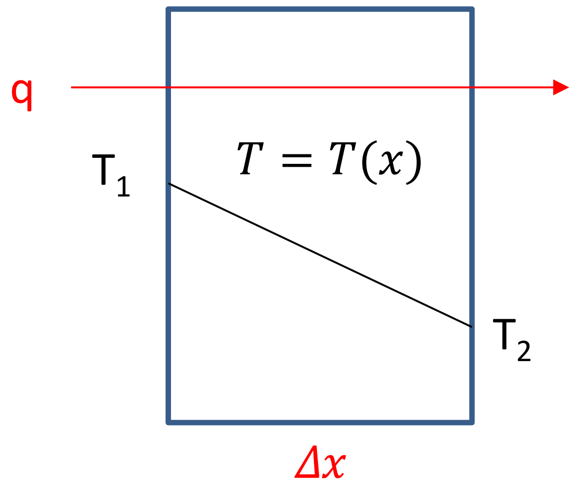
Chapter 10. Heat Transfer by Conduction

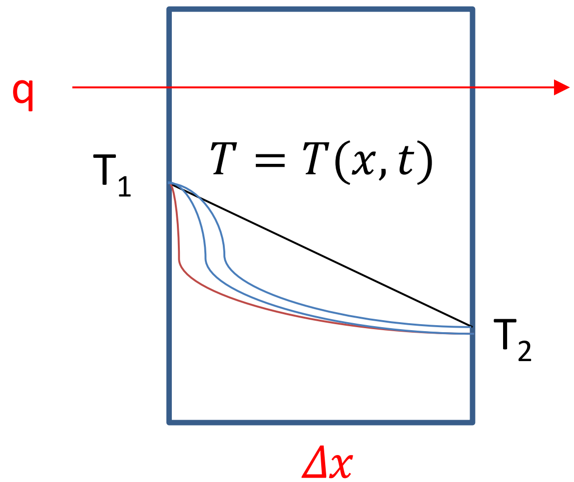
Heat transfer : transfer of K.E. by molecular interaction

<Assumptions>

1) **Conduction in solids**
2) **One-dimensional heat flow**
3) **Homogeneous solid**

1. Fourier's law





Steady & 1-D flow (x-direction)

1. Fourier's law

$$q \propto A \cdot \frac{1}{\Delta x} \cdot \Delta T$$

$$q = -kA\frac{\Delta T}{\Delta x} \quad q = \frac{-\Delta T}{(\Delta x / kA)} \qquad \boxed{q = -kA\frac{dT}{dx}}$$

q : rate of heat flow ***A*** : surface area ***T*** : temperature
x : distance normal to surface ***k*** : thermal conductivity

*** Thermal conductivity (열전도도) *k***

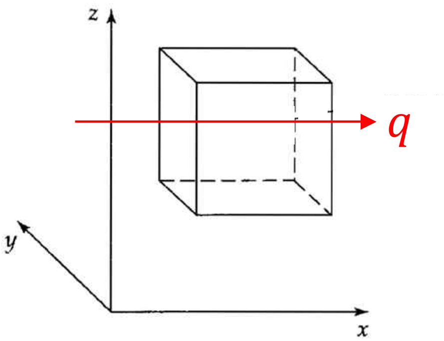
$k = k(T)$
$k = \mathrm{a} + \mathrm{b}\ T$

2. Heat conduction equation

Heat Balance

$$\overset{q_{in}\ (q_x)}{\begin{pmatrix}\text{Rate of}\\ \text{Heat Flow in}\end{pmatrix}} - \overset{q_{out}\ (q_{x+dx})}{\begin{pmatrix}\text{Rate of}\\ \text{Heat Flow out}\end{pmatrix}} + \overset{q_{gen}\ (\dot{q})}{\begin{pmatrix}\text{Rate of}\\ \text{Heat Generation}\end{pmatrix}} = \overset{q_{acc}\ .}{\begin{pmatrix}\text{Rate of}\\ \text{Heat Accumulation}\end{pmatrix}}$$

1) Cartesian coordination



$\dot{q}$ ~ Heat generated per unit volume $[\mathrm{W/m^3}]$

$$q_{in} - q_{out} + \dot{q} = \frac{\partial H}{\partial t}$$

$$H = mC_pT$$

$$dH = d(mC_pT) = d(\rho V C_pT)$$

$$q_x - q_{x+dx} + \dot{q}(dxdydz) = \frac{\partial(\rho C_pT)}{\partial t}dxdydz$$

cf) Taylor's series expansion

$$f(x+dx) = f(x) + \frac{1}{1!}\frac{\partial f(x)}{\partial x}\Delta x + \frac{1}{2!}\frac{\partial f^2(x)}{\partial x^2}\Delta x^2 + \frac{1}{3!}\frac{\partial f^3(x)}{\partial x^3}\Delta x^3 + \cdots$$

