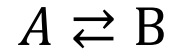


Runge-Kutta method: Chem Reaction Eng

예제 Chemical reaction engineering

반응물(A)가 생성물(B)로 1차 가역 반응에 의하여 진행된다.



정반응의 반응상수는 k_1 이며, 역반응의 반응상수는 k_2 이다. 위 반응에 대한 물질수지식을 세우면

$$-\frac{dC_A}{dt} = k_1 C_A - k_2 C_B, \quad -\frac{dC_B}{dt} = k_2 C_B - k_1 C_A$$

$C_{A0} = 1, C_{B0} = 0, k_1 = 10, k_2 = 5$ 라고 하면

$$-\frac{dC_A}{dt} = 10C_A - 5C_B, \quad -\frac{dC_B}{dt} = 5C_B - 10C_A$$

참고로 해석해를 구하면 다음과 같다.

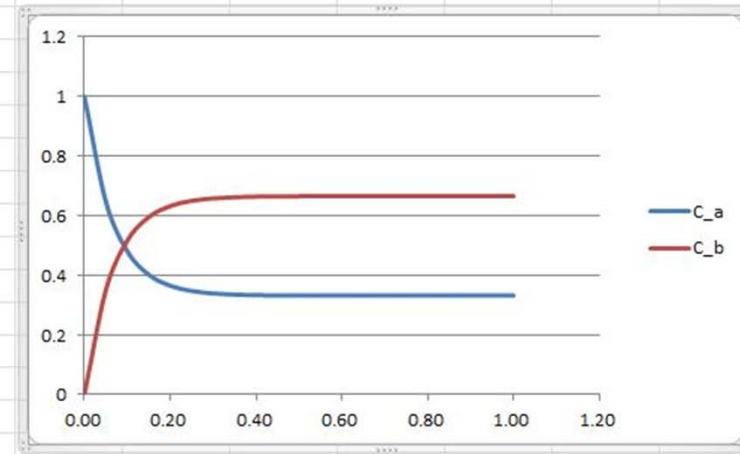
$$C_A = C_{A0} - m\{1 - e^{-(k_1+k_2)t}\}$$
$$C_B = C_{B0} + m\{1 - e^{-(k_1+k_2)t}\} \quad \left(m = \frac{k_1 C_{A0} - k_2 C_{B0}}{k_1 + k_2} \right)$$


예제 12-2

$$C_A = C_{A0} - m\{1 - e^{-(k_1+k_2)t}\} = \frac{2}{3}e^{-15t} + \frac{1}{3}$$

$$C_B = C_{B0} + m\{1 - e^{-(k_1+k_2)t}\} = \frac{2}{3}e^{-15t} - \frac{2}{3}$$

Analytical solution					
C_a=2/3*exp(-15*t)+1/3			C_b=-2/3*exp(-15*t)+2/3		
t	C_a		t	C_b	a+b
#VALUE!	1		#VALUE!	0	1
0.00	0.648244		0.05	0.351756	1
0.05	0.482087		0.10	0.517913	1
0.10	0.403599		0.15	0.596401	1
0.15	0.366525		0.20	0.633475	1
0.20	0.349012		0.25	0.650988	1
0.25	0.340739		0.30	0.659261	1
0.30	0.336832		0.35	0.663168	1
0.35	0.334986		0.40	0.665014	1
0.40	0.334114		0.45	0.665886	1
0.45	0.333702		0.50	0.666298	1
0.50	0.333508		0.55	0.666492	1
0.55	0.333416		0.60	0.666584	1
0.60	0.333372		0.65	0.666628	1
0.65	0.333352		0.70	0.666648	1
0.70	0.333342		0.75	0.666658	1
0.75	0.333337		0.80	0.666663	1
0.80	0.333335		0.85	0.666665	1
0.85	0.333334		0.90	0.666666	1
0.90	0.333334		0.95	0.666666	1
0.95	0.333334		1.00	0.666666	1
1.00	0.333334				




