3)
D_i = 0.0874 ft
D_o = 0.1096 ft
x_w = 0.0111 ft
$(D_o - D_i)/2$

. h & h_d: Table 11.1

What is the overall coefficient, based on the outside area of the inner pipe ? (즉, U_o=?)

(Ans.)

$$\bar{D}_L = \frac{D_o - D_i}{\ln(D_o / D_i)} = \ldots = 0.0983 \text{ft}$$

$$U_o = \quad \leftarrow \quad \text{from Eq.(11.37)}$$

$$= 80.9 \text{Btu/ft}^2 \cdot \text{h} \cdot {}^o\text{F}$$

*** Special cases**

In the special case that

Fouling effects are negligible

Metal wall is very thin (i.e., large-diameter thin-walled tube)

$\rightarrow \quad D_o / D_i \cong 1$

Then,

$$U_o = U_i = \frac{1}{1/h_o + x_w/k_m + 1/h_i}$$

Related problems: (Probs.) 11.1, 11.2 and 10.3.