If $\Delta x \to 0$ then,

$$\lim_{\Delta x \to 0} f(x+dx) = f(x+dx) = f(x) + \frac{\partial f(x)}{\partial x}dx$$

$$\Rightarrow \quad q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx$$

$$q_x - q_{x+dx} + \dot{q}(dxdydz) = \frac{\partial(\rho C_p T)}{\partial t}dxdydz$$

$$q_x - \left(q_x + \frac{\partial q_x}{\partial x}dx\right) + \dot{q}(dxdydz) = \frac{\partial(\rho C_p T)}{\partial t}dxdydz$$

Fourier's law $\quad q_x = -kA\frac{\partial T}{\partial x}$

$$-\frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)dx + \dot{q}(dxdydz) = \frac{\partial(\rho C_p T)}{\partial t}dxdydz$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q} = \frac{\partial(\rho C_p T)}{\partial t}$$

$$k\frac{\partial^2 T}{\partial x^2} + \dot{q} = \rho C_p\frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\rho C_p}$$

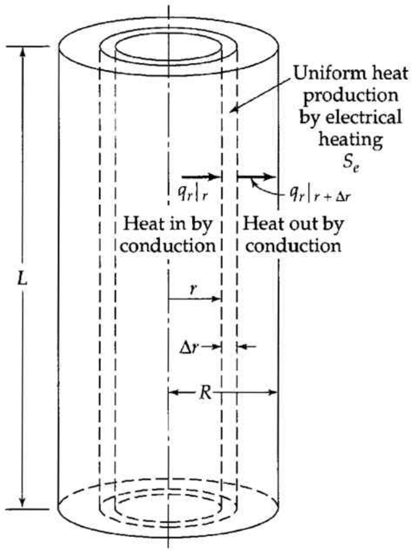
$$\boxed{\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\rho C_p}}$$

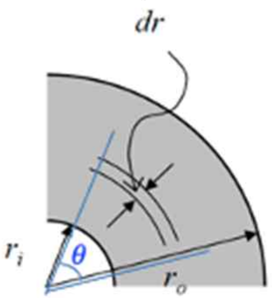
Heat conduction equation
for cartesian coordination

Where, $\alpha = \rho C_p$. Thermal conductivity $\left[\frac{\mathrm{m}^2}{\mathrm{s}}\right]$, $\left[\frac{\mathrm{f}^{\ 2}}{\mathrm{s}}\right]$

2. Heat conduction equation

2) Cylindrical coordination





$$q_{in} - q_{out} + \dot{q} = \frac{\partial(mC_pT)}{\partial t}$$

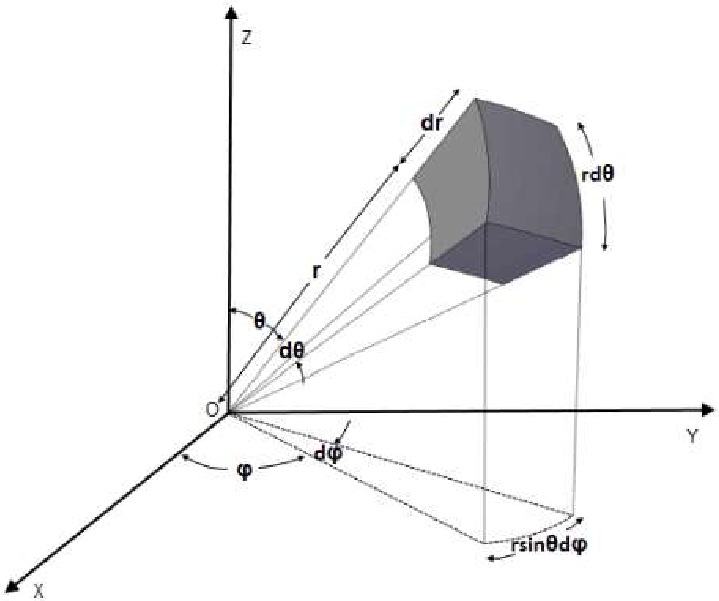
$$q_r - q_{r+dr} + \dot{q} = \frac{\partial(\rho C_pT)}{\partial t} \qquad q_{r+dr} = q_r + \frac{\partial q_r}{\partial r}dr$$

$$\boxed{\begin{aligned} \frac{\partial T}{\partial t} &= \frac{k}{\rho C_p}\frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\rho C_p} \\ \frac{\partial T}{\partial t} &= \alpha\frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\rho C_p} \end{aligned}}$$

Heat conduction equation
for cylindrical coordination

2. Heat conduction equation

3) Spherical coordination



$$q_{\text{in}} - q_{\text{out}} + \dot{q} = \frac{\partial\left(mC_pT\right)}{\partial t}$$

$$q_r - q_{r+dr} + \dot{q} = \frac{\partial\left(\rho C_pT\right)}{\partial t} \qquad q_{r+dr} = q_r + \frac{\partial q_r}{\partial r}dr$$