$$-\frac{dC_A}{dt} = 10C_A - 5C_B$$

$$-\frac{dC_B}{dt} = 5C_B - 10C_A$$



$$C_A + C_B = 1$$



$$-\frac{dC_A}{dt} = 10C_A - 5(1 - C_A)$$

$$-\frac{dC_B}{dt} = 5C_B - 10(1 - C_B)$$

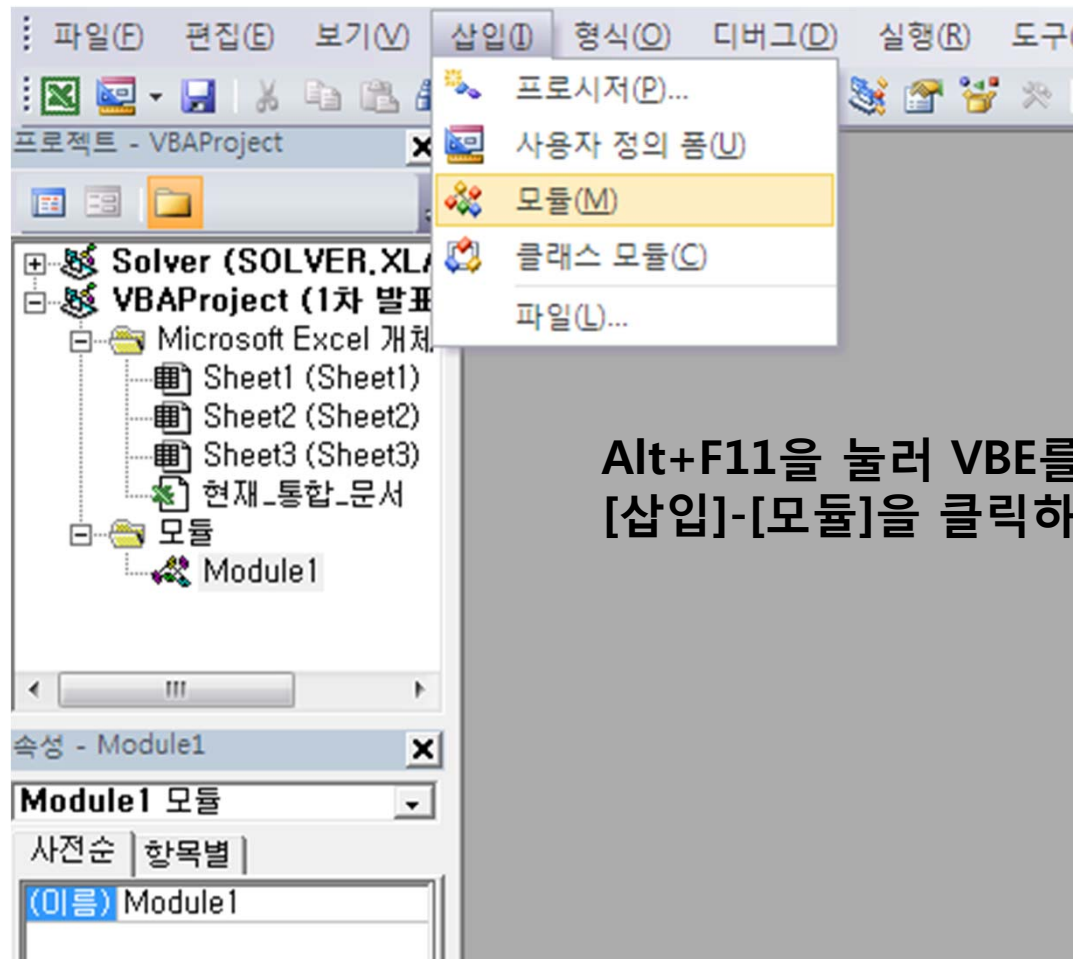
$$-\frac{dC_A}{dt} = 15C_A - 5$$

$$-\frac{dC_B}{dt} = 15C_B - 10$$

$$\frac{dC_A}{dt} = 5 - 15C_A$$

$$\frac{dC_B}{dt} = 10 - 15C_B$$

예제 12-2



Alt+F11을 눌러 VBE를 실행한 뒤
[삽입]-[모듈]을 클릭하여 새 모듈 생성

예제 12-2

```
(일반) funA  
Public Function funA(data1 As Single, data2 As Single)  
funA = 5 - 15 * data2  
End Function  
  
Public Function funB(data3 As Single, data4 As Single)  
funB = 10 - 15 * data4  
End Function
```

$$\text{funA} = 5 - 15 * \text{data2}$$

```
(일반) funB  
Public Function funA(data1 As Single, data2 As Single)  
funA = 5 - 15 * data2  
End Function  
  
Public Function funB(data3 As Single, data4 As Single)  
funB = 10 - 15 * data4  
End Function
```

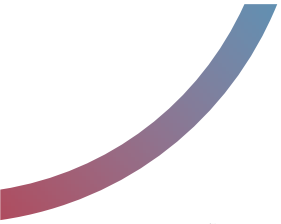
$$\text{funB} = 10 - 15 * \text{data4}$$

$$\text{data1} = t_{iA}, \text{data2} = y_{iA}, \text{data3} = t_{iB}, \text{data4} = y_{iB}$$

E10		fx							
	A	B	C	D	E	F	G	H	I
1									
2	Simultaneous ODEs by Runge-Kutta Method								
3	dA/dt = 5B - 10A, A(0) = 1								
4	dB/dt = 10A - 5B, B(0) = 0								
5									
6	4th order		hh =	0.1					
7			A0 =	1.0					
8			B0 =	0.0					
9	A	t _i	y _{i+1}	y _i	k ₁ = hf _A (t ₀ , y ₀)	k ₂ = hf _A (t ₀ + h/2, y _i + k ₁ /2)	k ₃ = hf _A (t ₀ + h/2, y _i + k ₂ /2)	k ₄ = hf _A (t ₀ + h, y _i + k ₃)	
10		0.00		1.0000000					
11		0.10							
12		0.20							
13		0.30							
14		0.40							
15		0.50							
16		0.60							
17		0.70							
18		0.80							
19		0.90							
20		1.00							
21									



Runge-Kutta 4th order	$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$
	$k_1 = h \cdot f(x_i, y_i)$ $k_2 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$ $k_3 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$ $k_4 = h \cdot f(x_i + h, y_i + k_3)$



