Part IV

ADVANCED ISSUES IN MPC

Contents

Chapter 1

STATE-SPACE MODEL PREDICTIVE CONTROL

1.1 SHORTCOMINGS OF CURRENT INDUSTRIAL MPC PRACTICE

- Truncated Step Response Model:
  - Many model coefficients have to be stored:

    Example) 5 x 5 system with 30 step response coefficients on each gives 750 coefficients.

    The problem is much worse for systems with mixed time scale dynamics (e.g. a high-purity distillation column) where sample time needs to be chosen according to the fast time-scale dynamics, but the settling time is determined by the slow time-scale dynamics.

    This limits the size of application.
  - Unstable systems cannot be handled.
  - Truncation error is unavoidable.

- Disturbance estimation:
  - Step disturbance is assumed and, thus, drift, ramp, oscillatory disturbances cause poor performance.
  - No cross channel update.
  - Unmeasured outputs are not updated.

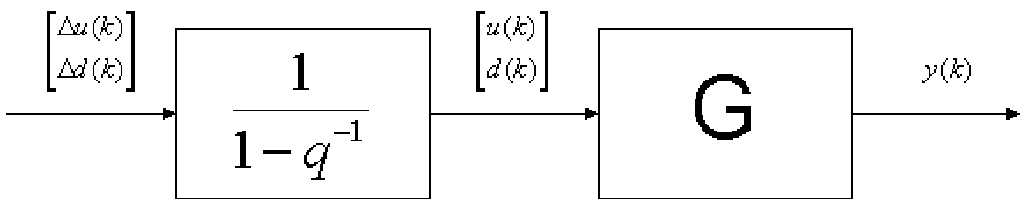
These shortcomings motivate development of

MPC based on a general state-space model.

1.2 STATE SPACE MPC

State Space Plant Model

Consider state space model of the plant obtained from either fundamental ODE's or system identification:



$$\begin{aligned} x(k+1) &= Ax(k) + B_u u(k) + B_d d(k) \\ y(k) &= Cx(k) \end{aligned}$$

$\Downarrow$ differencing

$$\begin{aligned} \Delta x(k+1) &= A\Delta x(k) + B_u \Delta u(k) + B_d \Delta d(k) \\ \Delta y(k) &= C\Delta x(k) \end{aligned}$$

$x(k)$: state
$u(k)$: control input
$y(k)$: measurement output
$d(k)$: measured disturbances

- The number of coefficients is reduced.

  Example) For 5 x 5 system with 10 states, only 200 coefficients need to be stored

- For appropriate choice of A, state space model can represent unstable process.
- No truncation error.

Prediction with State Space Plant Model

If we constrain that $\Delta u(k+m|k) = \cdots = \Delta u(k+p-1|k) = 0$,

$$\begin{bmatrix} \tilde{y}(k+1|k) \\ \tilde{y}(k+2|k) \\ \vdots \\ \tilde{y}(k+p|k) \end{bmatrix} = \begin{bmatrix} \Xi\Phi \\ \Xi\Phi^2 \\ \vdots \\ \Xi\Phi^p \end{bmatrix} X(k|k) + \begin{bmatrix} \Xi\Gamma_d \\ \Xi\Phi\Gamma_d \\ \vdots \\ \Xi\Phi^{p-1}\Gamma_d \end{bmatrix} d(k)$$
$$+ \begin{bmatrix} \Xi\Gamma_u & 0 & \cdots & 0 \\ \Xi\Phi\Gamma_u & \Xi\Gamma_u & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \Xi\Phi^{p-1}\Gamma_u & \Xi\Phi^{p-2}\Gamma_u & \cdots & \Xi\Phi^{p-m}\Gamma_u \end{bmatrix} \Delta\mathcal{U}(k)$$

Rewriting the above,

$$\Downarrow$$

$$\mathcal{Y}(k+1|k) = \mathcal{S}^X X(k|k) + \mathcal{S}^d \Delta d(k) + \mathcal{S}^{\mathcal{U}} \Delta\mathcal{U}(k)$$