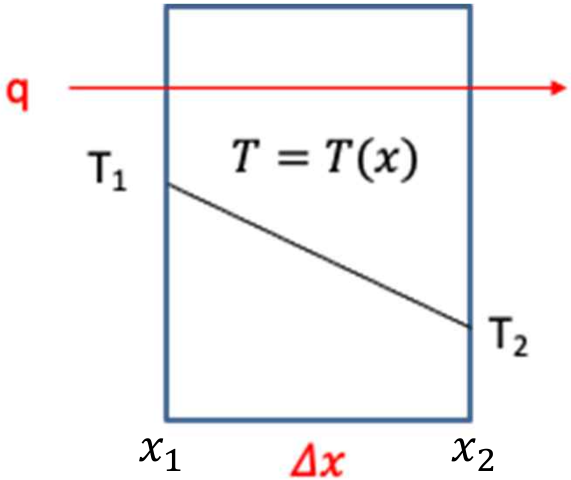
$$\boxed{\begin{aligned} \frac{\partial T}{\partial t} &= \frac{k}{\rho C_p}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right\} + \frac{\dot{q}}{\rho C_p} \\ \frac{\partial T}{\partial t} &= \alpha\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right\} + + \frac{\dot{q}}{\rho C_p} \end{aligned}}$$

Heat conduction equation
for spherical coordination

3. Steady-State Conduction

<assumption> : No heat generation

(1) Single layer slab



Heat conduction equation for cartesian coordination

$$\boxed{\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\rho C_p}}$$

$$0 = \frac{\partial^2 T}{\partial x^2} \xrightarrow{1^{st}\ \text{intg}} \frac{\partial T}{\partial x} = C_1$$

$$\xrightarrow{2^{nd}\ \text{intg}} T = C_1 x + C_2$$

B.C.

$@x = x_1, T = T_1$

$@x = x_2, T = T_2$

$$C_1 = \frac{T_1 - T_2}{x_1 - x_2} \qquad C_2 = T_1 - \left(\frac{T_1 - T_2}{x_1 - x_2}\right) x_1 = T_2 - \left(\frac{T_1 - T_2}{x_1 - x_2}\right) x_2$$

3. Steady-State Conduction

$$T(x) = \left(\frac{T_1 - T_2}{x_1 - x_2}\right)x + \left\{T_1 - \left(\frac{T_1 - T_2}{x_1 - x_2}\right)x_1\right\}$$

$$\frac{T(x) - T_1}{T_1 - T_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\boxed{\frac{T_1 - T(x)}{T_1 - T_2} = \frac{x - x_1}{x_2 - x_1}}$$

$$q = -kA\frac{dT}{dx}$$

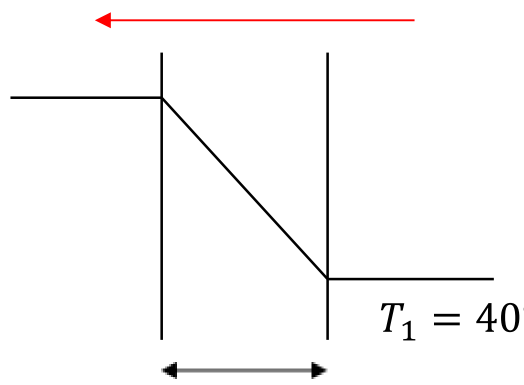
$$= -kA\frac{T_1 - T_2}{x_1 - x_2} = kA\frac{T_1 - T_2}{x_2 - x_1} = kA\frac{\Delta T}{\Delta x}$$

$$q/A = kA\frac{\Delta T}{\Delta x} \qquad q = \frac{\Delta T}{R} \qquad R = \frac{\Delta x}{kA}$$

Ex. 10.1) **A layer of pulverized cork (insulator)**

$q = ?$

$T_2 = 180°\text{F}$

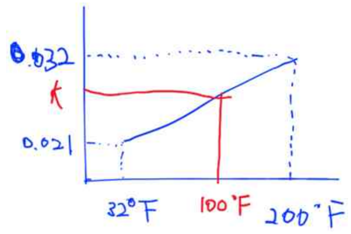


$A = 25\ \text{ft}^2$

$T_1 = 40°\text{F}$

$\Delta x = 0.5\ ft$

$$k = \begin{cases} 0.021\ \text{Btu/ft}\cdot\text{hr°F@40°F} \\ 0.032\ \text{Btu/ft}\cdot\text{hr°F@180°F} \end{cases}$$



$k = 0.026\ \text{Btu/ft}\cdot\text{hr°F}$

q (the rate of heat flow) ?

$$q = kA\frac{\Delta T}{\Delta x} = 0.026\left(\frac{\text{Btu}}{\text{ft hr °F}}\right)\cdot 25\left(\text{ft}^2\right)\frac{120(°\text{F})}{0.5\ (\text{ft})} = 182\frac{\text{Btu}}{\text{hr}}$$