E10		fx							
A	B	C	D	E	F	G	H		
1									
2	Simultaneous ODEs by Runge-Kutta Method								
3			dA/dt = 5B - 10A, A(0) = 1						
4			dB/dt = 10A - 5B, B(0) = 0						
5									
6		4th order	hh =	0.1					
7			A0 =	1.0					
8			B0 =	0.0					
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$	
10		0.00		1.0000000					
11		0.10							
12		0.20							
13		0.30							
14		0.40							
15		0.50							
16		0.60							
17		0.70							
18		0.80							
19		0.90							
20		1.00							
21									

=h*funA(B10,D10) 입력

$k_1 = h \cdot f(x_i, y_i)$

$k_2 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$

$k_3 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$

$k_4 = h \cdot f(x_i + h, y_i + k_3)$



E10		fx		=h*funA(B10,D10)					
	A	B	C	D	E	F	G	H	
1									
2	Simultaneous ODEs by Runge-Kutta Method								
3				dA/dt = 5B - 10A, A(0) = 1					
4				dB/dt = 10A - 5B, B(0) = 0					
5									
6			4th order	hh =	0.1				
7				A0 =	1.0				
8				B0 =	0.0				
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$	
10		0.00		1.0000000	-1.0000000				
11		0.10							
12		0.20							
13		0.30							
14		0.40							
15		0.50							
16		0.60							
17		0.70							
18		0.80							
19		0.90							
20		1.00							
21									

Sheet1 Sheet2 Sheet3

준비

F10 fx =h*funA(B10+h/2,D10+E10/2)

	A	B	C	D	E	F	G	H
1								
2		Simultaneous ODEs by Runge-Kutta Method						
3				dA/dt = 5B - 10A, A(0) = 1				
4				dB/dt = 10A - 5B, B(0) = 0				
5								
6			4th order	hh =	0.1			
7				A0 =	1.0			
8				B0 =	0.0			
9	A	t _i	y _{i+1}	y _i	k ₁ = hf _A (t ₀ , y ₀)	k ₂ = hf _A (t ₀ + h/2, y _i + k ₁ /2)	k ₃ = hf _A (t ₀ + h/2, y _i + k ₂ /2)	k ₄ = hf _A (t ₀ + h, y _i + k ₃)
10		0.00		1.0000000	-1.0000000	-0.2500000		
11		0.10						
12		0.20						
13		0.30						
14		0.40						
15		0.50						
16		0.60						
17		0.70						
18		0.80						
19		0.90						
20		1.00						
21								

=h*funA(B10+h/2,D10+E10/2) 입력

$$k_1 = h \cdot f(x_i, y_i)$$

$$k_2 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_i + h, y_i + k_3)$$

Sheet1 Sheet2 Sheet3

준비

G10		fx		=h*funA(B10+h/2,D10+F10/2)				
A	B	C	D	E	F	G	H	
1								
2	Simultaneous ODEs by Runge-Kutta Method							
3			dA/dt = 5B - 10A, A(0) = 1					
4			dB/dt = 10A - 5B, B(0) = 0					
5								
6		4th order	hh =	0.1				
7			A0 =	1.0				
8			B0 =	0.0				
9	A	t _i	y _{i+1}	y _i	k ₁ = hf _A (t ₀ , y ₀)	k ₂ = hf _A (t ₀ + h/2, y _i + k ₁ /2)	k ₃ = hf _A (t ₀ + h/2, y _i + k ₂ /2)	k ₄ = hf _A (t ₀ + h, y _i + k ₃)
10		0.00		1.0000000	-1.0000000	-0.2500000	-0.8125000	
11		0.10						
12		0.20						
13		0.30						
14		0.40						
15		0.50						
16		0.60						
17		0.70						
18		0.80						
19		0.90						
20		1.00						
21								

=h*funA(B10+h/2,D10+F10/2) 입력

$$k_1 = h \cdot f(x_i, y_i)$$

$$k_2 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_i + h, y_i + k_3)$$

H10		fx =h*funA(B10+h,D10+G10)						
A	B	C	D	E	F	G	H	
1								
2	Simultaneous ODEs by Runge-Kutta Method							
3			dA/dt = 5B - 10A, A(0) = 1					
4			dB/dt = 10A - 5B, B(0) = 0					
5								
6		4th order	hh =	0.1				
7			A0 =	1.0				
8			B0 =	0.0				
9	A	t _i	y _{i+1}	y _i	k ₁ = hf _A (t ₀ , y ₀)	k ₂ = hf _A (t ₀ + h/2, y _i + k ₁ /2)	k ₃ = hf _A (t ₀ + h/2, y _i + k ₂ /2)	k ₄ = hf _A (t ₀ + h, y _i + k ₃)
10		0.00		1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500
11		0.10						
12		0.20						
13		0.30						
14		0.40						
15		0.50						
16		0.60						
17		0.70						
18		0.80						
19		0.90						
20		1.00						
21								

=h*funA(B10+h,D10+G10) 입력

$$k_1 = h \cdot f(x_i, y_i)$$

$$k_2 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_i + h, y_i + k_3)$$



C10		fx						
A	B	C	D	E	F	G	H	
1								
2	Simultaneous ODEs by Runge-Kutta Method							
3			dA/dt = 5B - 10A, A(0) = 1					
4			dB/dt = 10A - 5B, B(0) = 0					
5								
6		4th order	hh =	0.1				
7			A0 =	1.0				
8			B0 =	0.0				
9	A	t _i	y _{i+1}	y _i	k ₁ = hf _A (t ₀ , y ₀)	k ₂ = hf _A (t ₀ + h/2, y _i + k ₁ /2)	k ₃ = hf _A (t ₀ + h/2, y _i + k ₂ /2)	k ₄ = hf _A (t ₀ + h, y _i + k ₃)
10		0.00		1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500
11		0.10						
12		0.20						
13		0.30						
14		0.40						
15		0.50						
16		0.60						
17		0.70						
18		0.80						
19		0.90						
20		1.00						
21								