1.3 DISTURBANCE ESTIMATION VIA STATE ESTIMATION

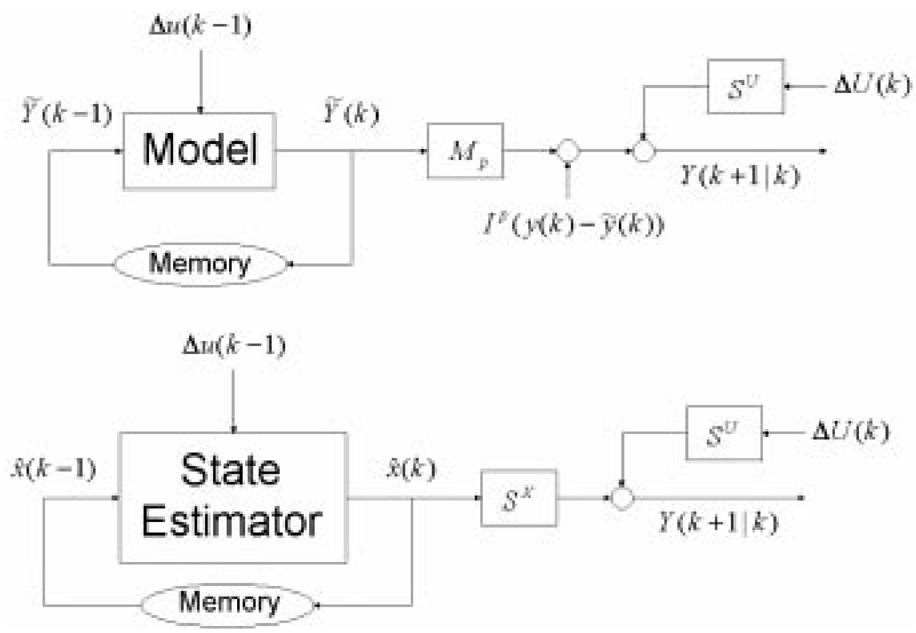
Motivation

In current industrial MPC algorithms,

- models are run open-loop
- feedback is entered into the prediction statically (no memory of the past feedbakc)

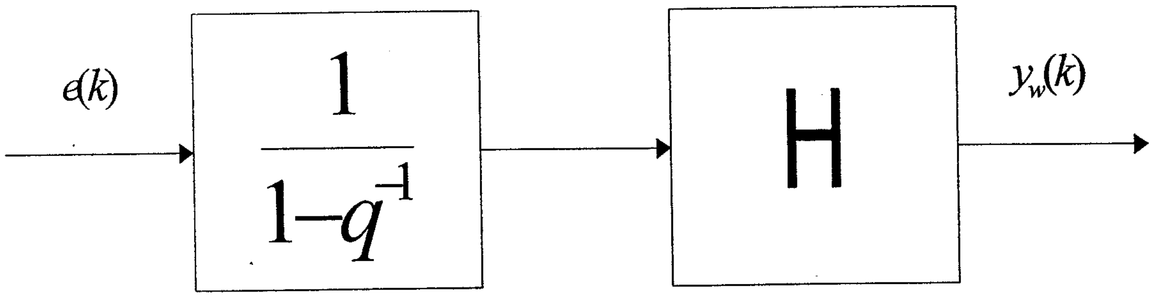
In disturbance estimation via state estimation

- unmeasured disturbance effects are included in the memory (state vector) and update is made directly to the states.
- fuller use of feedback measurement is allowed.



State Space Disturbance Model

State space disturbance model:



$$\begin{aligned}\Delta x_w(k+1) &= A_w \Delta x_w(k) + B_w e(k)\\ \Delta y_w(k) &= C_w \Delta x_w(k) + D_w e(k)\end{aligned}$$