14.4 Nonblack Surfaces

- Gray surfaces

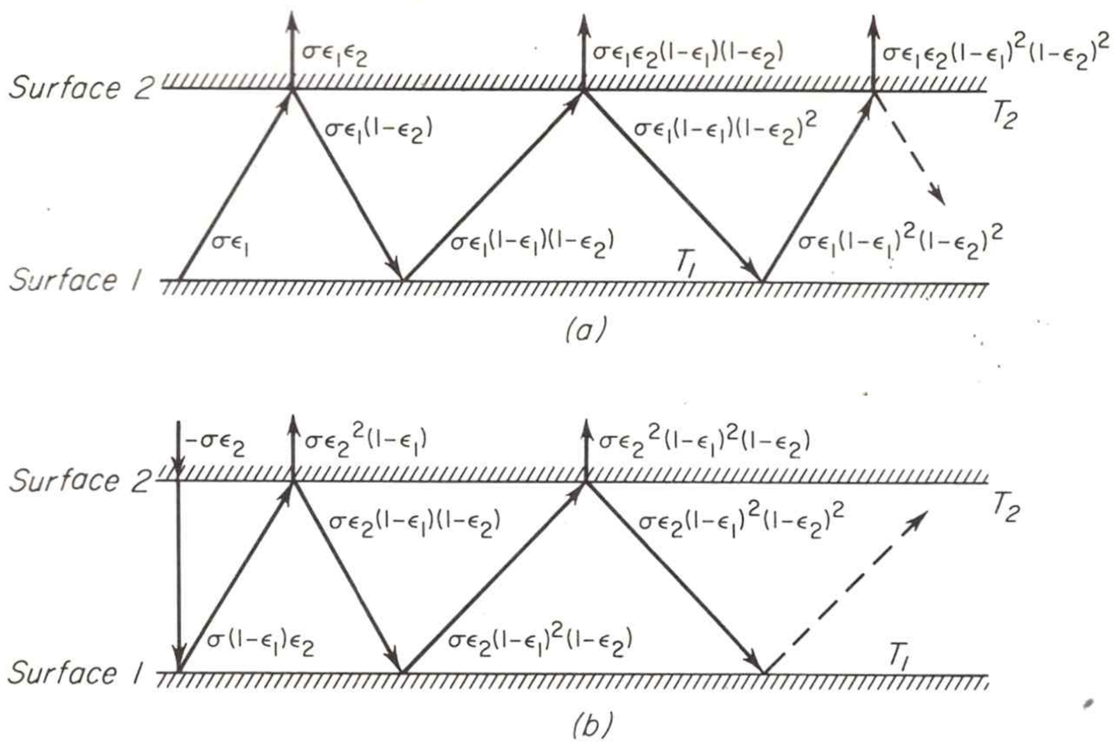
$$q_{12} = \sigma A_1 \mathcal{F}_{12}\left(T_1^4 - T_2^4\right) = \sigma A_2 \mathcal{F}_{21}\left(T_1^4 - T_2^4\right)$$

where, $\mathcal{F}_{12}, \mathcal{F}_{21}$: overall interchange factor, $f(\epsilon_1, \epsilon_2)$,

$$\sigma = 5.672 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m^2 \cdot K^4}} = 0.1713 \times 10^{-8} \frac{\mathrm{BTU}}{\mathrm{ft^2 \cdot hr \cdot {}^\circ F^4}}$$

14.4 Nonblack Surfaces

- Gray surfaces



14.4 Nonblack Surfaces

$$(a) q_{1\rightarrow 2} = \sigma T_1^4 \epsilon_1 \epsilon_2 [1 + (1-\epsilon_1)(1-\epsilon_2) + (1-\epsilon_1)^2(1-\epsilon_2)^2 + \cdots]$$

$$(b) q_{2\rightarrow 1} = \left[-\sigma\epsilon_2 + \sigma\epsilon_2^2(1-\epsilon_1) + \sigma\epsilon_2(1-\epsilon_1)^2(1-\epsilon_2)^2 + \cdots\right] T_2^4$$

$$= -\sigma T_2^4 \left[\epsilon_2 - \epsilon_2^2(1-\epsilon_1) + \sigma\epsilon_2^2(1-\epsilon_1)^2(1-\epsilon_2)^2 - \cdots\right]$$

$$= -\sigma T_2^4 \{\epsilon_2 - \epsilon_2^2(1-\epsilon_1)[1 + (1-\epsilon_1)(1-\epsilon_2) + (1-\epsilon_1)^2(1-\epsilon_2)^2 + \cdots]\}$$