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

=D10+(E10+2*F10+2*G10+H10)/6 입력

Sheet1 Sheet2 Sheet3

준비



C10		fx = =D10+(E10+2*F10+2*G10+H10)/6						
A	B	C	D	E	F	G	H	
1								
2	Simultaneous ODEs by Runge-Kutta Method							
3			dA/dt = 5B - 10A, A(0) = 1					
4			dB/dt = 10A - 5B, B(0) = 0					
5								
6		4th order	hh =	0.1				
7			A0 =	1.0				
8			B0 =	0.0				
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$
10		0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500
11		0.10						
12		0.20						
13		0.30						
14		0.40						
15		0.50						
16		0.60						
17		0.70						
18		0.80						
19		0.90						
20		1.00						
21								

C19까지 자동 채우기



C10 fx =D10+(E10+2*F10+2*G10+H10)/6

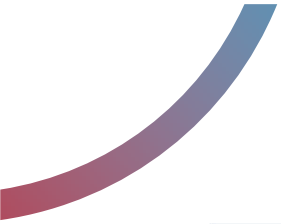
	A	B	C	D	E	F	G	H	
1									
2			Simultaneous ODEs by Runge-Kutta Method						
3				dA/dt = 5B - 10A, A(0) = 1					
4				dB/dt = 10A - 5B, B(0) = 0					
5									
6			4th order	hh =	0.1				
7				A0 =	1.0				
8				B0 =	0.0				
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$	
10		0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500	
11		0.10	0.0000000						
12		0.20	0.0000000						
13		0.30	0.0000000						
14		0.40	0.0000000						
15		0.50	0.0000000						
16		0.60	0.0000000						
17		0.70	0.0000000						
18		0.80	0.0000000						
19		0.90	0.0000000						
20		1.00							
21									

Sheet1 Sheet2 Sheet3

준비 평균: 0.0515625 개수: 10 합계: 0.515

D11		fx		=C10					
A	B	C	D	E	F	G	H		
1									
2	Simultaneous ODEs by Runge-Kutta Method								
3	dA/dt = 5B - 10A, A(0) = 1								
4	dB/dt = 10A - 5B, B(0) = 0								
5									
6		4th order	hh =	0.1					
7			A0 =	1.0					
8			B0 =	0.0					
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$	
10		0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500	
11		0.10	0.5156250	0.5156250					
12		0.20	0.0000000						
13		0.30	0.0000000						
14		0.40	0.0000000						
15		0.50	0.0000000						
16		0.60	0.0000000						
17		0.70	0.0000000						
18		0.80	0.0000000						
19		0.90	0.0000000						
20		1.00							
21									

D11셀에 '=C10'입력



D11		fx		=C10					
A	B	C	D	E	F	G	H		
1									
2	Simultaneous ODEs by Runge-Kutta Method								
3			$dA/dt = 5B - 10A, A(0) = 1$						
4			$dB/dt = 10A - 5B, B(0) = 0$						
5									
6		4th order	hh =	0.1					
7			A0 =	1.0					
8			B0 =	0.0					
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$	
10		0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500	
11		0.10	0.5156250	0.5156250					
12		0.20	0.0000000						
13		0.30	0.0000000						
14		0.40	0.0000000						
15		0.50	0.0000000						
16		0.60	0.0000000						
17		0.70	0.0000000						
18		0.80	0.0000000						
19		0.90	0.0000000						
20		1.00							
21									

D20까지 자동 채우기



D11		fx		=C10					
A	B	C	D	E	F	G	H		
1									
2	Simultaneous ODEs by Runge-Kutta Method								
3			dA/dt = 5B - 10A, A(0) = 1						
4			dB/dt = 10A - 5B, B(0) = 0						
5									
6		4th order	hh =	0.1					
7			A0 =	1.0					
8			B0 =	0.0					
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$	
10		0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500	
11		0.10	0.5156250	0.5156250					
12		0.20	0.5156250	0.5156250					
13		0.30	0.5156250	0.5156250					
14		0.40	0.5156250	0.5156250					
15		0.50	0.5156250	0.5156250					
16		0.60	0.5156250	0.5156250					
17		0.70	0.5156250	0.5156250					
18		0.80	0.5156250	0.5156250					
19		0.90	0.5156250	0.5156250					
20		1.00		0.5156250					
21									

Sheet1 Sheet2 Sheet3

준비

평균: 0.5156250 개수: 10 합계: 5.156



E10		fx		=h*funA(B10,D10)				
A	B	C	D	E	F	G	H	
1								
2	Simultaneous ODEs by Runge-Kutta Method							
3			dA/dt = 5B - 10A, A(0) = 1					
4			dB/dt = 10A - 5B, B(0) = 0					
5								
6		4th order	hh =	0.1				
7			A0 =	1.0				
8			B0 =	0.0				
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$
10		0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500
11		0.10	0.5156250	0.5156250				
12		0.20	0.5156250	0.5156250				
13		0.30	0.5156250	0.5156250				
14		0.40	0.5156250	0.5156250				
15		0.50	0.5156250	0.5156250				
16		0.60	0.5156250	0.5156250				
17		0.70	0.5156250	0.5156250				
18		0.80	0.5156250	0.5156250				
19		0.90	0.5156250	0.5156250				
20		1.00		0.5156250				
21								

E10:H10 영역 드래그

준비 | Sheet1 | Sheet2 | Sheet3 | 평균: -0.4609375 | 개수: 4 | 합계: -1.843



E10		fx		=h*funA(B10,D10)				
A	B	C	D	E	F	G	H	
1								
2	Simultaneous ODEs by Runge-Kutta Method							
3			dA/dt = 5B - 10A, A(0) = 1					
4			dB/dt = 10A - 5B, B(0) = 0					
5								
6		4th order	hh =	0.1				
7			A0 =	1.0				
8			B0 =	0.0				
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$
10		0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500
11		0.10	0.5156250	0.5156250				
12		0.20	0.5156250	0.5156250				
13		0.30	0.5156250	0.5156250				
14		0.40	0.5156250	0.5156250				
15		0.50	0.5156250	0.5156250				
16		0.60	0.5156250	0.5156250				
17		0.70	0.5156250	0.5156250				
18		0.80	0.5156250	0.5156250				
19		0.90	0.5156250	0.5156250				
20		1.00		0.5156250				
21								

E10:H19 영역을 자동 채우기로 셀을 채운다



E10		fx		=h*funA(B10,D10)				
A	B	C	D	E	F	G	H	
1								
2	Simultaneous ODEs by Runge-Kutta Method							
3			dA/dt = 5B - 10A, A(0) = 1					
4			dB/dt = 10A - 5B, B(0) = 0					
5								
6		4th order	hh =	0.1				
7			A0 =	1.0				
8			B0 =	0.0				
9	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$
10		0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500
11		0.10	0.3831787	0.5156250	-0.2734375	-0.0683594	-0.2221680	0.0598145
12		0.20	0.3469629	0.3831787	-0.0747681	-0.0186920	-0.0607491	0.0163555
13		0.30	0.3370602	0.3469629	-0.0204444	-0.0051111	-0.0166111	0.0044722
14		0.40	0.3343524	0.3370602	-0.0055903	-0.0013976	-0.0045421	0.0012228
15		0.50	0.3336120	0.3343524	-0.0015286	-0.0003822	-0.0012419	0.0003343
16		0.60	0.3334095	0.3336120	-0.0004179	-0.0001045	-0.0003396	0.0000915
17		0.70	0.3333542	0.3334095	-0.0001143	-0.0000286	-0.0000929	0.0000250
18		0.80	0.3333390	0.3333542	-0.0000313	-0.0000078	-0.0000254	0.0000069
19		0.90	0.3333349	0.3333390	-0.0000086	-0.0000021	-0.0000069	0.0000019
20		1.00		0.3333349				
21								

Sheet1 | Sheet2 | Sheet3

준비 | 평균: -0.0634407 | 개수: 40 | 합계: -2.537628

		H33		fx					
9	A	B	C	D	E	F	G	H	
	A	t_i	y_{i+1}	y_i	$k_1 = hf_A(t_0, y_0)$	$k_2 = hf_A(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_A(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_A(t_0 + h, y_i + k_3)$	
10		0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500	
11		0.10	0.3831787	0.5156250	-0.2734375	-0.0683594	-0.2221680	0.0598145	
12		0.20	0.3469629	0.3831787	-0.0747681	-0.0186920	-0.0607491	0.0163555	
13		0.30	0.3370602	0.3469629	-0.0204444	-0.0051111	-0.0166111	0.0044722	
14		0.40	0.3343524	0.3370602	-0.0055903	-0.0013976	-0.0045421	0.0012228	
15		0.50	0.3336120	0.3343524	-0.0015286	-0.0003822	-0.0012419	0.0003343	
16		0.60	0.3334095	0.3336120	-0.0004179	-0.0001045	-0.0003396	0.0000915	
17		0.70	0.3333542	0.3334095	-0.0001143	-0.0000286	-0.0000929	0.0000250	
18		0.80	0.3333390	0.3333542	-0.0000313	-0.0000078	-0.0000254	0.0000069	
19		0.90	0.3333349	0.3333390	-0.0000086	-0.0000021	-0.0000069	0.0000019	
20		1.00		0.3333349					
21									
22	B	t_i	y_{i+1}	y_i	$k_1 = hf_B(t_0, y_0)$	$k_2 = hf_B(t_0 + h/2, y_i + k_1/2)$	$k_3 = hf_B(t_0 + h/2, y_i + k_2/2)$	$k_4 = hf_B(t_0 + h, y_i + k_3)$	
23		0.00	0.4843750	0.0000000	1.0000000	0.2500000	0.8125000	-0.2187500	
24		0.10	0.6168213	0.4843750	0.2734375	0.0683594	0.2221680	-0.0598145	
25		0.20	0.6530371	0.6168213	0.0747681	0.0186920	0.0607491	-0.0163555	
26		0.30	0.6629398	0.6530371	0.0204444	0.0051111	0.0166111	-0.0044723	
27		0.40	0.6656476	0.6629398	0.0055903	0.0013975	0.0045421	-0.0012228	
28		0.50	0.6663880	0.6656476	0.0015286	0.0003822	0.0012419	-0.0003343	
29		0.60	0.6665905	0.6663880	0.0004179	0.0001045	0.0003396	-0.0000915	
30		0.70	0.6666458	0.6665905	0.0001143	0.0000286	0.0000929	-0.0000250	
31		0.80	0.6666610	0.6666458	0.0000313	0.0000078	0.0000254	-0.0000067	
32		0.90	0.6666651	0.6666610	0.0000086	0.0000021	0.0000069	-0.0000019	
33		1.00		0.6666651					