14.4 Nonblack Surfaces

$$q_{12} = (a) + (b) = q_{1\to 2} + q_{2\to 1}$$

let $y = (1-\epsilon_1)(1-\epsilon_2)$,

$$q_{12} = \sigma T_1^4 \epsilon_1 \epsilon_2 \left(1 + y + y^2 + y^3 + \cdots\right) - \sigma T_2^4 \left[\epsilon_2 - \epsilon_2^2 (1-\epsilon_1)\left(1 + y + y^2 + y^3 + \cdots\right)\right]$$

since $y < 1$ then, $1 + y + y^2 + y^3 + \cdots = \frac{1}{1-y}$

$$q_{12} = \sigma T_1^4 \epsilon_1 \epsilon_2 \frac{1}{1-y} - \sigma T_2^4 \left[\epsilon_2 - \epsilon_2^2 (1-\epsilon_1) \frac{1}{1-y}\right]$$

$q_{12} = \sigma A_1 \mathcal{F}_{12}\left(T_1^4 - T_2^4\right) = \sigma A_2 \mathcal{F}_{21}\left(T_1^4 - T_2^4\right)$ 이므로,

$$\mathcal{F}_{12} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

14.5 Total heat transfer by conduction, convection, and radiation

$$\frac{q_T}{A} = \frac{q_c}{A} + \frac{q_r}{A} = h_c(T_w - T) + \sigma\epsilon_w(T_w^4 - T^4)$$

where, $\frac{q_T}{A}$: total heat flux, $\frac{q_c}{A}$: heat flux by conduction and convection

$\frac{q_r}{A}$: heat flux by radiation, h_c: convective heat transfer coefficient

ϵ_w: emissivity of surface, T_w: temperature of surface, T: surrounding Temp.

$$\frac{q_T}{A} = (h_c + h_r)(T_w - T)$$

h_r: radiation heat transfer coefficient

$$h_r \equiv \frac{q_r}{A(T_w - T)}$$

Example

1. 두 강철판이 평행하게 서로 마주보고 있다. 두 강철판은 온도차이가 있으며, 한쪽의 온도는 986 K 이고 나머지 한 쪽의 온도는 478 K 이다. 그리고 더운 쪽과 덜 더운 쪽의 방사율은 각각 0.7, 0.4 이다. 다음에 답하시오 (Boltzman상수= $5.672 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$)

(a) 두 강철판 사이에 전해지는 heat flux는?

(b) 두 강철판 사이에 같은 크기의 얇은 검은칠을 한 알루미늄판을 나란히 놓는다면, 그 알루미늄 판의 온도는 몇 ℃ 가 되겠는가?

- sol

$\epsilon_1 = 0.7$ 986 K

$\epsilon_2 = 0.4$ 478 K

Example

- 가정 :
  - 두 강철판은 gray body이
  - 두 강철판의 면적은 같다.
  - 강철판 사이에 재복사되는 (reradiated) 내화벽 (refractory wall)은 없다
-

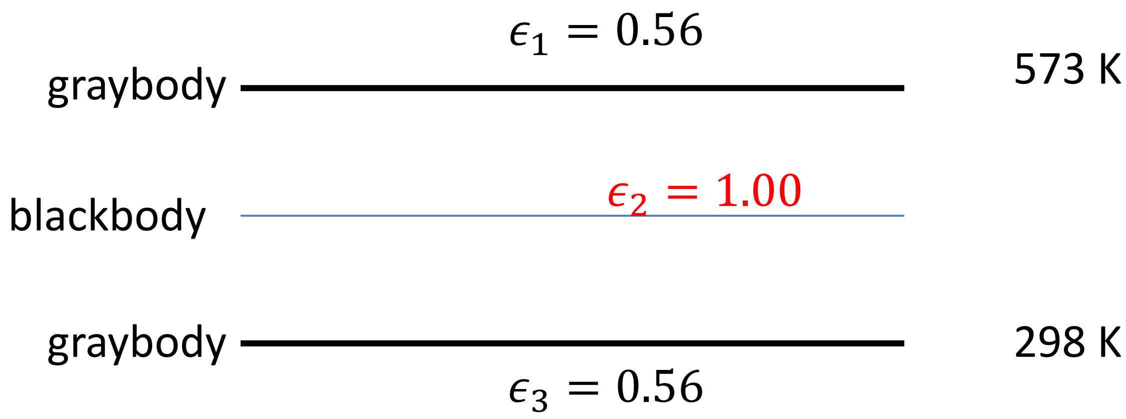
a) $q/A = \sigma \mathcal{F}_{12}\left(T_1^4 - T_2^4\right)$