funB를 사용해 B의 값들도 A와 같은 방식으로 셀의 값을 구한다



해석해와 수치해 비교

해석해 식

$$C_A = C_{A0} - m\{1 - e^{-(k_1+k_2)t}\} = \frac{2}{3}e^{-15t} + \frac{1}{3}$$

$$C_B = C_{B0} + m\{1 - e^{-(k_1+k_2)t}\} = \frac{2}{3}e^{-15t} - \frac{2}{3}$$

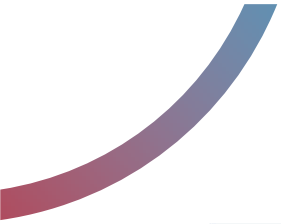


C4 fx = 1/3+2/3*EXP(-15*B4)

	A	B	C	D	E	F	G	H
1								
2			Analytical solution	Runge-Kutta method		Analytical solution	Runge-Kutta method	
3		t_i	C_A	C_A	error(%)	C_B	C_B	error(%)
4		0	1					
5		0.1						
6		0.2						
7		0.3						
8		0.4						
9		0.5						
10		0.6						
11		0.7						
12		0.8						
13		0.9						
14		1						
15								
16								

Sheet1 | Sheet2 | Sheet3

준비



C4 fx =1/3+2/3*EXP(-15*B4)

	A	B	C	D	E	F	G	H
1								
2			Analytical solution	Runge-Kutta method		Analytical solution	Runge-Kutta method	
3		t_i	C_A	C_A	error(%)	C_B	C_B	error(%)
4		0	1					
5		0.1	0.482086773					
6		0.2	0.366524712					
7		0.3	0.340739331					
8		0.4	0.334985835					
9		0.5	0.333702056					
10		0.6	0.333415607					
11		0.7	0.333351691					
12		0.8	0.333337429					
13		0.9	0.333334247					
14		1	0.333333537					
15								
16								

C14까지 드래그하여 자동 채우기

준비 Sheet1 Sheet2 Sheet3 평균: 0.411346474 개수: 11 합계: 4.524

	A	B	C	D	E	F	G	H
1								
2			Analytical solution	Runge-Kutta method		Analytical solution	Runge-Kutta method	
3		t_i	C_A	C_A	error(%)	C_B	C_B	error(%)
4		0	1					
5		0.1	0.482086773					
6		0.2	0.366524712					
7		0.3	0.340739331					
8		0.4	0.334985835					
9		0.5	0.333702056					
10		0.6	0.333415607					
11		0.7	0.333351691					
12		0.8	0.333337429					
13		0.9	0.333334247					
14		1	0.333333537					
15								
16								

앞에서 구한 Runge-Kutta 값 가져오기



D10		fx		1					
A	B	C	D	E	F	G	H		
Simultaneous ODEs by Runge-Kutta Method									
				dA/dt = 5B - 10A, A(0) = 1					
				dB/dt = 10A - 5B, B(0) = 0					
4th order				hh =	0.1				
				A0 =	1.0				
				B0 =	0.0				
A	t _i	y _{i+1}	y _i	k ₁ = hf _A (t ₀ , y ₀)	k ₂ = hf _A (t ₀ + h/2, y _i + k ₁ /2)	k ₃ = hf _A (t ₀ + h/2, y _i + k ₂ /2)	k ₄ = hf _A (t ₀ + h, y _i + k ₃)		
10	0.00	0.5156250	1.0000000	-1.0000000	-0.2500000	-0.8125000	0.2187500		
11	0.10	0.3831787	0.5156250	-0.2734375	-0.0683594	-0.2221680	0.0598145		
12	0.20	0.3469629	0.3831787	-0.0747681	-0.0186920	-0.0607491	0.0163555		
13	0.30	0.3370602	0.3469629	-0.0204444	-0.0051111	-0.0166111	0.0044722		
14	0.40	0.3343524	0.3370602	-0.0055903	-0.0013976	-0.0045421	0.0012228		
15	0.50	0.3336120	0.3343524	-0.0015286	-0.0003822	-0.0012419	0.0003343		
16	0.60	0.3334095	0.3336120	-0.0004179	-0.0001045	-0.0003396	0.0000915		
17	0.70	0.3333542	0.3334095	-0.0001143	-0.0000286	-0.0000929	0.0000250		
18	0.80	0.3333390	0.3333542	-0.0000313	-0.0000078	-0.0000254	0.0000069		
19	0.90	0.3333349	0.3333390	-0.0000086	-0.0000021	-0.0000069	0.0000019		
20	1.00		0.3333349						

**D10:D20 영역을 드래그하여
지정한 뒤 복사**

	A	B	C	D	E	F	G	H
1								
2			Analytical solution	Runge-Kutta method		Analytical	Runge-Kutta method	
3		t_i	C_A	C_A			C_B	error(%)
4		0	1					
5		0.1	0.482086773					
6		0.2	0.366524712					
7		0.3	0.340739331					
8		0.4	0.334985835					
9		0.5	0.333702056					
10		0.6	0.333415607					
11		0.7	0.333351691					
12		0.8	0.333337429					
13		0.9	0.333334247					
14		1	0.333333537					
15								
16								

- 잘라내기(D)
- 복사(C)
- 붙여넣기 옵션:
 - 123
- 선택하여 붙여넣기(S)...
- 복사한 셀 삽입(E)...
- 삭제(D)...
- 내용 지우기(N)
- 필터(F) >
- 정렬(O) >
- 메모 삽입(M)
- 셀 서식(F)...
- 드롭다운 목록에서 선택(K)...
- 내첨
 - 윗주 필드 표시(S)
 - 이름 정의(A)...
 - 하이퍼링크(I)...

D4셀에서 오른쪽 마우스 클릭하여 '값만 붙여넣기' 선택

	A	B	C	D	E	F	G	H
1								
2			Analytical solution	Runge-Kutta method		Analytical solution	Runge-Kutta method	
3		t_i	C_A	C_A	error(%)	C_B	C_B	error(%)
4		0	1	1				
5		0.1	0.482086773	0.5156250000				
6		0.2	0.366524712	0.3831787109				
7		0.3	0.340739331	0.3469629288				
8		0.4	0.334985835	0.3370601758				
9		0.5	0.333702056	0.3343523890				
10		0.6	0.333415607	0.3336119801				
11		0.7	0.333351691	0.3334095329				
12		0.8	0.333337429	0.3333541602				
13		0.9	0.333334247	0.3333390281				
14		1	0.333333537	0.3333348930				
15								
16								

	A	B	C	D	E	F	G	H
1								
2			Analytical solution	Runge-Kutta method		Analytical solution	Runge-Kutta method	
3		t_i	C_A	C_A	error(%)	C_B	C_B	error(%)
4		0	1	1				
5		0.1	0.482086773	0.5156250000				
6		0.2	0.366524712	0.3831787109				
7		0.3	0.340739331	0.3469629288				
8		0.4	0.334985835	0.3370601758				
9		0.5	0.333702056	0.3343523890				
10		0.6	0.333415607	0.3336119801				
11		0.7	0.333351691	0.3334095329				
12		0.8	0.333337429	0.3333541602				
13		0.9	0.333334247	0.3333390281				
14		1	0.333333537	0.3333348930				
15								
16								

$$\text{오차율(\%)} = \frac{|\text{수치해} - \text{해석해}|}{\text{해석해}} \times 100$$



E4 fx =ABS(D4-C4)/C4*100

	A	B	C	D	E	F	G	H
1								
2			Analytical solution	Runge-Kutta method		Analytical solution	Runge-Kutta method	
3		t_i	C_A	C_A	error(%)	C_B	C_B	error(%)
4		0	1	1	0			
5		0.1	0.482086773	0.5156250000	=ABS(D4-C4)/C4*100 입력			
6		0.2	0.366524712	0.3831787109				
7		0.3	0.340739331	0.3469629288				
8		0.4	0.334985835	0.3370601758				
9		0.5	0.333702056	0.3343523890				
10		0.6	0.333415607	0.3336119801				
11		0.7	0.333351691	0.3334095329				
12		0.8	0.333337429	0.3333541602				
13		0.9	0.333334247	0.3333390281				
14		1	0.333333537	0.3333348930				
15								
16								

Sheet1 Sheet2 Sheet3

준비



E4 fx =ABS(D4-C4)/C4*100

	A	B	C	D	E	F	G	H
1								
2			Analytical solution	Runge-Kutta method		Analytical solution	Runge-Kutta method	
3		t_i	C_A	C_A	error(%)	C_B	C_B	error(%)
4		0	1	1	0			
5		0.1	0.482086773	0.5156250000	6.956885859			
6		0.2	0.366524712	0.3831787109	4.543758752			
7		0.3	0.340739331	0.3469629288	1.8264982			
8		0.4	0.334985835	0.3370601758	0.619232468			
9		0.5	0.333702056	0.3343523890	0.194884245			
10		0.6	0.333415607	0.3336119801	0.058897526			
11		0.7	0.333351691	0.3334095329	0.017351626			
12		0.8	0.333337429	0.3333541602	0.005019153			
13		0.9	0.333334247	0.3333390281	0.001434222			
14		1	0.333333537	0.3333348930	0.000406716			
15								
16								