$$\mathcal{F}_{12} = \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1} = \frac{1}{1/0.7 + 1/0.4 - 1} = 0.34$$
$$q/A = (5.672 \times 10^{-8}) \cdot 0.34 \cdot (986^4 - 478^4) = 17220.56 \text{ W/m}^2$$

b) 얇은 알루미늄판을 black body로 가정하면 $\epsilon_a = 1$

$$\mathcal{F}_{1\text{a}} = \frac{1}{1/\epsilon_1 + 1/\epsilon_a - 1} = \frac{1}{1/0.7 + 1/1 - 1} = 0.7$$
$$\mathcal{F}_{\text{a}2} = \frac{1}{1/\epsilon_a + 1/\epsilon_2 - 1} = \frac{1}{1/1 + 1/0.4 - 1} = 0.4$$
$$q_{1\text{a}} = q_{\text{a}2}$$
$$\sigma \text{A} \mathcal{F}_{1\text{a}}\left(T_1^4 - T_a^4\right) = \sigma \text{A} \mathcal{F}_{\text{a}2}\left(T_a^4 - T_2^4\right)$$
$$T_a = \sqrt[4]{\frac{\mathcal{F}_{1\text{a}} T_1^4 + \mathcal{F}_{\text{a}2} T_2^4}{\mathcal{F}_{1\text{a}} + \mathcal{F}_{\text{a}2}}} = \sqrt[4]{\frac{0.7 \times 986^4 + 0.4 \times 478^4}{0.7 + 0.4}} = 887.52\ K = 614.52℃$$

Example 14.1



a) $T_2 = ?$

2 is blackbody → $\epsilon_2 = 1.00$

$$q_{12} = q_{23} \quad \therefore \text{ 2 is blackbody}$$

$$q_{12} = \sigma A_1 \mathcal{F}_{12}\left(T_1^4 - T_2^4\right)$$

$$q_{23} = \sigma A_2 \mathcal{F}_{23}\left(T_2^4 - T_3^4\right)$$

$$\sigma A_1 \mathcal{F}_{12}\left(T_1^4 - T_2^4\right) = \sigma A_2 \mathcal{F}_{23}\left(T_2^4 - T_3^4\right)$$

$$\mathcal{F}_{12} = \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1} = \frac{1}{1/0.56 + 1/1.0 - 1} = 0.56$$

$$\mathcal{F}_{23} = \frac{1}{1/\epsilon_2 + 1/\epsilon_3 - 1} = \frac{1}{1/0.56 + 1/1.0 - 1} = 0.56$$

$$T_1^4 - T_2^4 = T_2^4 - T_3^4$$

$$T_2 = \sqrt[4]{\frac{T_1^4 + T_3^4}{2}} = 490\text{ K}$$

Example 14.1

b) Heat flux ?

$$q_{12} = \sigma A_1 \mathcal{F}_{12}\left(T_1^4 - T_2^4\right)$$

$q_{12}/A = \sigma \mathcal{F}_{12}\left(T_1^4 - T_2^4\right)$
$= (5.672 \times 10^{-8})[\mathrm{W/m^2 \cdot K^4}] \cdot 0.56 \cdot (573^4 - 490.41^4)[\mathrm{K}]$
$= 1586.85\ \mathrm{W/m^2}$

$q_{23} = \sigma A_2 \mathcal{F}_{23}\left(T_2^4 - T_3^4\right)$
$= (5.672 \times 10^{-8})[\mathrm{W/m^2 \cdot K^4}] \cdot 0.56 \cdot (790^4 - 298^4)[\mathrm{K}]$
$= 1586.73\ \mathrm{W/m^2}$

$$q_{12} \approx q_{23}$$

*) 만약 2 가 없다면,

$$q_{13}/A = \sigma A_1 \mathcal{F}_{13}\left(T_1^4 - T_3^4\right)$$

$$\mathcal{F}_{13} = \frac{1}{1/\epsilon_2 + 1/\epsilon_3 - 1} = \frac{1}{1/0.56 + 1/1.0 - 1} = 0.39$$

$\frac{q_{13}}{A} = \sigma A_1 \mathcal{F}_{13}\left(T_1^4 - T_3^4\right) =$
$= (5.672 \times 10^{-8})[\mathrm{W/m^2 \cdot K^4}] \cdot 0.39 \cdot (573^4 - 298^4)[\mathrm{K}]$
$= 2210.17\ \mathrm{W/m^2}$