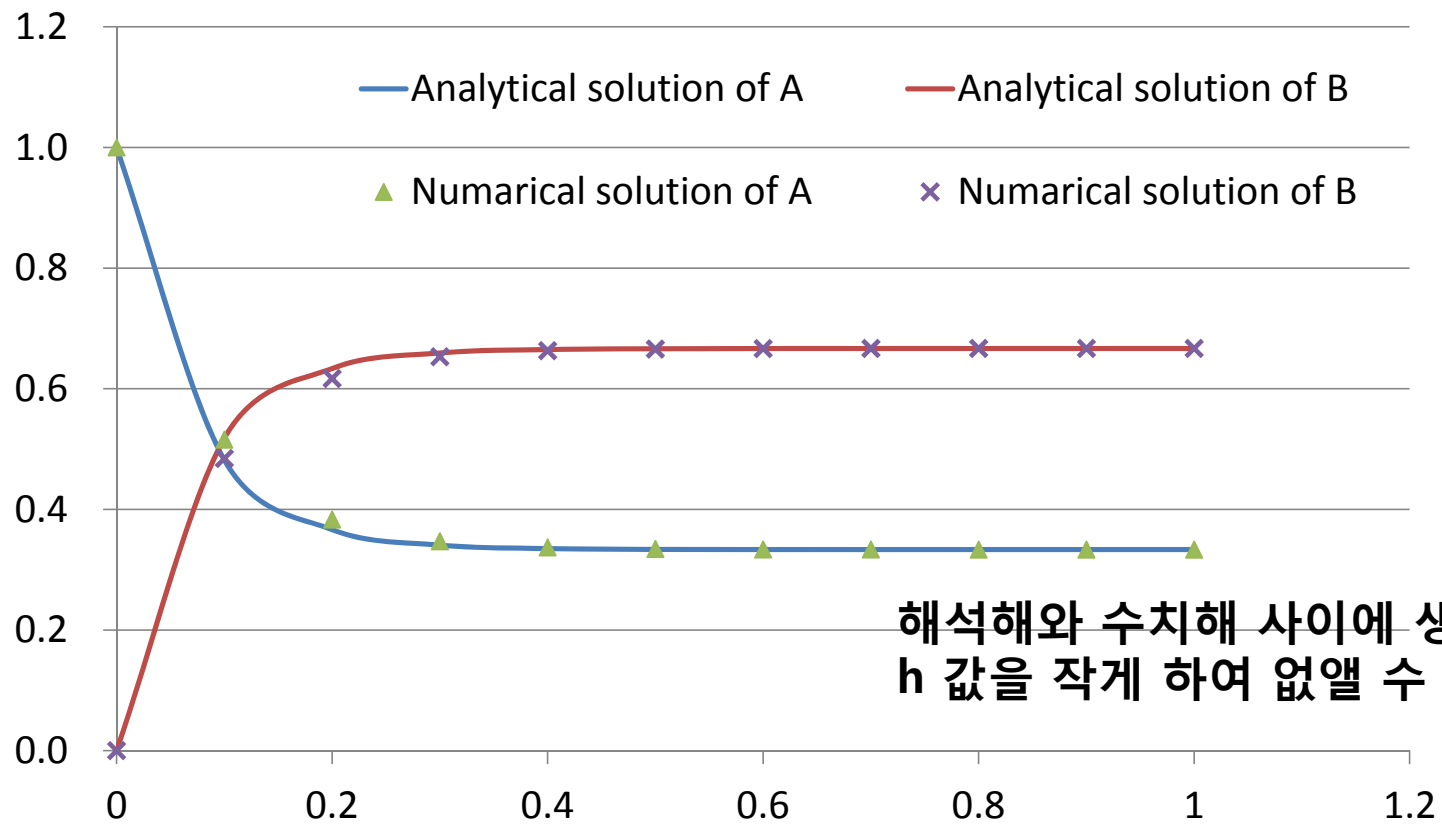
E14까지 드래그하여 자동 채우기

준비 Sheet1 Sheet2 Sheet3 평균: 1.293124433 개수: 11 합계: 14.224

B의 오차도 A와 같은 방법으로 구한다
 (단, B의 첫 번째 오차는 분모가 0이므로 함수이용 x)

	A	B	C		D		E		F
		t_i	C_A	C_A	error(%)	C_B	C_B	error(%)	
1									
2			Analytical solution	Runge-Kutta method		Analytical solution	Runge-Kutta method		
3		t_i	C_A	C_A	error(%)	C_B	C_B	error(%)	
4		0	1	1	0	0	0	0	
5		0.1	0.482086773	0.5156250000	6.956885859	0.517913227	0.4843750000	6.47564589	
6		0.2	0.366524712	0.3831787109	4.543758752	0.633475288	0.6168212891	2.628989483	
7		0.3	0.340739331	0.3469629288	1.8264982	0.659260669	0.6530370712	0.944026853	
8		0.4	0.334985835	0.3370601758	0.619232468	0.665014165	0.6629398167	0.311925461	
9		0.5	0.333702056	0.3343523890	0.194884245	0.666297944	0.6656475961	0.097606129	
10		0.6	0.333415607	0.3336119801	0.058897526	0.666584393	0.6663880050	0.029461903	
11		0.7	0.333351691	0.3334095329	0.017351626	0.666648309	0.6665904522	0.008678765	
12		0.8	0.333337429	0.3333541602	0.005019153	0.666662571	0.6666458249	0.002511858	
13		0.9	0.333334247	0.3333390281	0.001434222	0.666665753	0.6666609794	0.000715997	
14		1	0.333333537	0.3333348930	0.000406716	0.666666463	0.6666651070	0.000203358	
15									
16									

해석해와 수치해 비교 그래프



해석해와 수치해 사이에 생기는 오차는
h 값을 작게 하여 없앨 수 있다